12

R-1066

MOORING MECHANICS
A COMPREHENSIVE COMPUTER STUDY

Volume II

Three Dimensional Dynamic Analysis of Moored and Drifting Buoy Systems

by
Narender K. Chhabra

December 1976

The Charles Stark Draper Laboratory, Inc.
Cambridge, Massachusetts 02139

A gen
buoy s
dimen
imul
esea

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

(14) REPORT NUMBER: R-1066--Vol-2	2. GOVT ACCESSION NO. (9)
3. RECIPIENT'S CATALOG NUMBER: Final Report, Vol. II	
(6) 4. TITLE (and Subtitle): MOORING MECHANICS - A Comprehensive Computer Study - Three Dimensional Dynamic Analysis of Moored and Drifting Buoy Systems.	5. TYPE OF REPORT & PERIOD COVERED: FINAL
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): (10) Narender K. Chhabra	8. CONTRACT OR GRANT NUMBER(s): (15) N00014-75-C-1065
9. PERFORMING ORGANIZATION NAME AND ADDRESS: The Charles Stark Draper Laboratory, Inc. Cambridge, Massachusetts 02139	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: NORDA National Space Technology Laboratories Bay St. Louis, Mississippi 39520	(11) 12. REPORT DATE: December 1976
	13. NUMBER OF PAGES: 287
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
(13) 278p.	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

1. Oceanographic Systems Simulations
2. Mooring Systems
3. Cable Dynamics
4. Buoy Dynamics
5. Drogued Buoys
6. Mathematical Modelling
7. Computer Simulations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Ocean currents and surface waves may induce serious errors in oceanographic measurements obtained from moored and drifting buoy systems. A general, computationally efficient solution for the dynamics of moored buoy systems, free drifting buoys, and drogued buoy systems in three-dimensional space is described in this report. Time-domain computer simulations of four specific configurations in various environments are presented. The mathematical model of one of these configurations, a

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Subsurface mooring, was evaluated and improved using full scale ocean test data. The modal of the surface mooring configuration studied will soon also be evaluated using recent test data.

R-1066

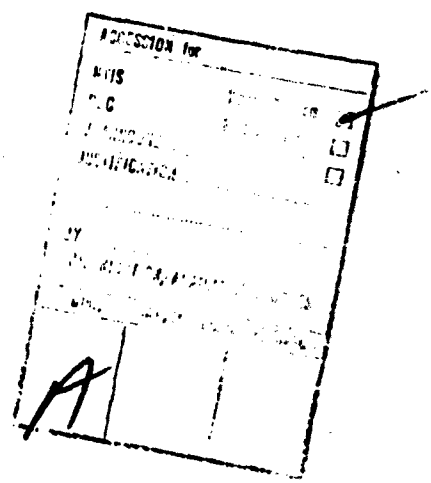
MOORING MECHANICS
A COMPREHENSIVE COMPUTER STUDY

Volume II

Three Dimensional Dynamic Analysis of Moored and Drifting Buoy Systems

by

Narend : K. Chhabra



December 1976

Approved: Philip N. Bowditch

Philip N. Bowditch
Head, Scientific
Research Department

The Charles Stark Draper Laboratory, Inc.
Cambridge, Massachusetts 02139

ACKNOWLEDGMENT

The author welcomes this opportunity to thank all those at the Charles Stark Draper Laboratory, Inc. who had valuable contributions to the work described. Special thanks are due to Mr. John Dahlen for his helpful guidance and comments during preparation of this report. To Dr. Jeffrey Lozow for his help during mathematical model formulations; Mr. James Scholten for his help during computer simulations; and Mr. William Vachon for his comments during the preparation of this report. Finally, the author wishes to thank Miss Cheryl Gibson and Mrs. Catherine Hall for doing such an excellent job in typing this report.

This report was prepared under CSDL Project 53-68800, sponsored by the Ocean Science and Technology Division of the Office of Naval Research, Department of the Navy, through contract N00014-75-C-1065.

ABSTRACT

Ocean currents and surface waves may induce serious errors in oceanographic measurements obtained from moored and drifting buoy systems. A general, computationally efficient solution for the dynamics of moored buoy systems, free drifting buoys, and drogued buoy systems in three-dimensional space is described in this report. Time-domain computer simulations of four specific configurations in various environments are presented. The mathematical model of one of these configurations, a subsurface mooring, was evaluated and improved using full scale ocean test data. The model of the surface mooring configuration studied will soon also be evaluated using recent test data.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

NOMENCLATURE

$\bar{A}$	Acceleration vector.
A_D	Area used in drag calculations.
$\bar{A}_{Ec}$	Acceleration of point c in the E frame.
A_N, A_T	Normal and Tangential areas for a cylindrical body.
$\bar{A}_R$	Relative acceleration vector.
a	Point of attachment between the surface buoy and the mooring line.
$\overline{AM}$	Added mass force vector.
c,x',y',z'	Body coordinate system, Frame B.
c	Mid point of the mooring line element.
{CF}	Array of constant nodal forces.
C_D	Drag coefficient.
C_N, C_T	Added mass coefficients, normal and tangential directions, surface buoy.
C_{DN}, C_{DT}	Normal and tangential drag coefficients for a cylindrical body.
C_{DP}	Pressure drag coefficient for the end plate of a spar buoy.
CDIN,CDIT, CDIA	Appropriate drag constants corresponding to $\overline{DN}$, $\overline{DT}$, and $\overline{DA}$.
$\overline{DA}$	Viscous drag force on a body inserted in a mooring line.
$\overline{DF}$	Viscous drag force vector.
$\overline{DM}$	Moment vector due to viscous drag force.
$\overline{DN}, \overline{DT}$	Normal and tangential drag forces per unit stretched length of the mooring line.

$\overline{DN}_n, \overline{DT}_n, \overline{DA}_n$	Viscous drag forces acting on the n^{th} node of the mooring line.
dB	Mass of water displaced by a differential disk of the surface buoy, $\rho S_o \, dz'$.
dM	Mass of a differential disk of the surface buoy.
$\overline{F}$	Total force vector acting on the body.
$\{F_A\}$	Array of additional forces and moments.
$\{F_D\}$	Array of wave exciting forces and moments.
$\overline{FF}$	Froude Krylov exciting force vector.
$\{F_G\}$	Array of hydrostatic and gravitational forces and moments.
$\overline{FH}$	Hydrostatic pressure force acting on the mooring line, per unit stretched length.
g	Acceleration due to gravity.
H	Length of the surface buoy.
h_o	Instantaneous draft of the surface buoy, measured up to the mean free surface.
h_1	Spar buoy drum height.
h_2	Spar buoy mast height.
h_i	h_o plus the wave elevation component.
$\overline{IF}$	Inertia force per unit stretched length of the mooring line.
$\overline{IF}_b$	Inertia force vector of the inserted body.
I_{xx}, I_{yy}	Moments of inertia of the spar buoy in roll and pitch about c.
$\hat{i}_x, \hat{i}_y, \hat{i}_z$	Unit vectors of o,x,y,z.
$\hat{i}_{x'}, \hat{i}_{y'}, \hat{i}_{z'}$	Unit vectors of c,x',y',z'.

$\bar{K}$	Wave number vector.
K_x	Wave number component, $\lvert\bar{K}\rvert \cos \beta$.
K_y	Wave number component, $\lvert\bar{K}\rvert \sin \beta$.
k_n	Stiffness coefficient of the n^{th} segment.
ℓ, u_1, u_2, u_3	Integration limits as defined.
M	Total mass of the spar buoy.
[M]	Matrix of tensors of inertia and added inertia.
m_a	Added mass of the inserted body, shape other than cylindrical.
m	Mass of the inserted body.
m_{dn}	Mass of water displaced by the n^{th} mass.
m_n, m_t	Added mass components (normal and tangential) due to cylindrical bodies.
N	Number of nodes or segments of a mooring line.
$\overline{NB}$	Net bouyancy of the inserted body.
n	Arbitrary line segment, or lumped mass.
o,x,y,z	Fixed coordinate system, frame E.
p	Hydrostatic pressure of the fluid.
P_i	$= \int_{\ell}^{u_2} dB(z'_p)^i$
p	Location of the centroid of the differential disk of a surface buoy.
Q_i	$= \int_{\ell}^{u_2} dBe^{Kz_w}(z'_p)^i$
$\bar{R}$	Displacement vector.
$\bar{R}_{oc}$	Displacement vector from point o to point c.

r	Radius of the spar buoy or the reduced radius of the mooring line.
r_1, r_2	Radii of the tuned spar buoy.
$\bar{S}$	Spatial vector in the x-y (horizontal) plane, describing propagation of the surface wave.
S_o	Cross-sectional area of the surface buoy.
S_{o1}, S_{o2}	Cross-sectional areas of the tuned spar buoy.
s	spatial coordinate of the mooring line.
$\bar{T}$	The tension vector.
Te	Effective tension variable along the mooring line.
$\overline{TM}$	Moment of the tension force $\bar{T}$.
$\overline{TF}$	Tension force vector on a mooring line node.
t	Time variable.
$\bar{U}$	Fluid velocity vector.
$\overline{UR}$	Relative fluid velocity vector.
$\overline{UNR}, \overline{UTR}$	Normal and tangential components of $\overline{UR}$.
$\bar{V}$	Velocity vector.
$\hat{\ }$	Unit vector along the mooring line.
$\{V\}$	Array of translation and rotation rates.
V_{Bw}	Velocity of water particle w in frame B.
V_b	Tangential relative fluid velocity at bottom of the tuned spar.
V_{ox}, V_{oy}, V_{oz}	Components of surface current.
V_s	Tangential relative fluid velocity at step of the tuned spar, also static velocity of the drogued drifting buoy system.

$\bar{W}$	Gravitational [illegible] element of [illegible] angular velocity [illegible]
W_a	Weight in air per [illegible] at the mooring line.
W_w	Weight in water, [illegible] per unit [illegible] line.
$\bar{W}^n, \bar{W}^t$	Normal and tangential [illegible]
w	Location of the particle of water.
$\bar{Y}$	Displacement vector of the mooring line.
$\ddot{\bar{Y}}^n, \ddot{\bar{Y}}^t$	Normal and tangential [illegible] of the mooring line element.
z_c	Distance between [illegible] buoy.
ϕ	Velocity potential function.
ω	Wave frequency.
β	Angle between wave direction and the x-axis.
ζ_o	Wave amplitude.
ζ	Wave surface elevation.
ϕ	Roll angle of the surface buoy [illegible] for mooring line inclination.
θ	Pitch angle of the surface buoy.
ψ	Yaw angle of the surface buoy.
ρ	Water density.
ΔL_n	Change in length [illegible]
α	Added mass coefficient for the [illegible] the spar buoy.

1.0 INTRODUCTION

Oceanographic measurements from moored buoy systems are contaminated by mooring motion. Numerous articles have been published in the last three years pointing out quantitatively the errors introduced by the surface wave field. Large vertical excursions have been experienced at great depths by instruments located on the synthetic rope portions of surface-buoyed mooring lines (WUNSCH and DAHLEN, 1974). Observations during the POLYMODE experiment showed subsurface mooring lines to have undergone hundreds of meters of vertical excursions. For any operational moored instrument system we need two models whose accuracy can be established with known confidence: a mooring system model from which the motion environment of instruments can be predicted, and a model of the instrument motion response characteristics from which the measurement error can be estimated. Possession of these models is required for more effective mooring systems design and for optimum interpretation of oceanographic measurements obtained from moored systems. The first of these two models is the main subject of this report.

An alternative to the moored approach is the use of surface-trackable drogued drifting buoys. Such a

buoy system employs a high drag device (or drogue) at some depth, tethered to a trackable buoy at the surface. The major impediments in the widespread use of this approach have been inadequate component and system design. Both of these impediments can be removed by the development of dynamic modeling of drogued buoys. A part of this report deals with the dynamic modeling of drogued buoys.

Dynamic models of drogued buoys do not exist except in primitive form, while such models of moored buoys and free-drifting buoys are further along. This difference stems from the fact that drogued buoy systems are dynamically very complex, and they have only recently been considered essential to major programs. Even though many dynamic mathematical models for moored buoys and free-drifting buoys are available, not much has been done toward the evaluation of these models using full-scale ocean test data. It is the purpose of this report to present a general, computationally efficient approach for analyzing moored buoy systems, free drifting buoys, and drogued buoy systems and to simulate this analysis on the computer so that these models can then be readily evaluated with full scale ocean test data.

This report constitutes parts 3 and 4 of a four part report. Parts 1 and 2 were published in

volume 1 and dealt with the "Three Dimensional Static Analysis and Design of Single Point Taut and Slack Moored Buoy Systems" (CHHABRA, 1973).

1.1 Background

We at Charles Stark Draper Laboratory Inc. have evaluated one of the mathematical models presented in this report. The mathematical model of a subsurface mooring system (Section 2.2.1 and simulation 5.2 in this report) has been shown previously to predict the mooring motion forced by ocean currents of periods greater than 15 min. (CHHABRA, DAHLEN and FROIDEVAUX, 1974). The model was evaluated in a full scale ocean test that provided experimental data on mooring response and ocean current forces. The ocean test was conducted jointly with Woods Hole Oceanographic Institution on R. V. Chain cruise 107. We obtained a record of the motion of an acoustic transceiver near the top of the subsurface mooring line at the 500-m depth. The transceiver sent out a sound pulse every minute and recorded the four return times of replies from four near-bottom acoustic transponders at 5460m. By comparison with this and other data from precision pressure recorders, tensiometers, and inclinometers, the mooring model had been found to predict well the observed motions. The r.m.s. difference

between the experimental and predicted trajectory of the acoustic transceiver was 11.8m, about 10% of the mean excursion. This mathematical model was evaluated a second time by data from the central mooring (Mooring No. 1, Station 481) of the Mid-Ocean Dynamics Experiment (MODE). In that study (CHHABRA, 1976), the current record from the topmost vector-averaging current meter on the central mooring of the MODE experiment was corrected for the effects of mooring motion, and power spectra of the uncorrected and corrected signals were compared. The correction was < 1 cm s^{-1}, for that mooring line (vertical excursion < 12 m). Creep in the synthetic portion of the mooring line was also identified.

In addition, we are planning to evaluate a second mathematical model presented in this report. The mathematical model is of a tuned spar buoy (35 ft. long, 1 ft. dia. at W.L.) tethered to a subsurface mooring line by a stiff buoyant line. An instrument line is hanging from the base of the spar (Sections 2.1.1, 2.2.2, 2.3 and simulation 5.3). This configuration was recently tested in the ocean during the October 1976 ONR/NDBO Mooring Dynamics Experiment. A total of fifteen motion sensing instruments (4 Force Vector Recorders, 6 Temperature/Pressure Recorders, 1 POPMIP, and 4 Acoustic

Beacons) were attached along this mooring system. The evaluation and improvement of this mathematical model would be done by comparing measured responses with those computed by the computer simulation for the measured/observed environment.

The above mentioned evaluation and improvement is planned to be completed in CY77. If such an evaluation and improvement is done, the only mathematical model to be evaluated and improved from the analysis presented in this report would be the drifting drogued buoys. We plan to do that task in the near future.

1.2 Assumptions, Capabilities, and Limitations

Surface gravity waves treated in this report have a single frequency and amplitude, propagating in a single direction. It is assumed that the surface buoy is oscillating in the path of small (i.e. amplitude of wave train much less than its wavelength) incident surface waves which are long in relation to the body dimension in the direction of wave propagation. Wave direction has been generalized to include any arbitrary direction in the horizontal plane. For the three-dimensional analysis of surface buoys, we also assume small displacements of buoy when compared to vertical dimension of the buoy. Hence only the first powers of

small quantities are retained for the three-dimensional surface buoy analysis. Moments of inertia about all axes in a horizontal plane are taken equal. Even though moment of inertia about the longitudinal axis of the spar buoy is negligible, it is included for numerical computational purposes.

In all mathematical models an earth fixed frame is assumed to be the valid inertial frame. Viscous drag forces are computed based on the square drag law. For cylindrical shapes these forces are assumed to act in directions normal and tangential to the longitudinal axis; and for any other shape they act in the direction of the relative flow. For the case of a tuned spar viscous drag forces and 'added mass' due to the bottom base and the step are also included. The ideal fluid damping (wave-damping) as derived by Newman (1963) for a spar buoy was found to be negligible and hence is neglected.

All mooring lines are considered elastic. In the continuous line formulation of mooring lines, nonlinear elasticity dependent on the prior loading history as explained in volume 1 (CHHABRA, 1973) is considered. For the lumped parameter formulation; a linear stress-strain curve is derived from the nonlinear curves for the range of stresses in study. Dynamic effects on these stress-

strain curves are neglected. Internal damping forces in the mooring lines are assumed negligible when compared with viscous drag and stiffness forces. The hydrostatic pressure forces are treated more rigorously than they are in the traditional method employed in most previous studies. As it turned out this new method changed only the stress distribution along the mooring system, while the configuration of the mooring system remained essentially unchanged. Instruments or buoyancy packages attached along the mooring lines are treated as concentrated forces which have length and give rise to forces as explained in Section 2.2. A time varying current profile of any nature and shape can be inputted to the computer programs. A list of current profiles used in various simulations is given in volume 1 (CHHABRA, 1973). In its present form no allowance for shrinkage or creep of the synthetic ropes is taken into account, but creep was identified in CHHABRA (1976). Elongations due to rotation of non-torque balanced cables is also not considered. In the mooring line models viscous drag forces due to the velocity field generated by the surface waves are included. Wave-damping forces are neglected. In the continuous line formulation of the mooring line, exciting forces exerted on the line by the wave system are neglected; whereas in the lumped

parameter formulation, exciting forces are taken equal to the 'Froude-Krylov' forces as it is assumed that the presence of the mooring line does not disturb the wave particle motion. For added mass purposes, it is assumed for the continuous line formulation that the body (continuous cylindrical line) motion accelerates fluid only in the direction normal to its longitudinal axis.

2.0 THEORETICAL ANALYSIS - MATHEMATICAL MODELS

This section derives all the equations of motion pertinent to the analysis presented in this report. Surface floats are analyzed in Subsection 2.1, mooring line in 2.2, their attachment in Section 2.3, and a window shade drogue in Section 2.4. The mooring line analysis includes all subsurface floats, instruments etc. attached to the mooring line.

2.1 Surface Buoys

As is well known, the analysis of the wave induced response of floating bodies is, in general, a most formidable task. Initially the action of the fluid must be decomposed into real (viscous) and ideal (inviscid) effects. Each effect then must be modeled as to its interaction with the floating body. Further, it is convenient to assume that the ocean waves are "gentle" enough so as to permit first order linear surface wave theory to serve as a foundation for the calculation of the wave exciting forces.

The ideal fluid problem is still so difficult in general that only the simplest body shapes are amenable to rigorous analytical solution (potential flow theory). This solution involves the determination of the (velocity) potential function $\Phi(\bar{S},t)$ which must

satisfy: 1) Laplace's equation, 2) the kinematic boundary condition on the moving body surface, 3) the free surface (and bottom) boundary condition(s) and 4) the radiation condition at great distance from the body.

Assuming this potential, $\Phi(\bar{S},t)$, has been determined by some means, the next step is to substitute it into the Bernoulli's expression for fluid pressure which in turn is integrated over the immersed portion of the body to yield the instantaneous force and moment vectors. In principle this is a straight-forward procedure. In practice it is hardly ever possible to find a tractable potential function which satisfies the above four conditions. In a few notable cases, however, for simple geometries and small body motions, rigorous solutions have been worked out. In particular Newman (1963) has derived the linearized equations of motion for a vertical, cylindrical spar buoy responding to the influence of a unidirectional wave train. The axisymmetry of the buoy as well as its postulated slenderness was greatly exploited in the work to yield manageable results. In Newman's work as well as in others of comparable rigor, the computed potential is the result of an intricate distribution of singularities (sources, sinks, dipoles, etc.) within or over the wetted body surface.

From these studies, it turns out that the linearized

hydrodynamic forces and moments in general body geometries may be decomposed into constituents proportional to body acceleration, body velocity, displaced fluid acceleration, and the displaced fluid velocity. In general, rigorous integration of the fluid pressure, both hydrostatic and wave, over the body surface yields: 1) the 'Froude-Krylov" force from the pressure distribution due to the undisturbed wave system; 2) the "diffraction" force from disturbance of the waves by the presence of the body; 3) the force due to the motion of the body; and 4) the hydrostatic pressure force. The Froude-Krylov force and the diffraction force in combination are also known as the exciting forces exerted on the body by the wave system. The Froude-Krylov force equals in value to the product of the mass of the displaced fluid times the acceleration of the local undisturbed fluid particles. The diffraction force and the force due to the motion of the body both yield "added mass" coefficients proportional to accelerations and wave-damping coefficients proportional to velocities of the fluid and the body respectively. As shown in Chung (1976), the added mass coefficients are same (opposite signs) for both the fluid and body accelerations. Also the two wave-damping coefficients have the same (opposite signs) value.

Hence, these two forces may be combined to represent an "added mass" force proportional to the relative acceleration (fluid acceleration minus the body acceleration), and a wave-damping force proportional to the relative velocity. Other forces (apart from the fluid pressure forces) acting on the body are: 1) the weight of the body; and 2) the actual unbalanced force which accelerates the body. In the real (viscous) fluid another force, the viscous drag force, acts on the submerged portion of the body.

Analysis in this section includes all the above mentioned forces, as derived from the potential flow theory, except the following deviations: 1) the ideal fluid damping (wave-damping) is omitted; 2) the added mass force is treated slightly differently, and 3) the viscous drag forces are added on to the general equations of motion.

The ideal fluid damping is omitted here for expediency. Later analysis of specific problem addressed in this report showed the wave damping force, as derived by Newman (1963) for a spar buoy, to be negligible compared to the viscous drag force on the submerged body. The added mass force, which is proportional to the relative acceleration is separated into two terms; an added mass force proportional to the longitudinal component

of the relative acceleration, and an added mass force proportional to the transverse component of the relative acceleration. Each of these two terms is multiplied by a different constant coefficient, depending on the shape of the floating body. These coefficients and the coefficients used to derive the viscous drag force are experimentally determined hydrodynamic coefficients.

As will be shown below, our general equations of motion reduce to the ones given in Rudnick (1967) for a particular value of these coefficients. Rudnick in his analysis used the reasoning of Lamb (1945) who treats a uniform two-dimensional flow across a long circular cylinder.

Formulation of the General Equations of Motion: Derivation of the equations of motion for surface piercing buoys in a train of regular harmonic ocean waves is given. In addition a surface current (not due to surface waves) is present. Surface buoys are considered as rigid bodies with six degrees of freedom. The problem under consideration is represented schematically in Figure 2.1. An earth fixed cartesian coordinate system (z positive upwards) is situated at the undisturbed level of the free surface. Call it the frame E with origin at o;

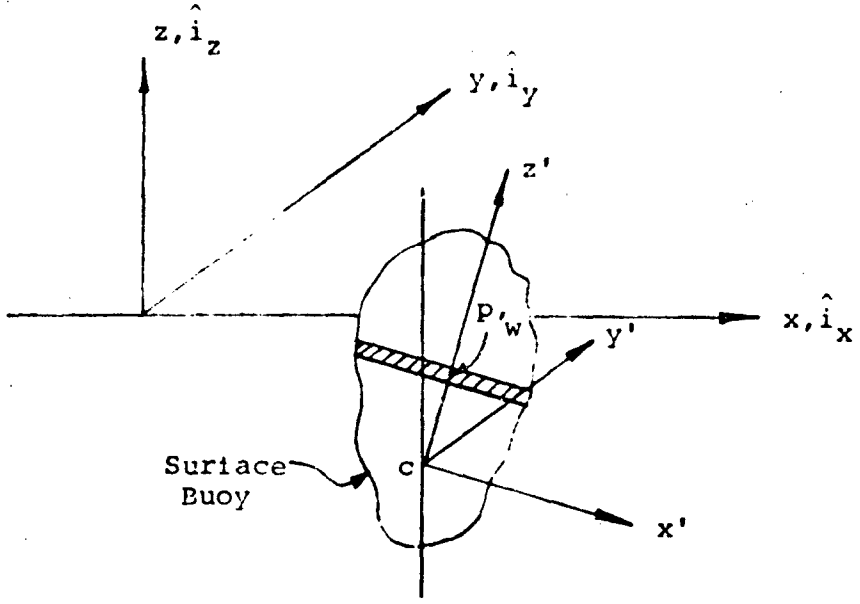


Figure 2.1 Coordinate System for Surface Piercing Buoys

and $\hat{i}_x, \hat{i}_y$, and $\hat{i}_z$ the unit vectors along x,y, and z respectively. Frame B is a body (surface buoy) fixed cartesian coordinate system with its origin at the centroid (c) of the surface buoy, and x',y',z' being parallel to x,y,z respectively when the buoy is in the upright and non-rotating position. In this analysis $\bar{R}$ represents a displacement vector; $\bar{V}$, a velocity vector, and $\bar{A}$, an acceleration vector. It is assumed that the surface buoy is oscillating in the path of small (i.e., amplitude of wave train much less than its wavelength) incident surface waves which are long in relation to the body dimensions in the direction of wave propagation. Point

p is the location of the centroid of a differential disk (height = dz') of the surface buoy (Figure 2.1).

First, the wave direction will be generalized to include any arbitrary direction in the x-y plane. To this end let the wavenumber vector $\bar{K}$ specify the direction of propagation (Figure 2.2) where:

$$\bar{K} = \frac{\omega^2}{g} (\cos\beta\ \hat{i}_x + \sin\beta\ \hat{i}_y)$$

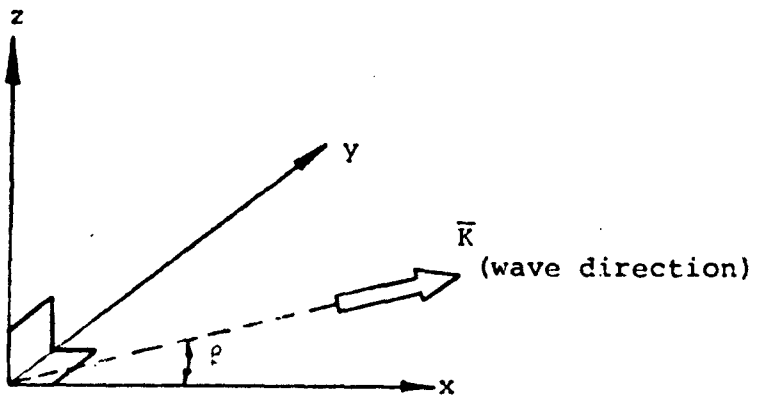


Figure 2.2 - Generalized Wave Direction

The generalized velocity potential field of the incident wave system may now be written:

$$\phi = \frac{g\xi_o}{\omega} e^{|\bar{K}|z} \cos(\bar{K}.\bar{S} - \omega t)$$

where; $\bar{S} = \hat{i}_x x + \hat{i}_y y$; and the wave surface elevation ξ is

given by:

$$\xi = - \xi_o \sin(\bar{K}.\bar{\Sigma} - \omega t)$$

Here, g is the acceleration due to gravity, and ξ_o is the amplitude of the incident wave of frequency ω. In the earth fixed frame (frame E), assumed to be a valid inertial frame for this problem, we define the fluid particle acceleration vector:

$$\bar{A}_{Ew} = \hat{i}_x \dot{\phi}_x + \hat{i}_y \dot{\phi}_y + \hat{i}_z \dot{\phi}_z$$

where w is the particle of water next to the differential disk of centroid p. It is assmed that A_{Ew} is constant over the entire differential disk. Components of A_{Ew} at point p are given by:

$$\dot{\phi}_x = \frac{d}{dt} \frac{\partial \phi}{\partial x} = g K_x \xi_o \, e^{|\bar{K}| z_w} \cos(\bar{K}.\bar{\Sigma} - \omega t)$$

$$\dot{\phi}_y = \frac{d}{dt} \frac{\partial \phi}{\partial y} = g K_y \xi_o \, e^{|\bar{K}| z_w} \cos(\bar{K}.\bar{\Sigma} - \omega t)$$

and $$\dot{\phi}_z = \frac{d}{dt} \frac{\partial \phi}{\partial z} = g |\bar{K}| \xi_o \, e^{|\bar{K}| z_w} \sin(\bar{K}.\bar{\Sigma} - \omega t)$$

where; $K_x = |\bar{K}| \cos\beta$; $K_y = |\bar{K}| \sin\beta$, and the higher order terms in $\dot{\phi}_z$ have been neglected.

Now; $\bar{R}_{op} = \bar{R}_{oc} + \bar{R}_{cp}$

$$\therefore \quad \bar{V}_{Ep} = [\dot{\bar{R}}_{op}]_E = [\dot{\bar{R}}_{oc}]_E + \bar{W}_{EB} \times \bar{R}_{cp}$$

and $$\bar{A}_{Ep} = [\ddot{\bar{R}}_{oc}]_E + [\dot{\bar{W}}_{EB}]_E \times \bar{R}_{cp} + \bar{W}_{EB} \times (\bar{W}_{EB} \times \bar{R}_{cp})$$

where; $\bar{W}_{EB}$ is the angular velocity of the buoy.

Let us also define the relative acceleration vector of the fluid particles with respect to the buoy ($\bar{A}_R$), at the location of point p as:

$$\bar{A}_R \equiv \bar{A}_{Ew} - \bar{A}_{Ep}$$

also let, $\bar{A}_{RT} = (\bar{A}_R \cdot \hat{i}_z,)\hat{i}_z,$

and $$\bar{A}_{RN} = \bar{A}_R - \bar{A}_{RT}$$

The vector force equations of motion may now be written as:

$$\int_{\ell}^{u_1} dM\ \bar{A}_{Ep} = \int_{\ell}^{u_2} dB\ \bar{A}_{Ew} + C_N \int_{\ell}^{u_2} dB\ \bar{A}_{RN} + C_T \int_{\ell}^{u_2} dB\ \bar{A}_{RT} - \int_{\ell}^{u_3} dB\ \bar{g} + \int_{\ell}^{u_1} dM\ \bar{g} \qquad (2.1)$$

where the viscous drag forces and the tension forces (due to the attached mooring lines) are left out for later introduction.

Here; $\ell = -Z_c$, $u_1 = H - Z_c$, $u_2 = h_o - Z_c$, and $u_3 = h_i - Z_c$. Z_c is the distance between c and the bottom of the buoy. H is the length of the buoy, h_o is the instantaneous draft of the buoy measured up to the mean free surface, and h_i is h_o plus the wave elevation component. Distances Z_c, H, h_o and h_i are measured along $\hat{i}_{z'}$. dM is the mass of the differential disk (Figure 2.1) of height dz' and dB is the mass of water displaced by this disk. C_N AND C_T are the appropriate hydrodynamic constants.

In equation (2.1) the integrals in order of their appearance represent: 1) the actual unbalanced force which accelerates the buoy; 2) the Froude-Krylov force; 3) the normal component of the added mass force; 4) the tangential component of the added mass force; 5) the hydrostatic pressure force; and 6) the weight of the buoy. This equation reduces to the equation in Rudnick (1967), which was derived for a spar buoy, for $C_N = 1.0$, and $C_T = 0.0$.

The vector moment equation can now be written similarly as: (moments about C.G.)

$$\int dM(\bar{R}_{cp} \times \bar{A}_{Ep}) = \int dB(\bar{R}_{cp} \times \bar{A}_{Ew}) +$$

$$+ C_N \int dB(\bar{R}_{cp} \times \bar{A}_{RN}) +$$

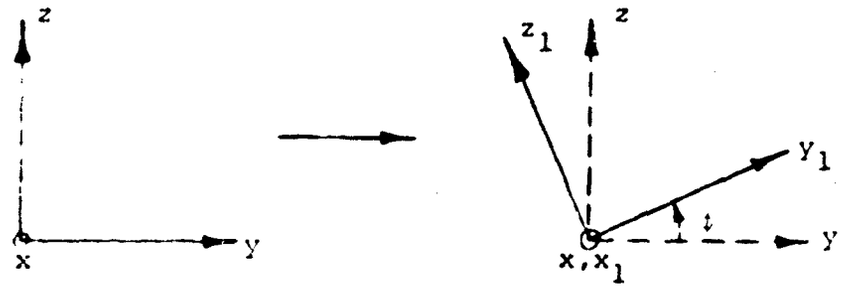
$$+ C_T \int dB(\bar{R}_{cp} \times \bar{A}_{RT}) - \qquad (2.2)$$

$$- \int dB(\bar{R}_{cp} \times \bar{g}) + \int dM(\bar{R}_{cp} \times \bar{g})$$

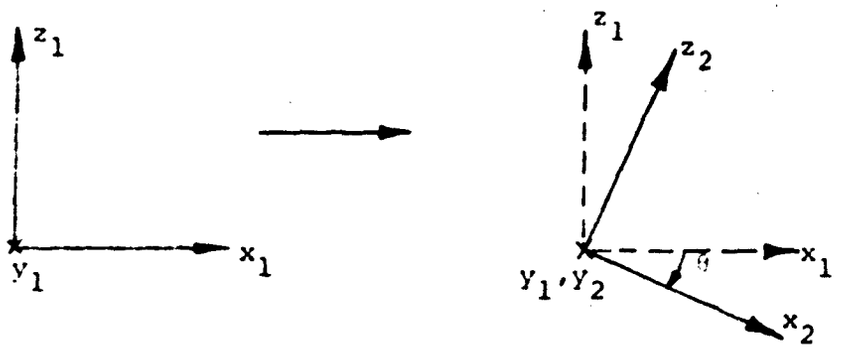
It is also necessary to determine an appropriate transformation scheme to relate vectors in the E frame to vectors in the B frame. A sequence of rotations (Figure 2.3) about the body x',y',z' axes (roll = ϕ, pitch = θ, and yaw = ψ) respectively yield the following transformation.

$$\begin{Bmatrix} \hat{i}_{x'} \\ \hat{i}_{y'} \\ \hat{i}_{z'} \end{Bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi - \cos\psi\sin\theta\cos\phi \\ -\sin\psi\cos\theta & \cos\psi\cos\phi - \sin\psi\sin\theta\sin\phi & \cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ \sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{Bmatrix} \hat{i}_x \\ \hat{i}_y \\ \hat{i}_z \end{Bmatrix}$$

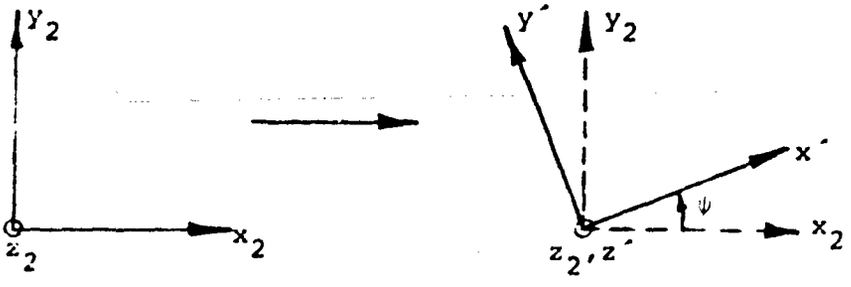
The inverse of this transformation matrix is given by its transpose. The force and moment equations (2.1) and (2.2) may be combined and written in matrix form as:



Step 1 - Roll (ϕ): Rotate y,z about x to obtain x_1, y_1, z_1



Step 2 - Pitch (θ): Rotate x_1, z_1 about y_1 to obtain x_2, y_2, z_2



Step 3 - Yaw (ψ): Rotate x_2, y_2 about z_2 to obtain x', y', z'

Figure 2.3 Transformation Matrix Rotations

$$[M] \cdot \frac{d}{dt} \{V\} = \{F_D\} + \{F_G\} + \{F_A\} \qquad (2.3)$$

Here the matrix [M] is composed of the tensors of inertia and added inertia. The {V} column vector has three translation rates and three angular rates. The vector $\{F_D\}$ contains the wave exciting forces and moments. The vector $\{F_G\}$ comprises the hydrostatic pressure and weight restoring forces and moments (i.e. gravitational and buoyancy effects); and $\{F_A\}$ is any additional forces and moments including viscous drag forces and moments introduced next and tension forces/moments from attached mooring lines introduced in Section 2.3.

Viscous drag forces and moments. A square drag law, where viscous drag forces are proportional to the square of the relative fluid veloicty is used. In Figure 2.1

$$\bar{R}_{ow} = \bar{R}_{oc} + \bar{R}_{cp} + \bar{R}_{pw}$$

$$\therefore \bar{V}_{Ew} = \bar{V}_{Ec} + \bar{W}_{EB} \times \bar{R}_{cp} + [\dot{\bar{R}}_{pw}]_E$$

If point p and w are overlapping then:

$$[\dot{\bar{R}}_{pw}]_E = [\dot{\bar{R}}_{pw}]_B = \bar{V}_{Bw} = \bar{V}_{Ew} - \bar{V}_{Ec} - \bar{W}_{EB} \times \bar{R}_{cp}$$

Here; $\bar{V}_{Ew} = \hat{i}_x (V_{ox} + \phi_x) + \hat{i}_y (V_{oy} + \phi_y) + \hat{i}_z (V_{oz} + \phi_z)$ and $\bar{V}_{Bw}$ is the relative velocity of water seen by the buoy at the local point p. Hence the viscous drag force on the submerged portion of the buoy is given by:

$$\overline{DF} = \frac{1}{2} \rho C_D \int_{\ell}^{u_2} dA_D |\bar{V}_{Bw}| \bar{V}_{Bw} \qquad (2.4)$$

where, ρ is the water density, C_D the drag coefficient, and dA_D the appropriate differential area. The viscous drag moment can similarly be written as:

$$\overline{DM} = \frac{1}{2} \rho C_D \int_{\ell}^{u_2} dA_D |\bar{V}_{Bw}| (\bar{R}_{cp} \times \bar{V}_{Bw}) \qquad (2.5)$$

Limits of integration for both (2.4) and (2.5) are the entire submerged depth of the buoy.

In the remainder of Section 2.1 the equations (2.1), (2.2), (2.4), and (2.5) presented above will be specialized to the specific problems at hand. The matrices of equation (2.3) will then be derived and presented for these specific problems.

2.1.1 Spar Buoy

A tuned spar buoy (Figure 2.4) will be analyzed in this section to obtain general spar buoy equations.

Note: Shown for zero rotation and translation.

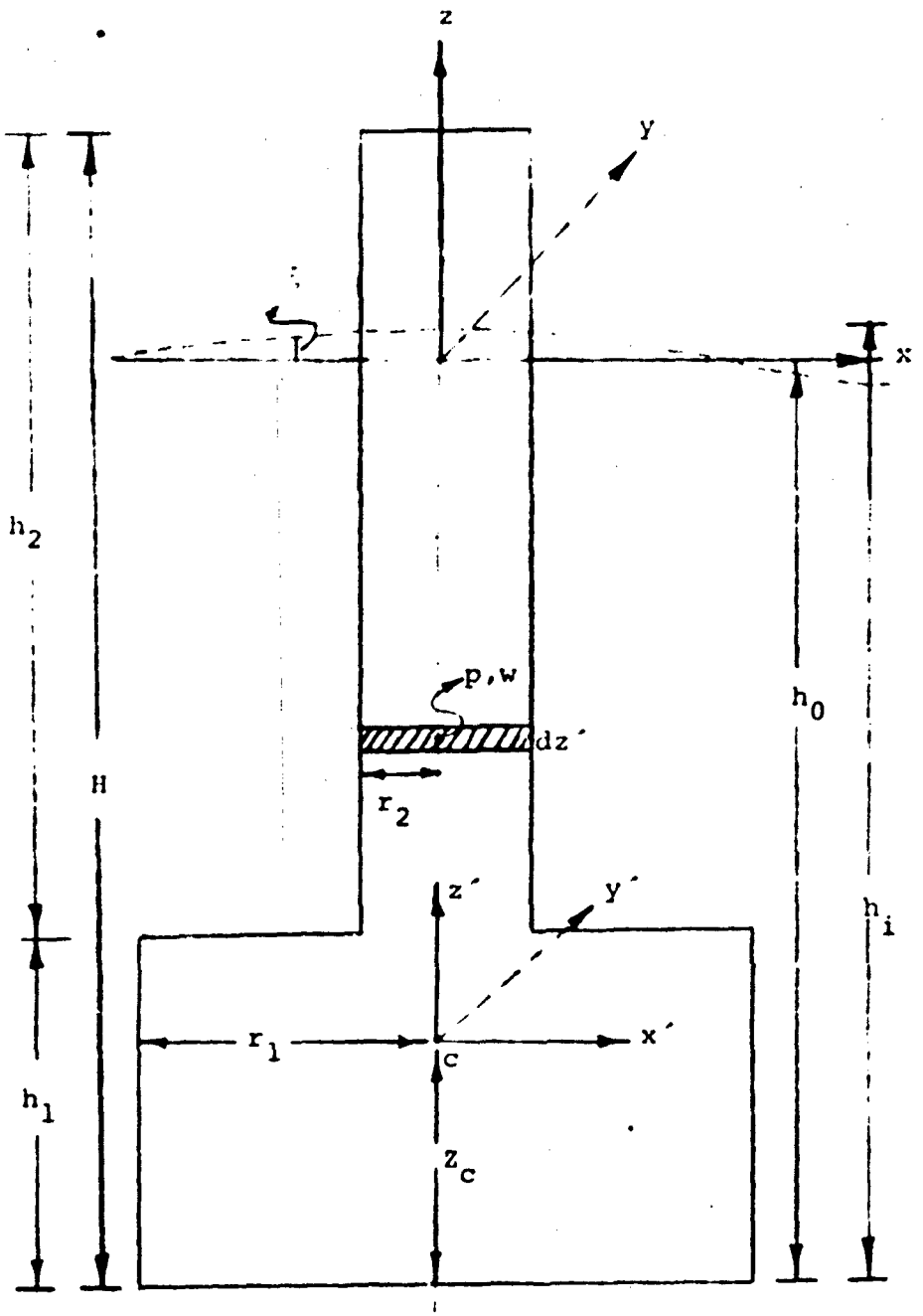


Figure 2.4 - A Tuned Spar Buoy

cylindrical spar buoy equations can then be obtained as a special case of the tuned spar buoy. Both two-dimensional and three-dimensional analysis will be presented. We will find the equations of motion of its response for a train of surface gravity waves having a single frequency and amplitude, propagating in any single direction. Variation in wave acceleration and velocity over the horizontal buoy dimensions are neglected. In addition in the three-dimensional analysis, all rotational angles (roll, pitch, and yaw) are restricted to be small.

Two-Dimensional Analysis: In this analysis $\bar{K}$ is a scaler with $\gamma = 0$; and $\bar{S}$ is replaced by the scaler x which is the direction of propagation. Also, $\bar{A}_{Ew} = i_x \ddot{x} + i_z \ddot{z}$.

$$\bar{A}_{RN} = (\bar{A}_R \cdot \hat{i}_{x'})\hat{i}_{x'}$$

and the appropriate transformation with $\zeta = 0$, and $\psi = 0$ becomes

$$\begin{Bmatrix} \hat{i}_{x'} \\ \hat{i}_{z'} \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \hat{i}_x \\ \hat{i}_z \end{Bmatrix}$$

Now let, $[\ddot{\bar{R}}_{oc}]_E = \hat{i}_x \ddot{x}_c + \hat{i}_z \ddot{z}_c$

$$\bar{W}_{EB} = \hat{i}_y \dot{\theta}$$

and $$\bar{R}_{cp} = z_p' \hat{i}_{z'}$$

$$\therefore \bar{A}_{Ep} = \hat{i}_x [\ddot{x}_c + z_p' (\ddot{\theta} \cos\theta - \dot{\theta}^2 \sin\theta)]$$

$$+ \hat{i}_z [\ddot{z}_c + z_p' (-\ddot{\theta} \sin\theta - \dot{\theta}^2 \cos\theta)]$$

Also, $\bar{A}_{Ew} = \zeta_o^2 e^{Kz_w} [\hat{i}_x \cos(Kx_c - \omega t) + \hat{i}_z \sin(Kx_c - \omega t)]$

Define; $P_i = \int_{\ell}^{u_2} dB (z_p')^i \qquad i = 0,1,2$

and $Q_i = \int_{\ell}^{u_2} dB\, e^{Kz_w} (z_p')^i \qquad i = 0,1$

Integrals of equation (2.1) can now be written as:

1. $\int dM\, \bar{A}_{Ep} = M[\hat{i}_x \ddot{x}_c + \hat{i}_z \ddot{z}_c]$

   Where M is the total mass of the buoy.

2. $\int dB\, A_{Ew} = \omega^2 \zeta_o Q_o [\hat{i}_x \cos(Kx_c - \omega t) + \hat{i}_z \sin(Kx_c - \omega t)]$

3. $C_N \int dB\ \bar{A}_{RN} = C_N\ \hat{i}_x\{\omega^2\xi_o Q_o[\cos(Kx_c - \omega t)\ \cos^2\theta -$

$- \sin(Kx_c - \omega t)\ \sin\theta\ \cos\theta] +$

$+ P_o(\ddot{z}_c\ \sin\theta\ \cos\theta - \ddot{x}_c\ \cos^2\theta) - P_1\ \cos\theta\ddot{\theta}\} +$

$+ C_N\ \hat{i}_z\{\omega^2\xi_o Q_o[\sin(Kx_c - \omega t)\sin^2\theta - \cos(Kx_c - \omega t)$

$\sin\theta\ \cos\theta] + P_o(\ddot{x}_c\ \sin\theta\ \cos\theta - \ddot{z}_c\sin^2\theta) +$

$+ P_1\sin\theta\ddot{\theta}\}$

4. $C_T \int dB\ \bar{A}_{RT} = C_T\ \hat{i}_x\{\omega^2\xi_o Q_o[\cos(Kx_c - \omega t)\ \sin^2\theta +$

$+ \sin(Kx_c - \omega t)\sin\theta\ \cos\theta] +$

$+ P_o(-\ddot{x}_c\ \sin^2\theta - \ddot{z}_c\ \sin\theta\ \cos\theta) +$

$+ P_1\sin\theta\dot{\theta}^2\} + C_T\ \hat{i}_z\{\omega^2\xi_o Q_o[\cos(Kx_c - \omega t)\sin\theta\cos\theta +$

$+ \sin(Kx_c - \omega t)\cos^2\theta] + P_o(-\ddot{x}_c\sin\theta\cos\theta -$

$- \ddot{z}_c\ \cos^2\theta) + P_1\ \cos\theta\dot{\theta}^2\}$

5. $-\int_{\ell}^{u_3} dB\ \bar{g} = g\int_{\ell}^{u_2} dB\ \hat{i}_z + g\int_{o}^{-\xi_o\sin(Kx_c-\omega t)/\cos\theta} dB\ \hat{i}_z$

Also; $dB = \rho S_o dz'$, where S_o is the cross-sectional area of the disk of height dz'. Then;

$$-\int_{\ell}^{u_3} dB\ \bar{g} = \hat{i}_z[P_o g - \rho S_{o2} g \xi_o \sin(Kx_c - \omega t)/\cos\theta]$$

where; $S_{o2} = \pi r_2^2$

6. $\int dM\bar{g} = - Mg\ \hat{i}_z$

Integrals of equation (2.2) can also be written as:

1. $\int dM(\bar{R}_{cp} \times \bar{A}_{Ep}) = \int dM\ z'^2\ \ddot{\theta}\ \hat{i}_y = I_{yy}\ \ddot{\theta}\ \hat{i}_y$

2. $\int dB(\bar{R}_{cp} \times \bar{A}_{Ew}) = \hat{i}_y\{\omega^2 \xi_o Q_1[\cos(Kx_c - \omega t)\cos\theta -$

$- \sin(Kx_c - \omega t)\sin\theta]\}$

3. $C_N \int dB(\bar{R}_{cp} \times \bar{A}_{RN}) = C_N\ \hat{i}_y\{\omega^2 \xi_o Q_1[\cos(Kx_c - \omega t)\cos\theta -$

$- \sin(Kx_c - \omega t)\sin\theta] - P_1\cos\theta\ \ddot{x}_c +$

$+ P_1\sin\theta\ \ddot{z}_c - P_2\ddot{\theta}\}$

4. $C_T \int dB(\bar{R}_{cp} \times \bar{A}_{RT}) = 0$

5. $-\int_{\ell}^{u_3} dB(\bar{R}_{cp} \times \bar{g}) \cong - \int_{\ell}^{u_2} dB(\bar{R}_{cp} \times \bar{g})$

$= \hat{i}_y(-P_1 g \sin\theta)$

6. $\int dM(\bar{R}_{cp} \times \bar{g}) = 0$

Viscous Drag Forces and Moments: For a tuned spar buoy the viscous drag forces are assumed to be acting in normal and tangential directions proportional to their respective drag coefficients and areas.

$$\bar{V}_{Bw} = \hat{i}_x[V_{ox} + \Phi_x - \dot{x}_c - z_p'\dot{\theta}\cos\theta] +$$

$$+ \hat{i}_z[V_{oz} + \Phi_z - \dot{z}_c + z_p'\dot{\theta}\sin\theta]$$

$$= \hat{i}_{x'}[(V_{ox} + \Phi_x - \dot{x}_c)\cos\theta$$

$$- (V_{oz} + \Phi_z - \dot{z}_c)\sin\theta - z_p'\dot{\theta}]$$

$$+ \hat{i}_{z'}[(V_{ox} + \Phi_x - \dot{x}_c)\sin\theta$$

$$+ (V_{oz} + \Phi_z - \dot{z}_c)\cos\theta]$$

$$= \hat{i}_{x'} V_{Bwx'} + \hat{i}_{z'} V_{Bwz'}$$

Drag force components can now be written:

$$DF_{x'} = \rho C_{DN} \int r|V_{Bwx'}|V_{Bwx'}\, dz'$$

$$DF_{z'} = \rho C_{DT} \pi \int r|V_{Bwz'}|V_{Bwz'}\, dz'$$

$$+ \frac{1}{2}\rho C_{Dp}\pi\{r_1^2|V_b|V_b + (r_1^2 - r_2^2)|V_s|V_s\}$$

Here; C_{DN} and C_{DT} are normal and tangential drag coefficients. C_{DP} is the pressure drag coefficient due to the bottom, and the step, of the tuned spar buoy.

$$V_s = V_{Bwz'} \quad (z' = h_1 - z_c)$$

and $$V_b = V_{Bwz'} \quad (z' = -z_c)$$

Also; $$DM = \rho C_{DN} \int r |V_{Bwx'}| V_{Bwx'} \, z_p' \, dz'$$

Using the transformation matrix:

$$\begin{Bmatrix} DF_x \\ DF_z \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} DF_{x'} \\ DF_{z'} \end{Bmatrix}$$

Added Mass Force Due to the Step and Bottom Base of the Tuned Spar: In addition to the forces mentioned above, an added mass force proportional to the tangential body acceleration ($\bar{A}_{Et}$) and acting perpendicular to the step and the bottom end of the tuned spar is considered. Lamb (1945) in his analysis of a cylindrical spar moving in still water, gives an added mass coefficient equal to $4/3\rho r^3$. Following representation is used in this analysis for this force ($\overline{AM}$).

$$\overline{AM} = \frac{4}{3}\alpha\rho\left[r_1^3\,\bar{A}_{Et}\big|_{z'=-z_c} + (r_1-r_2)^3\,\bar{A}_{Et}\big|_{z'=h_1-z_c}\right] \tag{2.6}$$

where; $\bar{A}_{Et} = (\bar{A}_{Ep}\cdot\hat{i}_{z'})\hat{i}_{z'}$, and α is a constant.

$\overline{AM}$ is added to the left hand side of equation (2.1). $\bar{R}_{cp} \times \overline{AM}$ would be equal to zero.

Combining all these forces and moments; the matrix coefficients of equation (2.3) can be written as:

$$M_{11} = M + P_o(C_N\cos^2\theta + C_T\sin^2\theta) + \frac{4}{3}\alpha\rho[r_1^3 + (r_1-r_2)^3]\sin^2\theta$$

$$M_{12} = P_o\sin\theta\cos\theta(C_T - C_N) + \frac{4}{3}\alpha\rho[r_1^3 + (r_1-r_2)^3]\sin\theta\cos\theta$$

$$M_{13} = C_N P_1\cos\theta$$

$$M_{21} = P_o\sin\theta\cos\theta(C_T - C_N) + \frac{4}{3}\alpha\rho[r_1^3 + (r_1-r_2)^3]\sin\theta\cos\theta$$

$$M_{22} = M + P_o(C_N\sin^2\theta + C_T\cos^2\theta) + \frac{4}{3}\alpha\rho[r_1^3 + (r_1-r_2)^3]\cos^2\theta$$

$$M_{23} = - C_N P_1 \sin\theta$$

$$M_{31} = C_N P_1 \cos\theta$$

$$M_{32} = - C_N P_1 \sin\theta$$

$$M_{33} = I_{yy} + C_N P_2$$

$\{V\} = [\dot{x}_c, \dot{z}_c, \dot{\theta}]$; which is a column vector.

$$F_{D1} = \omega^2 \xi_o Q_o [\cos(Kx_c - \omega t)(1 + C_N \cos^2\theta + C_T \sin^2\theta) + \sin(Kx_c - \omega t) \sin\theta\cos\theta (C_T - C_N)]$$

$$F_{D2} = \omega^2 \xi_o Q_o [\sin(Kx_c - \omega t)(1 + C_N \sin^2\theta + C_T \cos^2\theta) + \cos(Kx_c - \omega t) \sin\theta\cos\theta (C_T - C_N)]$$

$$F_{D3} = \omega^2 \xi_o Q_1 [\cos(Kx_c - \omega t)\cos\theta - \sin(Kx_c - \omega t)\sin\theta](1 + C_N)$$

$$F_{G1} = 0.$$

$$F_{G2} = P_o g - \rho S_{o2} g \xi_o \sin(Kx_c - \omega t)/\cos\theta - Mg$$

$$F_{G3} = - P_1 g \sin\theta$$

$$F_{A1} = C_T P_1 \sin\theta\dot{\theta}^2 + DF_x - \frac{4}{3}\alpha\rho[r_1^3+(r_1-r_2)^3]z_c\sin\theta\dot{\theta}^2$$

$$F_{A2} = C_T P_1 \cos\theta\dot{\theta}^2 + DF_z + \frac{4}{3}\alpha\rho[r_1^3+(r_1-r_2)^3]$$

$$(h_1-z_c)\cos\theta\dot{\theta}^2$$

and $F_{A3} = DM$

Three-Dimensional Analysis: For small angles the appropriate transformation matrix reduces to:

$$\begin{Bmatrix} \hat{i}_{x'} \\ \hat{i}_{y'} \\ \hat{i}_{z'} \end{Bmatrix} = \begin{bmatrix} 1 & \cdot & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{Bmatrix} \hat{i}_x \\ \hat{i}_y \\ \hat{i}_z \end{Bmatrix}$$

In this analysis we let $C_N = 1.0$, $C_T = 0.0$, and define:

$$[\ddot{\bar{R}}_{oc}]_E = \hat{i}_x \ddot{x}_c + \hat{i}_y \ddot{y}_c + \hat{i}_z \ddot{z}_c$$

$$\bar{W}_{EB} = \hat{i}_x\dot{\phi} + \hat{i}_y\dot{\theta} + \hat{i}_z\dot{\psi}$$

and $\bar{R}_{cp} = z'_p \hat{i}_{z'}$

$$\therefore \bar{A}_{Ep} = \hat{i}_x(\ddot{x}_c + z'_p\ddot{\theta} + \hat{i}_y(\ddot{y}_c - z'_p\ddot{\cdot}) + \hat{i}_z\ddot{z}_c$$

and; $$\bar{A}_{EW} = \omega^2 \xi_o e^{|\bar{K}|z_w} [\hat{i}_x \cos\cdot\cos(\bar{K}\cdot\bar{S} - t) +$$

$$\hat{i}_y \sin\cdot\cos(\bar{K}\cdot\bar{S} - t) + \hat{i}_z \sin(\bar{K}\cdot\bar{S} - t)]$$

Integrals of equation (2.1) can now be written as:

1. $$\int dM\, \bar{A}_{Ep} = M(i_x\ddot{x}_c + \hat{i}_y\ddot{y}_c + \hat{i}_z\ddot{z}_c)$$

2. $$\int dB\, \bar{A}_{EW} = \omega^2 \xi_o Q_o [\hat{i}_x \cos\cdot\cos(\bar{K}\cdot\bar{S} - t) +$$

$$\hat{i}_y \sin\cdot\cos(\bar{K}\cdot\bar{S} - t) + i_z \sin(\bar{K}\cdot\bar{S} - t)]$$

also; $$\bar{A}_{RN} = \hat{i}_x(\ddot{\cdot}_x - \ddot{x}_c - z'\ddot{\theta} + \ddot{z}_c) +$$

$$i_y(\ddot{\cdot}_y - \ddot{y}_c + z'\ddot{\cdot} - \ddot{\cdot}_c) + \hat{i}_z(\ddot{x}_c - \ddot{\cdot}_c)$$

3. $$\int dB\, \bar{A}_{RN} = i_x(\omega^2 \xi_o \cos\cdot Q_o \cos(\bar{K}\cdot\bar{S} - t) - P_o\ddot{x}_c - P_1\ddot{\theta} +$$

$$P_o\ddot{z}_c) + i_y(\omega^2 \xi_o \sin\cdot Q_o \cos(\bar{K}\cdot\bar{S} - t) -$$

$$P_o\ddot{y}_c + P_1\ddot{\cdot} - P_o\ddot{\cdot}_c) + i_z P_o(\ddot{x}_c - \ddot{y}_c)$$

4. $$-\int_{\ell}^{u_3} dB\, \bar{g} = \int_{\ell}^{u_2} dB\, g\, i_z + \int_{o}^{-\xi_o \sin(\bar{K}\cdot\bar{S} - \omega t)} dB\, g\, i_z$$

$$= i_z[P_o g - (S_{o2} g \xi_o \sin(\bar{K}\cdot\bar{S} - \omega t)]$$

5. $\int dM\,\bar{g} = -\,Mg\,\hat{i}_z$

Similarly integrals of equation (2.2) can be written as:

1. $\int dM(\bar{R}_{cp} \times \bar{A}_{Ep}) = \hat{i}_x\, I_{xx}\ddot{\phi} + \hat{i}_y\, I_{yy}\ddot{\theta}$

where; $I_{xx} = I_{yy} = \int_o^H dM\, z'^2$

2. $\int dB(\bar{R}_{cp} \times \bar{A}_{Ew}) = \hat{i}_x\{-\omega^2 \xi_o Q_1 \sin\beta\cos(\bar{K}\cdot\bar{S}-\omega t)\}$

$+ \hat{i}_y\{\omega^2 \xi_o Q_1 \cos\beta\cos(\bar{K}\cdot\bar{S}-\omega t)\}$

3. $\int dB(\bar{R}_{cp} \times \bar{A}_{RN}) = \hat{i}_x\{-\omega^2 \xi_o Q_1 \sin\beta\cos(\bar{K}\cdot\bar{S}-\omega t) + P_1\ddot{Y}_c - P_2\ddot{\phi}$

$+ P_1\phi\ddot{z}_c\} + \hat{i}_y\{\omega^2 \xi_o Q_1 \cos\beta\cos(\bar{K}\cdot\bar{S}-\omega t)$

$- P_1\ddot{x}_c - P_2\ddot{\theta} + P_1\theta\ddot{z}_c\} - \hat{i}_z P_1\{\theta\ddot{Y}_c + \phi\ddot{x}_c\}$

4. $-\int_\ell^{u_3} dB(\bar{R}_{cp} \times \bar{g}) = -\int_\ell^{u_2} dB(\bar{R}_{cp} \times \bar{g})$

$= \hat{i}_x(-P_1\phi g) + \hat{i}_y(-P_1\theta g)$

5. $\int dM(\bar{R}_{cp} \times \bar{g}) = 0$

Viscous drag Forces and Moments:

$$\overline{V}_{Bw} = \hat{i}_x[V_{ox} + \phi_x - \dot{x}_c - z'_p\dot{\theta}] +$$

$$\hat{i}_y[V_{oy} + \phi_y - \dot{y}_c + z'_p\dot{\phi}] +$$

$$\hat{i}_z\,[V_{oz} + \phi_z - \dot{z}_c]$$

$$= \hat{i}_{x'}[V_{ox}+\phi_x-\dot{x}_c-z'_p\dot{\theta}+V_{oy}\psi-\dot{y}_c\psi-V_{oz}\theta+\dot{z}_c\theta]$$

$$+ \hat{i}_{y'}[-V_{ox}\psi+\dot{x}_c+V_{oy}+\phi_y-\dot{y}_c+z'_p\dot{\phi}+V_{oz}\phi-\dot{z}_c\phi]$$

$$+ i_{z'}[V_{ox}\theta-\dot{x}_c\theta-V_{oy}\phi+\dot{y}_c\phi+V_{oz}+\phi_z-\dot{z}_c]$$

$$= \hat{i}_{x'}\,V_{Bwx'} + \hat{i}_{y'}\,V_{Bwy'} + \hat{i}_{z'}\,V_{Bwz'}$$

Hence; $$DF_{x'} = \rho C_{DN}\int r\sqrt{V_{Bwx'}^2 + V_{Bwy'}^2}\;(V_{Bwx'})dz'$$

$$DF_{y'} = \rho C_{DN}\int r\sqrt{V_{Bwx'}^2 + V_{Bwy'}^2}\;(V_{Bwy'})dz'$$

and, $$DF_{z'} = \rho C_{DT}\pi\int r|V_{Bwz'}|V_{Bwz'}\,dz' +$$

$$\frac{1}{2}\,\rho C_{DP}\pi\{r_1^2|V_b|V_b + (r_1^2 - r_2^2)|V_s|V_s\}$$

Using the transformation matrix:

$$\begin{Bmatrix} DF_x \\ DF_y \\ DF_z \end{Bmatrix} = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \begin{Bmatrix} DF_{x'} \\ DF_{y'} \\ DF_{z'} \end{Bmatrix}$$

Also $\overline{DM} = \int(\bar{R}_{cp} \times d\overline{DF})$

$$= \hat{i}_{x'} (DM_{x'}) + \hat{i}_{y'} (DM_{y'})$$

Therefore:

$$\begin{Bmatrix} DM_x \\ DM_y \\ DM_z \end{Bmatrix} = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \begin{Bmatrix} DM_{x'} \\ DM_{y'} \\ 0 \end{Bmatrix}$$

Combining all these forces and equation (2.6) of additional added mass, we can write the non-zero elements of matrices in equation (2.3) as:

$$M_{11} = M + P_o = M_{22}$$

$$M_{13} = 4/3\ \alpha\rho[r_1^3 + (r_1-r_2)^3]\theta - P_c\theta = M_{31}$$

$$M_{15} = P_1 = M_{51}$$

$$M_{23} = P_o\phi - 4/3\ \alpha\rho[r_1^3 + (r_1-r_2)^3]\phi = M_{32}$$

$$M_{24} = -P_1 = M_{42}$$

$$M_{33} = M + 4/3\ \alpha\rho[r_1^3 + (r_1-r_2)^3]$$

$$M_{43} = -P_1\phi$$

$$M_{44} = I_{xx} + P_2$$

$$M_{53} = -P_1\theta$$

$$M_{55} = I_{yy} + P_2$$

$$M_{61} = P_1\phi \qquad M_{62} = P_1\theta$$

$\{V\} = [\dot{x}_c, \dot{y}_c, \dot{z}_c, \dot{\phi}, \dot{\theta}, \dot{\psi}]$ is a column vector.

$$F_{D1} = 2\omega^2\xi_o Q_o \cos\beta\cos(\bar{K}\cdot\bar{S}-\omega t)$$

$$F_{D2} = 2\omega^2\xi_o Q_o \sin\beta\cos(\bar{K}\cdot\bar{S}-\omega t)$$

$$F_{D3} = \omega^2\xi_o Q_o \sin(\bar{K}\cdot\bar{S}-\omega t)$$

$$F_{D4} = -2\omega^2\xi_o Q_1 \sin\beta\cos(\bar{K}\cdot\bar{S}-\omega t)$$

$$F_{D5} = 2\omega^2\xi_o Q_1 \cos\beta\cos(\bar{K}\cdot\bar{S}-\omega t)$$

$$F_{G3} = P_o g - \rho S_{o2} g \zeta_o \sin(\bar{K}\cdot\bar{\xi} - \omega t) - Mg$$

$$F_{G4} = - P_1 A_1 g$$

$$F_{G5} = - P_1 A_1 g$$

and $\{F_A\} = [DF_x, DF_y, DF_z, DM_x, DM_y, DM_z]$ is a column vector.

As can be seen from these matrix elements, the sixth degree of freedom corresponding to ψ does not drop out; but M_{66} equals zero. M_{66} is introduced in the equations for computational purposes. M_{66} is given by:

$$M_{66} = \frac{\rho\pi}{2}[r_1^4 h_1 + r_2^4 (H-h_1)]$$

which represents the moment of inertia of a tuned spar about its longitudinal axis.

2.1.2 Other Shapes

To obtain the velocity potential function ϕ for any shapes other than for simple geometries and small body motions is hardly ever possible. Hence to obtain any reasonable solution to this problem one has to depend on emperical approaches. Solution to these problems has thus been left out of this report. In order to use the

analysis and computer programs of this report, the readers will have to substitute their own elements for [M], $\{F_D\}$, $\{F_G\}$, and $\{F_A\}$ matrices of equation 2.3; pertinent to the particular surface buoy in question.

2.2 Mooring Line

A mooring line connects a surface or a subsurface buoy to the anchor. In this report, a line attached to a buoy but not to an anchor is also considered a mooring line. Such a line could be connecting a drogue with the surface buoy or be an instrument line hanging from a moored surface buoy. In general a mooring line is made of any type or number of materials (steel, nylon, dacron, etc.) and has any type or number of instruments (including subsurface floats) inserted along its length.

The mathematical model of mooring line dynamics will be formulated in two different approaches. The "first" approach is called the "continuous line formulation". In this approach the mooring line differential equations, with respect to the spatial coordinate s, are integrated incrementally down the mooring line, to obtain its dynamic equilibrium at any instant of time. Velocities and accelerations of the mooring line differential elements and the instruments (including subsurface floats) inserted in the mooring line are computed by differentiating

positions found by the dynamic equilibriums. This approach was used to model the low frequency motion of a subsurface mooring system, as presented later in the report. Exciting forces exerted on the mooring line by the wave system are neglected and so are the wave-damping forces. Viscous drag effects due to the velocity field generated by the surface waves is included. This approach was found to be computationally inefficient, and hard to solve numerically (due to the differentiation of positions to find velocities and accelerations) for the high frequency motion of a surface moored system.

For the high frequency motion of a surface moored system a "second" approach called the "lumped parameter formulation" is presented. Here exciting forces exerted on the mooring line by the wave system are taken equal to the "Froude-Krylov" forces, as it is assumed that the presence of the mooring line does not disturb the wave particle motion. Again, viscous drag effects due to the velocity field generated by the surface waves are included, and the wave-damping forces are neglected. In both approaches, a velocity profile (could-be time varying) can be present along with the surface wave. Both formulations are presented in three-dimensions and can be reduced to two-dimensions when needed.

2.2.1 Continuous Line Formulation

On a differential element of a continuous mooring line the forces acting are: (a) the constant force due to gravitational attraction, (b) the variable tensile forces transmitted from the adjoining elements, and (c) the variable pressure (normal) and shear (tangential) forces applied by the fluid. The fluid forces can be broken down into (1) the hydrostatic pressure force, (2) the pressure force due to the acceleration of the fluid by the element (the so-called added mass force), and (3) the viscous drag forces due to fluid relative velocity, which have both pressure (normal) and shear (tangential) components. Internal damping forces are neglected in this analysis as these are assumed small compared to tensile and viscous drag forces. Exciting and damping forces due to the wave system are also neglected. From Newton's second law of mechanics the above forces should equal the mass of the differential element multiplied by its acceleration. By computing these forces the differential equations for the mooring line dynamic equilibrium are derived as follows:

The mooring line is considered to be a cylindrical slender body. A free body diagram of a differential element of length ds of the mooring line is shown in Figure 2.5.

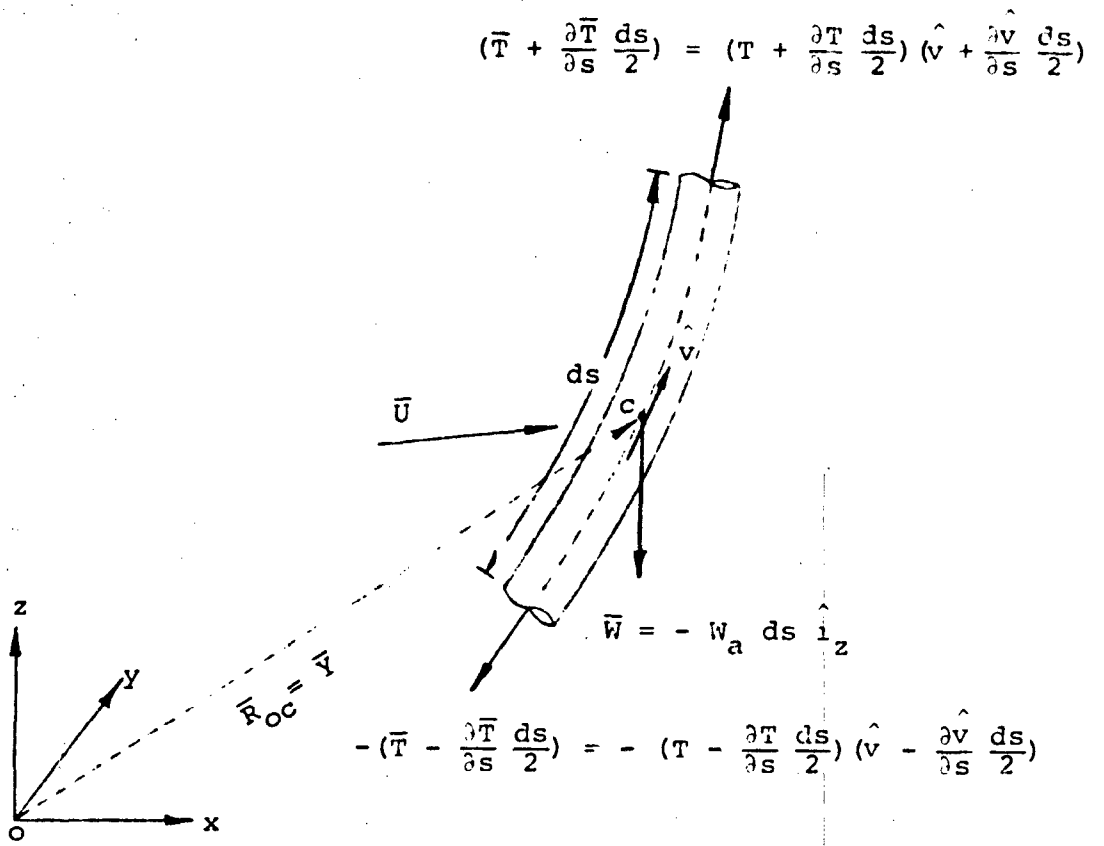


Figure 2.5 Mooring Line Differential Element

The internal tension and the inclination of the line segment change to keep all the above-mentioned forces in equilibrium. In Figure 2.5 $\hat{v}$ is a unit vector along the mooring line given by:

$$\hat{v} = \hat{i}_x \cos\phi_1 + \hat{i}_y \cos\phi_2 + \hat{i}_z \cos\phi_3$$

$\bar{T}$ is a tension vector, and $\bar{U}$ is a fluid velocity vector (obtained from the current profile and the velocity field generated by the surface wave). The relative velocity vector $\overline{UR}$ is given by:

$$\overline{UR} = \bar{U} - \dot{\bar{Y}}_c$$

where; $\bar{U} = \hat{i}_x U_x + \hat{i}_y U_y + \hat{i}_z U_z$

and, $\dot{\bar{Y}}_c = \hat{i}_x \dot{x}_c + \hat{i}_y \dot{y}_c + \hat{i}_z \dot{z}_c$ is the velocity of point c on the differential segment.

The viscous drag forces are computed according to the square drag law and are assumed to act in normal and tangential directions to the cylindrical line, proportional to their respective drag coefficients and areas.

Let: $\overline{UR} = \overline{UTR} + \overline{UNR}$

where; $\overline{UTR} = (\overline{UR} \cdot \hat{v})\hat{v}$

and, $\overline{UNR} = \overline{UR} - \overline{UTR}$

From $\overline{UNR}$ and $\overline{UTR}$, normal ($\overline{DN}$) and tangential ($\overline{DT}$) drag forces can be calculated using the pertinent drag coefficients (C_{DN} and C_{DT}). Representative areas of the differential element 'ds' can be calculated using the

reduced diameter and stretched length. These are given by dA_N and dA_T. The standard formulation is given by:

$$\overline{DN}ds = \rho/2\ (C_{DN})\ (dA_N)\ |\overline{UNR}|\overline{UNR}$$

and $$\overline{DT}ds = \rho/2\ (C_{DT})\ (dA_T)\ |\overline{UTR}|\overline{UTR}$$

The constant force due to gravitational attraction is $\overline{W} = -W_a ds\ \hat{i}_z$. Here, W_a is the weight in air per unit stretched length of the mooring line. $\overline{W}$ is resolved into normal ($\overline{WN}$) and tangential ($\overline{WT}$) components as:

$$\overline{WT} = (\overline{W}\cdot\hat{v})\hat{v} = -W_a ds\cos\phi_3\hat{v}$$

and $$\overline{WN} = \overline{W} - \overline{WT}$$
$$= -W_a ds(\hat{i}_z - \hat{v}\cos\phi_3)$$

The hydrostatic pressure force on the differential element by the surrounding fluid is given by the weight of the fluid displaced minus the hydrostatic pressure forces on the end cross-sections of the element. Or,

$$\overline{FH}ds = (W_a - W_w)ds\ \hat{i}_z - \pi r^2[\hat{v}(P_b - P_t) -$$
$$\frac{1}{2}\frac{\partial\hat{v}}{\partial s}\,ds\ (P_b + P_t)]$$

where, W_w is the weight in water per unit stretched length, and r is the reduced radius due to stretch of the mooring line. P_b and P_t are the hydrostatic pressures at the bottom and top ends of the differential element. Assume;

$$(P_b - P_t) = \rho g \, ds \cos\phi_3$$

$$\frac{P_b + P_t}{2} = P_c$$

and $$W_a - W_w = \rho g \pi r^2$$

$$\therefore \overline{FH} \, ds = (W_a - W_w) ds \left\{ (\hat{i}_z - \hat{v} \cos\phi_3) + \frac{P_c}{\rho g} \frac{\partial \hat{v}}{\partial s} \right\}$$

Here, P_c is the hydrostatic pressure at the midpoint c of the differential element.

For a continuous cylindrical body it is also assumed that the body motion accelerates water only in the direction normal to its longitudinal axis. The inertia forces due to body's (differential element) own acceleration and the so called "added mass" term can now be written as:

$$\overline{IF}\ ds = -\frac{W_a}{g}\ ds\ \ddot{\overline{Y}} - \frac{(W_a - W_w)}{g}\ ds\ \ddot{\overline{YN}}$$

where; $\ddot{\overline{YN}} = \ddot{\overline{Y}} - \ddot{\overline{YT}}$

$\ddot{\overline{YT}} = (\ddot{\overline{Y}} \cdot \hat{v})\hat{v}$; and the added mass of the differential element is assumed to be the mass of water displaced by this element.

Combining all these forces, the force equilibrium is written as:

$$\left(T + \frac{\partial T}{\partial s}\frac{ds}{2}\right)\left(\hat{v} + \frac{\partial \hat{v}}{\partial s}\frac{ds}{2}\right) - \left(T - \frac{\partial T}{\partial s}\frac{ds}{2}\right)\left(\hat{v} - \frac{\partial \hat{v}}{\partial s}\frac{ds}{2}\right) +$$

$$\overline{DN}\ ds + \overline{DT}\ ds - W_a\ ds\ \cos\phi_3\hat{v} -$$

$$W_a\ ds(\hat{i}_z - \hat{v}\ \cos\phi_3) +$$

$$(W_a - W_w)ds\{(\hat{i}_z - \hat{v}\ \cos\phi_3) + \qquad (2.7)$$

$$\frac{P_c}{\rho g}\frac{\partial \hat{v}}{\partial s}\Big\} - \frac{W_a}{g}\ ds\ (\ddot{\overline{YN}} + \ddot{\overline{YT}}) - \frac{(W_a - W_w)}{g}\ ds\ \ddot{\overline{YN}} = 0$$

Or, $$\left(\frac{\partial T}{\partial s}\right)\hat{v} = (-DT + W_a\ \cos\phi_3 + \frac{W_a}{g}\ \dot{\dot{\overline{YT}}})\hat{v} \qquad (2.8)$$

and
$$\left\{T + \frac{P_c(W_a - W_w)}{g}\right\} \frac{\partial \hat{v}}{\partial s} = -\overline{DN} + W_w(\hat{i}_z - \hat{v}\cos\phi_3) + \left\{\frac{2W_a - W_w}{g}\right\} \ddot{\overline{YN}} \qquad (2.9)$$

Also we can write:

$$\frac{\partial \overline{Y}}{\partial s} = \hat{v} \qquad (2.10)$$

Here T, the tension, $\hat{v}$, the unit vector along the mooring line, and $\overline{Y}$, the geometric displacement vector are the dependent variables of interest. Equations (2.8) and (2.9) can be simplified if we define a new variable called the effective tension (Te) similar to the one described in GOODMAN and BRESLIN (1976).

$$Te = T + \frac{P_c(W_a - W_w)}{\rho g} \qquad (2.11)$$

Differentiating Te with respect to s

$$\frac{\partial Te}{\partial s} = \frac{\partial T}{\partial s} + \frac{W_a - W_w}{\rho g}\frac{\partial P_c}{\partial s}$$

$$= \frac{\partial T}{\partial s} - (W_a - W_w)\cos\phi_3 \qquad (2.12)$$

Substituting (2.11) and (2.12) in (2.8) and (2.9) we obtain:

$$\frac{\partial Te}{\partial s} = - DT + W_w \cos\phi_3 + \frac{W_a}{g} \ddot{YT} \tag{2.13}$$

and

$$Te \frac{\partial \hat{v}}{\partial s} = - \overline{DN} + W_w(\hat{i}_z - \hat{v} \cos\phi_3) + \left(\frac{2W_a - W_w}{g}\right) \ddot{\overline{YN}} \tag{2.14}$$

Equations (2.13) and (2.14) are derived for a line of an arbitrary stretched length and of consequent reduced diameter. Also we assume, as reasoned in GOODMAN and BRESLIN (1976) for materials obeying Hooke's law, that the effective tension and not the actual tension controls the extensibility of the mooring line. The extension of the mooring line is caused by (1) pulling on the line due to tension and (2) squeezing of the line due to hydrostatic pressure. Volume I (CHHABRA, 1973) plotted non-linear curves between tension and elongation of various mooring cables and ropes. Using the effective tension instead of the actual tension, mooring line stretch and the consequent reduced diameter from some initial reference state are found as discussed in Volume I.

The analysis also allows for an arbitrary number of intermediate bodies such as sensor packages and subsurface floats inserted in the mooring line. For this case equations (2.13) and (2.14) are replaced by:

$$\overline{Te}\Big|_{n+1} = \overline{Te}\Big|_{n} + \overline{F} \qquad (2.15)$$

Here, $\overline{F}$ is the summatior of gravitational attraction, fluid, and inertia forces of the body inserted between n, the differential element above the body, and n+1, below it.

For cylindrical packages, the viscous drag representation remains similar to the one for cylindrical mooring line. For a spherical or any other shaped package this viscous drag force is computed as:

$$\overline{DA} = \rho/2 \; C_{Db} \; A_{Db} \; |\overline{UR}|\overline{UR}$$

where; C_{Db} and A_{Db} are the appropriate drag coefficient and area.

The gravitational force and the hydrostatic pressure force are combined to give:

$$\overline{NB} = - \; W_{wb} \; \hat{i}_z$$

where; W_{wb} is the weight in water of the inserted body.

Inertia forces for cylindrical packages are changed slightly from that of a continuous cylindrical mooring line. Here the assumption of the body motion accelerating water only in the direction normal to its longitudinal axis is dropped. Instead two added mass terms; one normal and the other tangential to the longitudinal axis are used. For cylindrical packages inertia force is given by:

$$\overline{IF}_b = - (m_b \ddot{\overline{Y}} + m_{nb} \ddot{\overline{YN}} + m_{tb} \ddot{\overline{YT}})$$

and for any other shape this force is:

$$\overline{IF}_b = - (m_b + m_{ab}) \ddot{\overline{Y}}$$

Here; m_b is the mass, m_{nb} and m_{tb} are the normal and tangential components of added mass for cylindrical bodies, and m_{ab} is the added mass for any other shaped body. Summation of gravitational attraction, inertia, and fluid forces gives $\overline{F}$. Integration of equations (2.10), (2.13), (2.14) and (2.15) along the spatial coordinate, s, of the mooring line gives the dynamic equilibrium of the

mooring system at discrete times. At any time these four equations can be solved for positions, $\bar{Y}$ (t), of the mooring system. Positions $\bar{Y}$ are calculated at short enough time intervals, Δts, so that velocities $\dot{\bar{Y}}$ and accelerations $\ddot{\bar{Y}}$ during these time intervals can be computed as piecewise constant. Accelerations are neglected at t_0 and t_1, velocities are neglected at t_0.

2.2.2 Lumped Parameter Formulation

In this formulation, the mooring line is reduced to a discretized dynamic system which is solved by a lumped parameter approach. All forces acting on the mooring line and the inserted packages are transferred to a fixed number of nodes (say N). If each node has q degrees of freedom then N x q simultaneous differential equations, q for each node, describing these nodes can be solved simultaneously by using any of the numerical integration techniques. Once again the forces transferred at each node (lumped mass) are the same as listed in 2.2.1, with the exception of Froude-Krylov forces which are not neglected here. The forces for the n^{th} mass for a N node system are derived next.

Consider the discretized dynamic system as shown in Figure 2.6, where an N-mass system is shown. In this figure $\hat{v}_n$ is a unit vector along the n^{th} segment

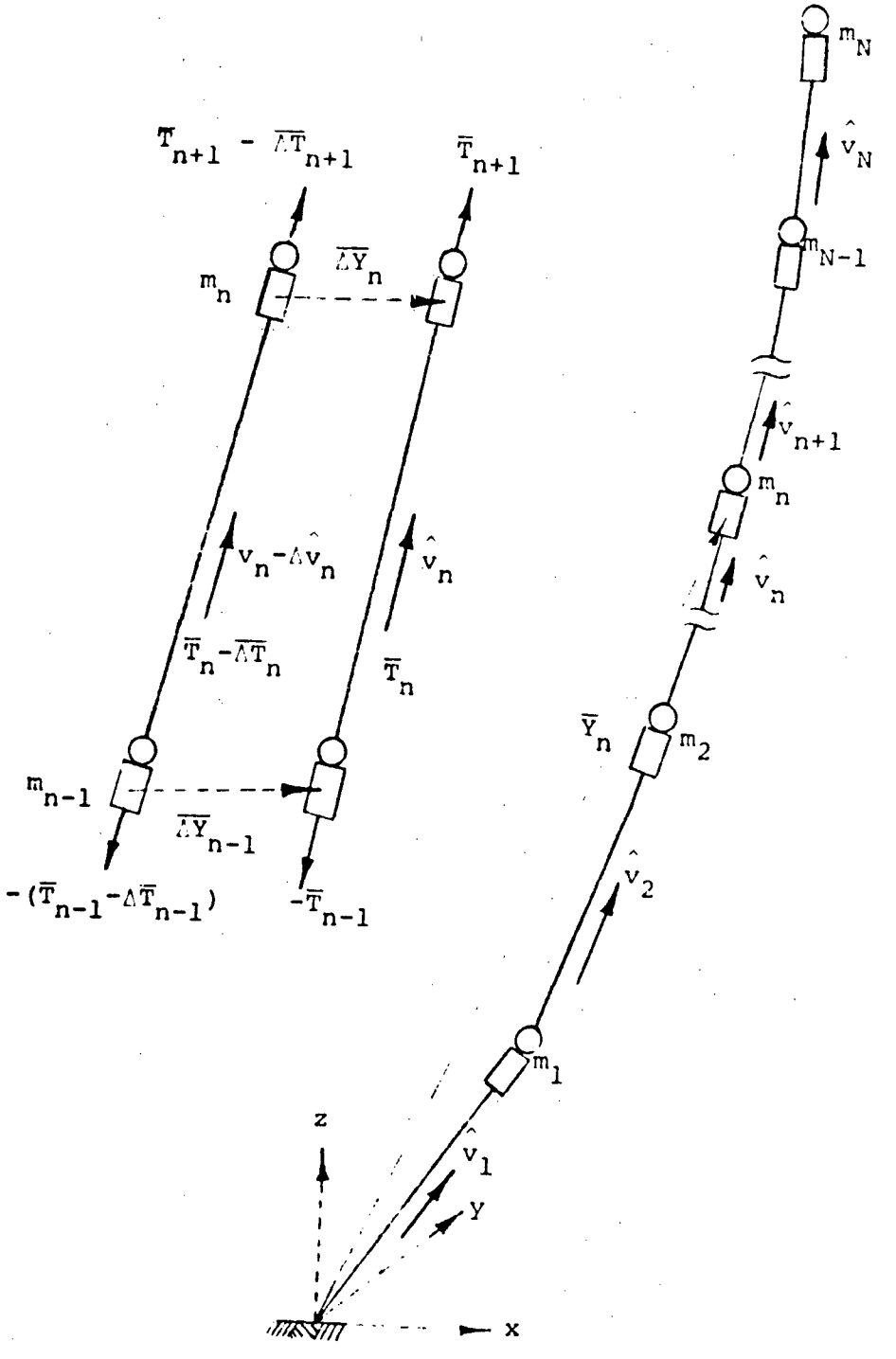


Figure 2.6 N-Mass System

(below the n^{th} mass). This can be written as:

$$\hat{v}_n = \hat{i}_x \cos\phi_{1n} + \hat{i}_y \cos\phi_{2n} + \hat{i}_z \cos\phi_{3n}$$

$\overline{T}_n$, $\overline{T}_{n+1}$ are the tension vectors, and $\overline{U}_n$ is the fluid velocity vector at the location of the n^{th} mass. A typical mass could consist of cylindrical components, spherical components, and any other shaped components. The transfer of all adjacent forces to a typical n^{th} mass is done as explained below.

Viscous drag forces acting on the mooring system have to be transferred at 'N' nodes. Any scheme devised for this transfer should take into account the different behavior of cylindrical and spherical components of the mooring system. Let $\overline{DN}_n$ be the drag force transferred to n^{th} node which is equivalent to contribution of normal drag forces on cylindrical components lumped at the n^{th} node. Similarly, $\overline{DT}_n$ is the drag force on n^{th} node equivalent to tangential drag forces on cylindrical components lumped at the n^{th} node. $\overline{DA}_n$ is the drag force acting on the n^{th} node in the direction of the relative velocity $\overline{UR}_n$, and is a contribution from spherical or any other shaped components lumped at the n^{th} node. These forces are given by :

$$\overline{DN}_n = (CDIN)_n |\overline{UNR}_n| \ \overline{UNR}_n$$

$$\overline{DT}_n = (CDIT)_n |\overline{UTR}_n| \ \overline{UTR}_n$$

$$\overline{DA}_n = (CDIA)_n |\overline{UR}_n| \ \overline{UR}_n$$

and $$\overline{DF}_n = \overline{DN}_n + \overline{DT}_n + \overline{DA}_n \qquad (2.16)$$

Here; $\overline{U}_n$ is computed from the current profile and the velocity of water due to surface waves at the location of the n^{th} mass which is exponentially attenuated with depth. CDIN, CDIT, and CDIA are the appropriate drag coefficients multiplied by respective areas.

The gravitational forces and the hydrostatic pressure forces are combined for all the transferred components to give:

$$\overline{NB}_n = - W_{wn} \hat{i}_z \qquad (2.17)$$

Inertia forces for the components, in general, are given by:

$$\overline{IF}_n = - (m_n + m_{an}) \ddot{\overline{Y}}_n - m_{nn} \ddot{\overline{YN}}_n - m_{tn} \ddot{\overline{YT}}_n \qquad (2.18)$$

This representation is similar to the one used for inserted bodies in the continuous line formulation.

The tensile forces acting on the n^{th} mass change in direction as well as magnitude with the motion of the nodes. The direction can be obtained by $\overline{Y}_n$ and $\overline{Y}_{n-1}$ (refer Figure 2.6); but as the present problem is one of large displacements the magnitude is determined at each integration step by updating its value at the beginning of the time step by the stiffness force due to the incremental deformation during this time step. Let the incremental change in length of the nth segment $\overline{\Delta L}_n$ be:

$$\overline{\Delta L}_n = \overline{\Delta Y}_n - \overline{\Delta Y}_{n-1}$$

For small $\overline{\Delta Y}_n$ and $\overline{\Delta Y}_{n-1}$;

$$\Delta L_n \cong \overline{\Delta L}_n \cdot \hat{v}_n =$$

$$\left[-\cos\phi_{1n} - \cos\phi_{2n} - \cos\phi_{3n} \cos\phi_{1n} \cos\phi_{2n} \cos\phi_{3n}\right] \begin{Bmatrix} \overline{\Delta Y}_{n-1} \\ \overline{\Delta Y}_n \end{Bmatrix}$$

If k_n (stiffness coefficient) is defined as the force required at node n for a unit displacement along $\hat{v}_n$; then the change in tension magnitude ΔT_n is:

$$\Delta T_n = k_n \Delta L_n$$

or; $$\bar{T}_n = (T_n - \Delta T_n)\hat{v}_n + \Delta T_n \hat{v}_n$$

Also, as tension vectors $\bar{T}_n$ and $\bar{T}_{n+1}$ are acting at the n^{th} mass, we have:

$$\overline{TF}_n = \bar{T}_{n+1} - \bar{T}_n \tag{2.19}$$

Froude-Krylov force on the submerged 'mass n' is assumed to be given by the product of mass of water displaced by the 'n^{th} mass' and the acceleration of water at that location which is exponentially attenuated with depth. Or,

$$\overline{FF}_n = m_{dn}\, g\xi_o e^{|\bar{K}| z_n} \left[\hat{i}_x K_x \cos(\bar{K}\cdot\bar{S}-\omega t) + \hat{i}_y K_y \cos(\bar{K}\cdot\bar{S}-\omega t) + \hat{i}_z |\bar{K}| \sin(\bar{K}\cdot\bar{S}-\omega t)\right] \tag{2.20}$$

where, m_{dn} is the mass of water displaced by the n^{th} mass.

Combining equations (2.16) through (2.20) for all nodes and taking into account all the boundary conditions, the force balance reduces in matrix notation to:

$$[M]\{\ddot{Y}\} = \{DF\}+\{CF\}+\{TF\}+[K]\{\Delta Y\}+\{FF\} \tag{2.21}$$

where; [M] is a matrix of appropriate inertia terms.

[K] is a matrix of stiffness terms.

{TF} is an array of tension components at the nodes.

{DF} is an array of appropriate viscous drag terms.

{CF} is an array of constant nodal forces (gravitational and hydrostatic).

{FF} is an array of Froude-Krylov forces.

$\ddot{\{Y\}}$ is an array of accelerations at the nodes.

and $\{\Delta Y\}$ is an array of incremental deformations of the nodes.

Derivation of Matrices: In equation (2.21), the order of each matrix and array is equal to the number of nodes (N) multiplied by degrees of freedom (q = 3) per node. For simplicity the masses were assumed to be lumped at 'N' nodes, in which case the off-diagonal terms in the mass matrix are zero, i.e., force at node 'i' due to an acceleration at node 'j' equals zero. A consistent mass matrix analysis can be done to allow for distributed mass.

Mass Matrix - [M]

$$[M] = \begin{bmatrix} M_1 & & & & & & \\ & M_2 & & & & & \\ & & M_3 & & & & \\ & & & \ddots & & & \\ & & & & M_n & & \\ & & & & & \ddots & \\ & & & & & & M_{N-1} \\ & & & & & & & M_N \end{bmatrix}$$

where; M_1, M_2, ... M_N are 3x3 matrices given by:

$$M_n = \begin{bmatrix} m_n+m_{an}+m_{nn}\sin^2\phi_{1n} & (m_{tn}-m_{nn})\cos\phi_{1n}\cos\phi_{2n} & (m_{tn}-m_{nn})\cos\phi_{1n}\cos\phi_{3n} \\ +m_{tn}\cos^2\phi_{1n} & & \\ (m_{tn}-m_{nn})\cos\phi_{2n}\cos\phi_{1n} & m_n+m_{an}+m_{nn}\sin^2\phi_{2n} & (m_{tn}-m_{nn})\cos\phi_{2n}\cos\phi_{3n} \\ & +m_{tn}\cos^2\phi_{2n} & \\ (m_{tn}-m_{nn})\cos\phi_{3n}\cos\phi_{1n} & (m_{tn}-m_{nn})\cos\phi_{3n}\cos\phi_{2n} & m_n+m_{an}+m_{nn}\sin^2\phi_{3n} \\ & & +m_{tn}\cos^2\phi_{3n} \end{bmatrix}$$

m_n = mass lumped at the n^{th} node.

m_{an} = added mass at n^{th} node due to non-cylindrical components

and m_{nn}, m_{tn} = Normal and tangential components of added mass at n^{th} node due to cylindrical components

Stiffness Matrix [K] - 4 Mass System

$$[K] = \begin{bmatrix} K^1 + K^2 & -K^2 & 0 & 0 \\ -K^2 & K^2 + K^3 & -K^3 & 0 \\ 0 & -K^3 & K^3 + K^4 & -K^4 \\ 0 & 0 & -K^4 & K^4 \end{bmatrix}$$

where; K^s are 3x3 matrices whose elements are given by

$$K^s_{ij} = -k_s cs_{i,s} cs_{j,s}$$

k_s = Stiffness coefficient for segment s.

and $cs_{i,s}$ = $\cos\phi_i$ of segment s.

{TF}, {DF}, {CF}, {FF}, $\{\ddot{Y}\}$ and {ΔY} are column matrices. These matrices can be written as

$$\{TF\} = \begin{Bmatrix} -(T_1 - \Delta T_1)\, CS_{1,1} + (T_2 - \Delta T_2)\, CS_{1,2} \\ -(T_1 - \Delta T_1)\, CS_{2,1} + (T_2 - \Delta T_2)\, CS_{2,2} \\ -(T_1 - \Delta T_1)\, CS_{3,1} + (T_2 - \Delta T_2)\, CS_{3,2} \\ \vdots \\ -(T_{N-1} - \Delta T_{N-1}) CS_{3,N-1} + (T_N - \Delta T_N) CS_{3,N} \\ -(T_N - \Delta T_N) CS_{1,N} \\ -(T_N - \Delta T_N) CS_{2,N} \\ -(T_N - \Delta T_N) CS_{3,N} \end{Bmatrix} \quad \{FF\} = \begin{Bmatrix} FF_{1,1} \\ FF_{2,1} \\ FF_{3,1} \\ \vdots \\ FF_{1,N} \\ FF_{2,N} \\ FF_{3,N} \end{Bmatrix}$$

$$\{DF\} = \begin{Bmatrix} DF_{1,1} \\ DF_{2,1} \\ DF_{3,1} \\ \cdot \\ \cdot \\ \cdot \\ DF_{1,N} \\ DF_{2,N} \\ DF_{3,N} \end{Bmatrix}; \quad \{CF\} = \begin{Bmatrix} CF_{1,1} \\ CF_{2,1} \\ CF_{3,1} \\ \cdot \\ \cdot \\ \cdot \\ CF_{1,N} \\ CF_{2,N} \\ CF_{3,N} \end{Bmatrix}; \quad \{\ddot{Y}\} = \begin{Bmatrix} \ddot{Y}_{1,1} \\ \ddot{Y}_{2,1} \\ \ddot{Y}_{3,1} \\ \cdot \\ \cdot \\ \cdot \\ \ddot{Y}_{1,N} \\ \ddot{Y}_{2,N} \\ \ddot{Y}_{3,N} \end{Bmatrix}; \quad \{\Delta Y\} = \begin{Bmatrix} \Delta Y_{1,1} \\ \Delta Y_{2,1} \\ \Delta Y_{3,1} \\ \cdot \\ \cdot \\ \cdot \\ \Delta Y_{1,N} \\ \Delta Y_{2,N} \\ \Delta Y_{3,N} \end{Bmatrix}$$

Equation (2.21) can be re-written as

$$\{\ddot{Y}\} = [M]^{-1} \{TF\} + [M]^{-1} [K] \{\Delta Y\} +$$

$$[M]^{-1} \{DF\} + [M]^{-1} \{FF\} + [M]^{-1} \{CF\}$$

These 3N second-order differential equations can be reduced to a set of 6N first-order differential equations which can be solved by any of the numerical methods.

2.3 Attachment Between Surface Buoy and a Mooring Line

Figure 2.7 shows the attachment between a surface buoy and a mooring line. Let the attachment point be 'a', which is fixed rigidly to the buoy.

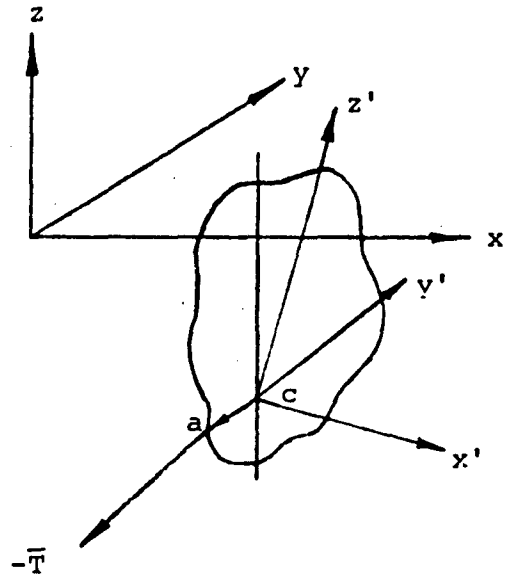


Figure 2.7 Attachment Between Surface Buoy and a Mooring Line

In addition to all the other forces acting on the surface buoy as outlined in Section 2.1; a tension vector $\overline{T}$ is acting at point a, which should be included in the force balance.

$$\overline{T} = T\hat{v} = T_x\hat{i}_x + T_y\hat{i}_y + T_z\hat{i}_z \qquad (2.22)$$

also let vector $\bar{R}_{ca}$ be given by:

$$\bar{R}_{ca} = \hat{i}_{x'}\, x_a' + \hat{i}_{y'}\, y_a' + \hat{i}_{z'}\, z_a'$$

Taking moment of $\bar{T}$ about point c we obtain:

$$\overline{TM} = - \bar{R}_{ca} \times \bar{T} \qquad (2.23)$$

for inclusion in the moment balance.

Two Dimensional Analysis:

$$\bar{R}_{ca} = \hat{i}_{x'}\, x_a' + \hat{i}_{z'}\, z_a'$$

$$= \hat{i}_x(x_a'\cos\theta + z_a'\sin\theta) + \hat{i}_z(z_a'\cos\theta - x_a'\sin\theta)$$

and; $$\overline{TM} = \hat{i}_y[T_z(x_a'\cos\theta + z_a'\sin\theta) - T_x(z_a'\cos\theta - x_a'\sin\theta)]$$

Three Dimensional Analysis:

$$\bar{R}_{ca} = \hat{i}_x(x_a' - \psi y_a' + \theta z_a') + \hat{i}_y(\psi x_a' + y_a' - \phi z_a') +$$

$$\hat{i}_z(-\theta x_a' + \phi y_a' + z_a')$$

And; $$\overline{TM} = \hat{i}_x[T_y(-\theta x'_a + \phi y'_a + z'_a) - T_z(\psi x'_a + y'_a - \phi z'_a)] +$$

$$\hat{i}_y[T_z(x'_a - \psi y'_a + \theta z'_a) - T_x(-\theta x'_a + \phi y'_a + z'_a)] +$$

$$\hat{i}_z[T_x(\psi x'_a + y'_a - \phi z'_a) - T_y(x'_a - \psi y'_a + \theta z'_a)]$$

Change in mooring line tension magnitude due to buoy rotation (coefficients in the stiffness matrix).

Two Dimensional Analysis:

Rotate $\overline{R}_{ca}$ through $\Delta\theta$ to obtain $\overline{R}_{ca_1}$

$$\overline{R}_{ca_1} = \begin{bmatrix} 1 & \Delta\theta \\ -\Delta\theta & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} x'_a \\ z'_a \end{Bmatrix}$$

For small rotations change in tension magnitude ΔT is given by

$$\Delta T = k(\overline{R}_{ca_1} - \overline{R}_{ca}).\hat{v}$$

where k is the stiffness coefficient for the line segment attached to the surface buoy.

If $$\hat{v} = \hat{i}_x \cos\phi + \hat{i}_z \sin\phi$$

$$\Delta T = k[\cos\phi(z'_a \cos\theta - x'_a \sin\theta) + \sin\phi(-z'_a \sin\theta - x'_a \cos\theta)]\Delta\theta$$

Three Dimensional Analysis:

$$\bar{R}_{ca_1} = \begin{bmatrix} 1 & -\Delta\psi & \Delta\theta \\ \Delta\psi & 1 & -\Delta\phi \\ -\Delta\theta & \Delta\phi & 1 \end{bmatrix} \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \begin{Bmatrix} x'_a \\ y'_a \\ z'_a \end{Bmatrix}$$

and;

$$\Delta T = k \begin{bmatrix} \cos\phi_2(\theta x'_a - \phi y'_a - z'_a) \cos\phi_1(-\theta x'_a + \phi y'_a + z'_a) \cos\phi_1(-\psi x'_a - y'_a + \phi z'_a) \\ +\cos\phi_3(\psi x'_a + y'_a - \phi z'_a) + \cos\phi_3(-x'_a + \psi y'_a - \theta z'_a) + \cos\phi_2(x'_a - \psi y'_a + \theta z'_a) \end{bmatrix}$$

$$\cdot \begin{Bmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{Bmatrix}$$

If more than one mooring lines are attached to the surface buoy, as is the case of simulation study in Section 5.3, force and moment of both these lines are to be included in the appropriate equations. It is important to note that within the definition of the coordinate frame (fixed at the anchor) if a mooring line hangs from the buoy (as is the case in 5.3), $-\bar{T}$ of this section would be replaced by $+\bar{T}$. Also in this case ΔT would be given by

$$\Delta T = -k(\bar{R}_{ca_1} - \bar{R}_{ca}) \cdot \hat{v}$$

2.4 Window Shade Drogue

Sometimes an alternative approach to mooring surface buoys is the use of surface trackable drogued drifting buoys. Such a buoy system employs a high drag device (or drogue) tethered to a trackable buoy at the surface. A drogue is subjected to the same forces (including the wave effects) as a differential element of the mooring line. A special case of drogues, the window shade drogue, as shown in Figure 2.8 is considered in this analysis. The drogue, modeled dynamically as a lumped parameter system (Figure 2.8), is reduced to strips. The mass, the acting forces, and the dynamic behavior of each strip is lumped at an assigned nodal point. Nodal masses are linked by suitably elastic lines.

The window shade drogue is a compliant sheet. Close observation of scale model drogues (Vachon, 1973) has revealed that it seems nearly locked to the local water mass insofar as motion normal to the sheet is concerned. However, in the tangential direction, it seems to slip through the local water mass rather easily. The forces were therefore described as follows: the wave forces (Froude-Krylov force; as it is assumed that there is no diffracted wave) acting on a material strip are estimated by calculating the force normal to the axis

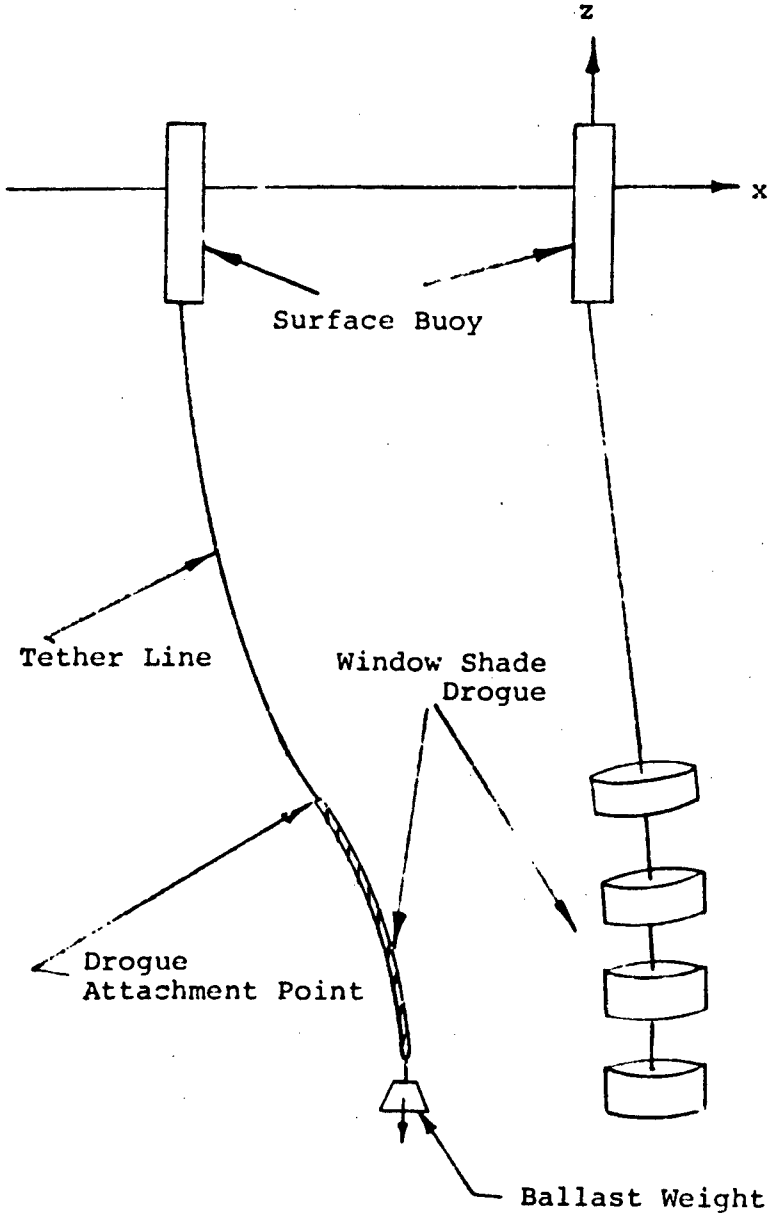


Figure 2.8 Drifting Drogued Buoy System

of revolution of a hypothetical cylindrical element, the diameter and length of which is equal to the breadth and length of the strip respectively. This force is given by the mass of the hypothetical water cylinder multiplied by the component of water acceleration normal to the cylinder's longitudinal axis. Or;

$$\overline{FF}_n = m_{dn} \cdot [\overline{A}_{Ew} - (\overline{A}_{Ew} \cdot \hat{v}_n)\hat{v}_n] \qquad (2.24)$$

where w is the particle of water next to the n^{th} node, and m_{dn} is the mass of the hypothetical water cylinder. For inertia forces m_{an} and m_{tn} equal zero; whereas m_{nn} equals m_{dn}. The viscous drag of this strip in the normal direction is calculated using the frontal area of the strip and the drag coefficient measured in water-filled quarry tests (Vachon, 1975). The elemental strips of the drogue are assumed subject to tangential viscous drag calculated using the area of the strip and a tangential drag coefficient measured during drop tests in the ocean (Vachon, 1975).

Gravitational, hydrostatic pressure, and tensile forces on a drogue strip are similar to the ones outlined in Section 2.2 for a discretized dynamic system.

3.0 METHOD OF SOLUTION

Section 2 outlined all the necessary ingredients of the mathematical models used in this report. In this section some details of how to use these ingredients, to obtain a solution for a specific problem on hand, are given. For illustrative purposes a specific case study, of a spar buoy at the surface and a mooring line connecting this buoy with the anchor, will be discussed. Method of solution for both the two-dimensional and the three-dimensional analysis of the moored system will be outlined in 3.1. Section 3.2 outlines the approximate initial conditions for the steady state analysis of a spar buoy freely floating in a surface wave.

Another case study of a spar buoy at the surface and connected to a window shade drogue at a given depth is discussed in section 3.3.

3.1 Moored System Analysis

Two-Dimensional:

In this case, the current profile is in the same plane as the train of surface gravity waves having a single frequency and amplitude; and its direction parallel or anti-parallel to the direction of surface wave propagation.

Static Solution:

For the static solution, the analysis is started at the spar buoy. The pertinent static equations for the spar buoy (Section 2.1) with an attached mooring line (Section 2.3) are:

$$DF_x - T_x = 0 \tag{3.1}$$

$$P_o g - Mg + DF_z - T_z = 0 \tag{3.2}$$

and

$$-P_1 g \sin\theta + DM + T_z(x'_a \cos\theta + z'_a \sin\theta) - T_x(z'_a \cos\theta - x'_a \sin\theta) = 0 \tag{3.3}$$

Here; we have three equations and four (T_x, T_z, θ, h_s) unknowns, where h_s is the static draft of the spar buoy. Now if we assume a value for h_s, the other three unknowns can be solved by equations (3.1) through (3.3). The solution for the three unknowns T_x, T_z, and θ is found by iterating on the pitch angle θ. If θ is known; DF_x, DF_z, and DM can be computed explicitly. P_o and P_1 can be calculated if h_s is known and thus T_x and T_z are given by equations (3.1) and (3.2) respectively. Moment balance can now be performed by using equation (3.3)

to obtain the error. This error is minimized by iterating on the pitch angle θ.

Having solved the buoy force and moment balances for a assumed h_s the mooring line can be analyzed next.

For the continuous line formulation, a numerical integration is carried down the mooring line by dividing it into a finite number of segments. In this representation equations (2.13), (2.14), and (2.10) for two-dimensions can be written as:

$$\Delta Te = (-DT + W_w \sin\phi)\Delta S \tag{3.4}$$

$$Te\Delta\phi = [DN + W_w \cos\phi]\Delta S \tag{3.5}$$

and

$$\Delta Y = \hat{i}_x \Delta S \cos\phi + \hat{i}_z \Delta S \sin\phi \tag{3.6}$$

Having found T_x and T_z (or T and ϕ) at the top of the mooring line equations (3.4) through (3.6) are used to find the configuration and the tension distribution along the mooring line. At the end of the mooring line the boundary condition of the ocean floor depth should be met. If not, the whole process is repeated with a new h_s; thus computing the correct value of h_s for the given boundary condition. Whenever the mooring line is

discontinued to insert an intermediate body, the discontinuity is taken as a lumped mass and resolved as done below for the lumped parameter approach.

For the lumped parameter formulation, the configuration and the tension distribution along the mooring line can be found by (Section 2.2):

$$\bar{T}_n = \bar{T}_{n+1} - W_{wn}\,\hat{i}_z + \overline{DF}_n \qquad (3.7)$$

where; $\bar{T}_n$ is the tension vector below the n^{th} mass and $\bar{T}_{n+1}$ above it.

Dynamic Solution:

Equations of Section 2.2 which are written for three-dimensional analysis can be easily reduced for the two dimensional problem. Using the lumped parameter approach for the mooring line; equations (2.21) are combined with equations (2.3), (2.22), and (2.23) to obtain a global matrix equation for the mooring/buoy system. Stiffness coefficients for the change in mooring line tension magnitude due to buoy rotation are obtained from Section 2.3. This matrix equation can be solved in the time domain using various numerical integration techniques. Integrals of P_i and Q_i, given in Section 2.1,

are computed exactly; whereas the integrals for viscous drag forces and moments DF_x, DF_z, and DM are computed numerically by dividing the spar buoy in disks of height dz'. The composit matrices are of order 2N + 3. Matrices [M], {TF}, {K}, {DF}, {FF}, {FD}, {FG}, and {FA} are computed at each time step of the integration.

For the continuous line formulation; the matrices of equation (2.3) combined with equations (2.22) and (2.23) can be solved for $\overline{T}$ in equation (2.22) if the buoy draft h_o is known. Then integration of equations (2.13), (2.14), (2.10), and (2.15) along the spatial coordinate, s, of the mooring line gives the dynamic equilibrium of the mooring system at discrete times. Iteration on the unknown h_o may be necessary to meet the boundary condition of the ocean floor depth.

Three Dimensional:

Here; the train of surface gravity waves having a single frequency and amplitude, can propagate in a general direction. The current profile is also three-dimensional and can vary, in magnitude and direction, with depth.

Static Solution:

The pertinent static equations for the spar buoy (Section 2.1) with an attached mooring line (Section 2.3) are:

$$-\int d\bar{B}\bar{g} + \int d\bar{M}\bar{g} + \overline{DF} - \bar{T} = 0 \qquad (3.8)$$

and;

$$-\int dB(\bar{R}_{cp} \times \bar{g}) + \overline{DM} + \overline{TM} = 0 \qquad (3.9)$$

Integrations for dB are over the static draft of the buoy, and integration for DM is over the length of the buoy.

Equations (3.8) and (3.9) can be written as:

$$P_o g \hat{i}_z - Mg \hat{i}_z + \overline{DF} - \bar{T} = 0 \qquad (3.10)$$

and

$$\begin{Bmatrix} \phi \\ \theta \\ \psi \end{Bmatrix} = [S]^{-1} \{F\} \qquad (3.11)$$

where; [S] is a 3 x 3 matrix with components given by:

$$S_{11} = P_o g \ell_1 + DF_z \ell_2 - T_y y'_a - T_z z'_a$$

$$S_{12} = T_y x'_a \; ; \quad S_{13} = T_z x'_a$$

$$S_{21} = T_x y'_a \; ; \quad S_{23} = T_z y'_a$$

$$S_{22} = P_o g \ell_1 + DF_z \ell_2 - T_x x'_a - T_z z'_a$$

$$S_{31} = - DF_x \ell_2 + T_x z'_a$$

$$S_{32} = - DF_y \ell_2 + T_y z'_a$$

$$S_{33} = - T_x x'_a - T_y y'_a$$

and $\{F\}$ is a 3 x 1 array

$$F_1 = - DF_y \ell_2 + T_y z'_a - T_z y'_a$$

$$F_2 = DF_x \ell_2 - T_x z'_a + T_z x'_a$$

$$F_3 = T_x y'_a - T_y x'_a$$

also $P_o = \int dB$; and ℓ_1, ℓ_2 are the distances between center of gravity and center of buoyancy and center of

drag respectively. For the static case we also assume (small angles):

$$DF_x \propto |V_{Bwx}| V_{Bwx}$$

$$DF_y \propto |V_{Bwy}| V_{Bwy}$$

and $$DF_z \propto |V_{Bwz}| V_{Bwz}$$

$\therefore$ $\bar{T}$ and ϕ, θ, and ψ can be solved explicitly from equations (3.10) and (3.11) if h_s is known. h_s is determined by iteration on the boundary condition of the ocean floor depth.

Mooring line solution is similar to the one presented for the two-dimensional analysis.

Dynamic Solution:

This dynamic solution is similar to the one for two-dimensional analysis except that the order of matrices in this case is 3N + 6.

3.2 Initial Conditions for the Steady-State Analysis of a Spar Buoy

Two-Dimensional Analysis:

Let $C_N = 1.0$, $C_T = 0.0$, and $x_c = 0.0$. Also assuming small pitch of the buoy and neglecting viscous

drag forces, the equations of motion for a spar buoy become:

$$2M\ddot{x}_c + P_1\ddot{\theta} = 2\omega^2\xi_o Q_o \cos\omega t \qquad (3.12)$$

$$M^*\ddot{z}_c + \rho S_{o2} g z = -\omega^2\xi_o Q_o \sin\omega t + \rho S_{o2} g \xi_o \sin\omega t \qquad (3.13)$$

and $$P_1\ddot{x}_c + (I_{yy} + P_2)\ddot{\theta} + P_1 g\theta = 2\omega^2\xi_o Q_1 \cos\omega t \qquad (3.14)$$

where, $M^* = M + 4/3\ \rho[r_1^3 + (r_1 - r_2)^3]$

Surge Equation 3.12:

Let $P_1\ddot{\theta} << 2M\ddot{x}_c$ and $x = C_1 \sin\omega t + C_2 \cos\omega t + C_3$. Now differentiating x twice and substituting in (3.12) we obtain

$$x = -\xi_o Q_o \cos\omega t/M + C_3$$

and $$\dot{x} = \omega\xi_o Q_o \sin\omega t/M + C_3 t$$

Hence $$\dot{x}(o) = 0$$

and $$x(o) = -\xi_o Q_o/M$$

Heave equation (3.13)

Let $z = C_1 \sin\omega t + C_2 \cos\omega t + C_3$

Differentiating twice and substituting in equation (3.13) we obtain:

$$C_2 = C_3 = 0$$

and $$C_1 = \xi_o(\rho S_{o2} g - \omega^2 Q_o)/(\rho S_{o2} g - M^* \omega^2)$$

Hence, $$z(o) = 0$$

$$\dot{z}(o) = \omega C_1$$

Pitch Equation 3.14:

Substituting $\ddot{x}_c$ from equation (3.12) into equation (3.14) we obtain:

$$A\ddot{\theta} + B\theta = C \cos\omega t \qquad (3.15)$$

where, $$A = (I_{yy} + P_2 - P_1^2/2M)$$

$$B = P_1 g$$

and $C = (2\omega^2 \xi_o Q_1 - P_1 \omega^2 \xi_o Q_o / M)$

Now let $\theta = C_1 \sin\omega t + C_2 \cos\omega t + C_3$

Differentiating θ twice and substituting in equation (3.15) gives:

$$C_1 = C_3 = 0$$

and $C_2 = C/(B - A\omega^2)$

Or, $$\theta = \frac{C}{B - A\omega^2} \cos\omega t$$

Hence,

$$\theta(o) = \frac{\omega^2 \xi_o (2Q_1 - P_1 Q_o / M)}{P_1 g - \omega^2 (I_{yy} + P_2 - P_1^2 / 2M)}$$

and $\dot{\theta}(o) = 0$

Three Dimensional Analysis:

Similarly; for the three dimensional case with wave direction at an angle β with the x-axis we have:

$$x(o) = - \xi_o \cos\beta \, Q_o/M$$

$$y(o) = - \xi_o \sin\beta \, Q_o/M$$

$$z(o) = 0.$$

$$\dot{x}(o) = \dot{y}(o) = 0$$

$$\dot{z}(o) = \frac{\omega\xi_o(\rho S_{o2} g - \omega^2 Q_o)}{\rho S_{o2} g - M^* \omega^2}$$

$$\phi(o) = - \frac{\omega^2 \xi_o \sin\beta (2Q_1 - P_1 Q_o/M)}{P_1 g - \omega^2 (I_{yy} + P_2 - P_1^2/2M)}$$

$$\theta(o) = \frac{\omega^2 \xi_o \cos\beta (2Q_1 - P_1 Q_o/M)}{P_1 g - \omega^2 (I_{yy} + P_2 - P_1^2/2M)}$$

$$\psi(o) = 0$$

$$\dot{\phi}(o) = \dot{\theta}(o) = \dot{\psi}(o) = 0.$$

3.3 Drifting Drogued Spar Buoy

In this analysis the drogue and the mooring line connecting it to the spar buoy are modeled dynamically as a lumped parameter system.

Two-Dimensional Analysis

Static Solution:

The pertinent static equations for the spar buoy (Section 2.1) with an attached mooring line (Section 2.3) are:

$$DF_x + T_x = 0 \tag{3.16}$$

$$P_o g - Mg + DF_z + T_z = 0 \tag{3.17}$$

and

$$-P_l g \sin\theta + DM - T_z(x'_a \cos\theta + z'_a \sin\theta) + T_x(z'_a \cos\theta - x'_a \sin\theta) = 0 \tag{3.18}$$

Here; we have three equations and five (V_s, h_s, T_x, T_z, θ) unknowns where V_s is the static velocity of the drogued drifting system. These equations are solved by iterating on V_s and θ. Assuming a value for V_s, T_x and T_z can be calculated by known weights, buoyancies, viscous drag forces, and elastic characteristics of the system beneath the spar. Now if a value for θ is assumed equation (3.17) is solved for h_s and the error in equation (3.18) is found. This error is minimized by iterations on θ. Next V_s is iterated upon to satisfy equation (3.16).

Dynamic Solution

This solution is similar to the lumped parameter solution discussed in Section 3.1.

Three Dimensional Analysis

Static Solution:

Here the pertinent equations (static) remain the same as Section 3.1 expect for replacing $-\overline{T}$ by $+\overline{T}$ and $+\ \overline{TM}$ by $-\overline{TM}$.

Also; following similar reasoning of small angles $\overline{T}$, ϕ, θ, and ψ can be solved explicitly from equations (3.10) and (3.11) with new $\overline{T}$ and $\overline{TM}$ if V_s is known. V_s is determined by iteration on the equation similar to (3.10).

Dynamic Solution

This solution is similar to the one in Section 3.1 .

4.0 COMPUTER PROGRAM DETAILS

This section will describe the various computer programs written for the theoretical analysis presented in Section 2 and using solution methods of Section 3. Description of the programs is general and specific details can be found in the program listings presented in Appendix A. All computer programs are written in FORTRAN IV and have been run for many case studies on the CSDL AMDAHL 470 V6 computer. Input data required for these programs is also explained in this section. Some of the simulations, along with the accompanying input data, are presented in Section 5.

Computer programs written for the surface moored and drifting systems are tabulated in Table 4.1. Programs for subsurface moored systems are tabulated in Table 4.2. Program listings for some of these computer programs are presented in Appendix A. Listings not presented are duplications of what is presented and can be obtained after minor modifications.

4.1 Surface Moored/Drifting Systems

Computer program SD3.FORT is coded for the three-dimensional analysis of surface moored and drifting systems presented in Section 2. Four case studies can be simulated by this program:

1. A freely floating spar buoy.

FILE NAME	MOORING SYSTEM	DEGREES OF FREEDOM	PHYSICAL FORCES	INPUT ENVIRONMENT (Forcing Function)
SD3.FORT	Cylindrical spar buoy analysis. Tuned spar buoy analysis. Cylindrical or tuned spar buoy anchored by a mooring line. Cylindrical or tuned spar buoy anchored by a mooring line and an instrument line hanging from the buoy. Drifting drogued buoy (cylindrical or tuned spar) analysis.	Surface buoy-six Mooring line-three	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
SD2.FORT	Same as above	Surface buoy-three Mooring line-two	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
BUOY.FORT	Cylindrical spar buoy analysis. Tuned spar buoy analysis.	Buoy-six	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
MDE.FORT	Cylindrical or tuned spar buoy anchored by a mooring line and an instrument line hanging from the buoy.	Surface buoy-six Mooring line-three	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
DDB.FORT	Drifting drogued buoy analysis (cylindrical or tuned spar)	Surface buoy-six Mooring line-three	All applicable forces of Section 2.	Surface wave and constant absolute current profile.

TABLE 4.1 Programs for Surface Moored/Drifting Systems

FILE NAME	MOORING SYSTEM	DEGREES OF FREEDOM	PHYSICAL FORCES	INPUT ENVIRONMENT (Forcing Function)
SSD3.FORT	Subsurface moored system.	Three	All applicable forces from Section 2.	Time varying relative current profile.
SSD31.FORT	Subsurface moored system.	Three	All applicable forces from Section 2.	Time varying absolute current profile.
SSS3.FORT	Subsurface moored system.	Three	All applicable forces from Section 2 except inertia forces.	Time varying relative current profile.

TABLE 4.2 Programs for Subsurface Moored Systems

2. A spar buoy anchored with a lumped parameter mooring line.
3. A spar buoy anchored with a lumped parameter mooring line and a lumped parameter mooring line hanging from the buoy.
4. A spar buoy attached to a window shade drogue.

SD2.FORT does the same simulations as SD3.FORT, but is coded for the two dimensional analysis of Section 2. Computer programs BUOY.FORT, MDE.FORT, and DDB.FORT are subsets of the computer program SD3.FORT. BUOY.FORT simulates case study 1, MDE.FORT simulates case study 3, and DDB.FORT simulates case study 4. All these programs use the lumped parameter formulation for the mooring line. Input data necessary for these programs is outlined below.

Input Data Required for Computer Program SD3.FORT (All units in F.P.S.)

General data:

NM: Number of nodes (masses). Equals one for case study 1.

NB: Number of the surface buoy. Starting from the anchor, the number of the node (anchor is not counted as a node) where

the buoy is located. NM = NB for case study 3. NB equals one for case studies 1 and 4.

ND: Number of the first node on the drogue.

NST: Number of steps. This is used for dividing the spar buoy in NST strips for integration of viscous drag forces and moments over the entire submerged buoy.

NC: Number of cycles, i.e. number of surface waves for which the simulations are to be done.

MOOR: Index for case study control. Equals zero for case study 1; one for case studies 2 and 3, and two for case study 4.

DEPTH: Ocean depth.

DT: Integration time step ΔT.

TMAX: Maximum time of simulation.

T2: Time interval for simulation printout.

Buoy Data:

RD1,RD2: Two radii (r_1 and r_2) of the tuned spar buoy. RD1 is for the drum and equals RD2 for a cylindrical spar.

ZCG: Distance ($Z_c > 0$) between centeroid of the spar base and its C.G.

RGYR: Radius of gyration of the buoy about any axis in the horizontal plane.

HMAX: Length of the spar buoy (H).

CDL: Normal viscous drag coefficient (C_{DN}) used for the spar. πC_{DT} is found by multiplying CDLx0.02

HST: Length of the drum for the tuned spar buoy (h_1). For cylindrical spar HST is some arbitrary number between zero and the submerged depth of the spar.

CON: Added mass coefficient (C_N) for the buoy. (Used & read in SD2.FORT only).

COT: Added mass coefficient (C_T) for the buoy. (Used & read in SD2.FORT only).

CDP: Viscous drag coefficient for the base and step of the tuned spar (C_{DP}).

ALPHA: Added mass coefficient for the base and step of the tuned spar (α).

Mooring Line Data:

W(I): Weight in water ($\omega_{\omega i}$) of components lumped at the I^{th} node. Equals weight in air for the buoy (NB^{th} node)

CM(I): Mass of components lumped at the I^{th} node (m_i).

CMS(I): Added mass of spherical components lumped at the I^{th} node (m_{ai}).

CMD(I): Mass of water displaced by components lumped at the I^{th} node (m_{di}).

CMN(I), CMT(I): Normal and tangential components of added mass of cylindrical components lumped at the I^{th} node (m_{ni} and m_{ti}).

SL(I): Slack length of the mooring line proceeding the I^{th} node (L_i).

EK(I): Elastic coefficient for SL(I) (k_i).

CDIN(I), CDIT(I): Normal and tangential viscous drag constants ($1/2\ \rho C_D A_D$) of cylindrical components lumped at the I^{th} node.

CDIA(I): Viscous drag constant of non-cylindrical components lumped at the I^{th} node.

DEP(I): Depth (less than zero) of the I^{th} node (approximate) (z_i).

Attachment Data:

XC1,YC1, ZC1: Coordinates of the attachment point between spar and the anchoring line in spar buoy coordinate frame (x', y', z').

XC2,YC2, ZC2: Coordinates of the attachment point between the spar and the instrument line in x', y', z' frame.

Current Profile Data:

V(IJ,I): Absolute velocity of water in J^{th} (x, y, and z) direction at the depth of I^{th} node.

Surface Wave Data:

WE: Frequency (ω) of the surface wave.

AMP: Amplitude (ξ_o) of the surface wave.

BETA: Wave direction (β).

All of the input data are read in the main program. The main program calls subroutines: (a) STATIC; which calculates the mean configuration or steady state (static) solution without any surface wave forcing. (b) BUOYS; which calculates the mass matrix [M] for the spar buoy. (c) DBDRG, which calculates the viscous drag

forces/moments on the spar buoy. (d) MINVER, which inverts a matrix. (e) MATRIX, which calculates the wave exciting plus hydrostatic forces on the spar buoy, the stiffness matrix [K], the remaining mass matrix for the mooring lines, viscous drag forces on the mooring lines, Froude Krylov forces on the mooring lines, and the tension force components at each node. It also integrates the differential equations of motion. (f) NEXT, which updates the geometry of the system.

Before finding the steady state dynamic response of a system to a sinusoidal surface wave, an equivalent current profile has to be determined to compute the mean configuration of the system. This current profile depends on the original current profile plus the contribution due to the surface wave. In effect it is a rectification of the time constant current profile by the oscillatory velocity field generated by the surface wave. From Section 2.1 we have:

$$\bar{V}_{Ew} = \hat{i}_x(V_{ox} + \Phi_x) + \hat{i}_y(V_{oy} + \Phi_y) + \hat{i}_z(V_{oz} + \Phi_z)$$

also; $$|\bar{V}_{Ew}|^2 = (V_{ox} + \Phi_x)^2 + (V_{oy} + \Phi_y)^2 + (V_{oz} + \Phi_z)^2$$

And as viscous drag forces determine the moored or drifting system configurations for the static solution, and these are proportional to $|\bar{V}_{Ew}|\bar{V}_{Ew}$, the square root of mean ($|\bar{V}_{Ew}|\bar{V}_{Ew}$) over a wave period (T) would give the equivalent rectified current vector at this location.

Let;
$$\bar{E} = \frac{1}{\omega T}\int_{o}^{\omega T} |\bar{V}_{Ew}|(\bar{V}_{Ew})\,d(\omega t)$$
$$= \hat{i}_x E_x + \hat{i}_y E_y + \hat{i}_z E_z$$

Now the equivalent current vector can be written as:

$$\bar{V}_{\varepsilon q} = \bar{E}/EE$$

where:
$$EE = \sqrt{|\bar{E}|}$$

This calculation to find $\bar{V}_{\varepsilon q}$, before a static solution is performed, is done in the main program. The numerical integration of differential equations in the subroutine MATRIX was performed by the rectangular rule, which was found adequate for all case studies simulated.

4.2 Subsurface Moored Systems

Computer program SSD3.FORT is coded for the three dimensional analysis of subsurface moored systems

(continuous line formulation) given in Section 2. The program simulates the response of a subsurface moored system to the time varying current fields. SSD3.FORT reads in the relative (relative to the mooring system) velocity profile in the three orthogonal x, y, and z directions. From these three components the viscous drag forces can be computed directly. These components of the relative velocity could be the ones measured by current sensors on the mooring line. Alternatively, SSD31.FORT reads in the absolute (relative to the ocean floor) velocity profile in the three orthogonal (x, y, and z) directions. Profile is read in as amplitudes of the sinusoidally varying currents. All components (varying with depth) are driven by a single frequency. In addition a surface wave of given amplitude, frequency, and direction generates velocity field exponentially attenuated in depth. The program computes the total absolute velocity and then substracts the mooring line velocity from this absolute velocity to compute the viscous drag forces. SSS3.FORT is the same as SSD3.FORT but it neglects all inertia forces of the mooring system. All three programs use the continuous line formulation of the mooring line, given in Section 2, as the mathematical model. Input data, except

for the input forcing function (velocity profiles and surface wave) are the same for all three programs and are detailed below:

General Data:

NI: Number of intermediate bodies (instrument clusters or inserted floats) to be included in the analysis.

NP: Number of mooring parts (each mooring part can have different properties).

NS: Number of segments the continuous line is to be divided in.

NPT: Number of dynamic equilibriums to be performed.

IKK: Number of locations the velocity profile changes along the ocean depth.

DEPTH: Depth of the ocean. (Meters)

DDT: Approximate depth of the mooring line top. (Meters)

ER: Used for iteration on the boundary condition of ocean depth. If DDT is known with reasonable accuracy, ER is not required. (Meters)

ER1: Used for error control in SSD31.FORT.

Instrument Data:

PI(J) Position of the J^{th} instrument in slack length distance from the top. (Meters)

SI(J): Length of the J^{th} instrument. Equals zero for non-cylindrical instruments. (Meters)

ZM(J): Mass of the J^{th} instrument (m_j). (Slugs)

ZMC(J): Normal added mass component (m_{nj}) for cylindrical instruments. (Slugs)

ZMV(J): Added mass (m_{aj}) of the J^{th} instrument for a non-cylindrical instrument. Tangential added mass component (m_{tj}) for cylindrical instruments. (Slugs)

F(3,J): Weight in water (W_{wj}) of the J^{th} instrument. (Pounds)

CDIN(J), CDIT(J): Normal and tangential viscous drag constants ($1/2\ \rho C_D A_D$) of the instruments. CDIT is not used for spherical instruments. (F.P.S. units)

Mooring Line Data:

DIAL(I): Nominaldiameter of the I^{th} mooring part. (Inches)

SLL(I): Slack length of the I^{th} mooring part. (Meters)

AWL(I): Weight in air per unit slack length of the I^{th} mooring part (W_a). (lbs/M)

WWL(I): Weight in water per unit slack length of the I^{th} mooring part (W_w). (lbs/M)

TPL(I): Transient peak load on the I^{th} mooring part. For materials which obey Hook's law this is substituted with the Young's modulus of elasticity. (Refer volume 1 CHHABRA, 1973). (lbs. or lbs/in^2)

CO1(I), CO2(I), PO1(I), PO2(I), AO(I): Constants for stress-strain relationships of mooring line materials. (CHHABRA, 1973). Co1 is used for jacket diameter (in.) in jacketed wire rope.

CDN(I), CDT(I): Normal and tangential drag coefficients of the I^{th} mooring part. (C_{DN} and C_{DT})

Current Profile Data:

D(I): Depths, greater than zero, where the current changes direction. I goes from 1 to IKK. (Meters)

V(I,J): For SSD3.FORT: Relative velocity in the I^{th} (x, y, and z) direction for the J^{th} zone. J goes from 1 to IKK + 1. (mm/sec)

For SSD31.FORT: Absolute velocity amplitude in the I^{th} direction for the J^{th} zone. Following data are used only in SSD31.FORT.

WE: Frequency of current profile amplitudes V(I,J). (Rad/sec)

DT: Time step for calculation of the mooring system dynamic equilibrium.

T2: Time step for simulation output.

TMAX: Maximum time for simulation.

WS: Surface wave frequency (ω). (Rad/sec)

AMP: Surface wave amplitude (ζ_o). (mm)

BETA: Surface wave direction (β). (Radians)

All of input data are read in the main program. The main program calls subroutine MOTION which calculates the dynamic equilibrium of the subsurface mooring system at discrete times. Subroutine MOTION calls subroutine FORCES, whenever an instrument package or a subsurface float is encountered in the mooring line incremental integration scheme. Both MOTION and FORCES call subroutine SPEED to find the current for viscous drag calculations.

5.0 CASE STUDIES/SIMULATIONS

This section presents computer simulations of specific case studies; using the computer programs outlined in Section 4. All simulations are done in the time-domain. Simulations of the freely floating spar buoy, as presented in Section 5.1, were combined to obtain the response amplitude operator (RAO) vs Kh (wave number multiplied by the appropriate buoy draft). Section 5.2 presents the subsurface moored system simulations. Section 5.3 gives simulations of a surface moored system, and Section 5.4 presents simulations of a drifting drogued buoy.

5.1 Spar Buoy

Both the two-dimensional computer program (SD2.FORT) and the three-dimensional computer program (SD3.FORT) are used to simulate each of the two spar buoys; one cylindrical, and the other tuned presented in Sections 5.1.1 and 5.1.2 respectively. RAO vs Kh plots are presented for the two-dimensional simulations of cylindrical and tuned spar buoys.

5.1.1 Cylindrical Spar

The spar buoy (Figure 5.1) used in these simulations was tested in a series of wave tank tests to

derive empirical response data for comparison with the computer simulations. This comparison has not been presented in this report. The dimensions

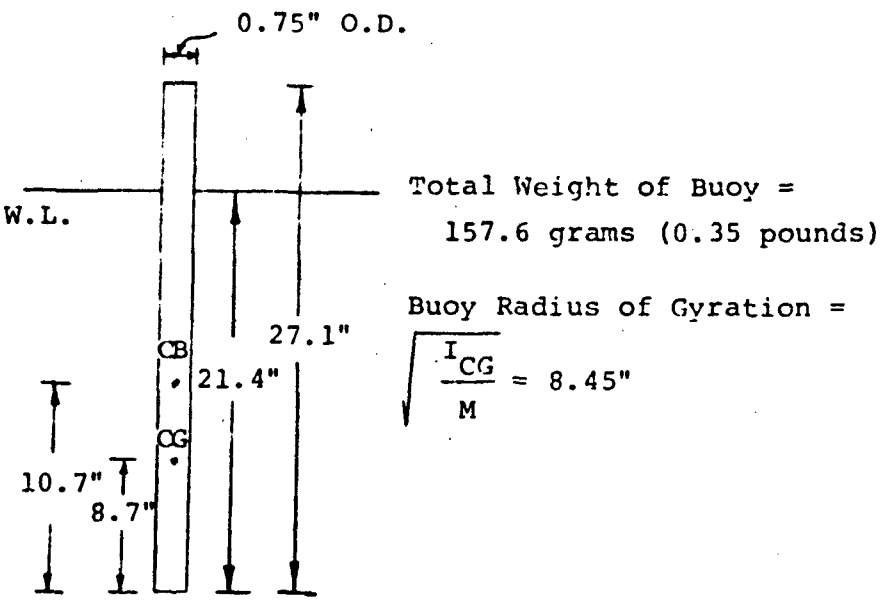


Figure 5.1 Scale Model Spar Buoy Description

and characteristics of this scale model spar buoy are shown in Figure 5.1.

Two Dimensional Simulations

Figures 5.2, 5.3, and 5.4 present the typical surge, heave, and pitch response of the buoy to surface waves of

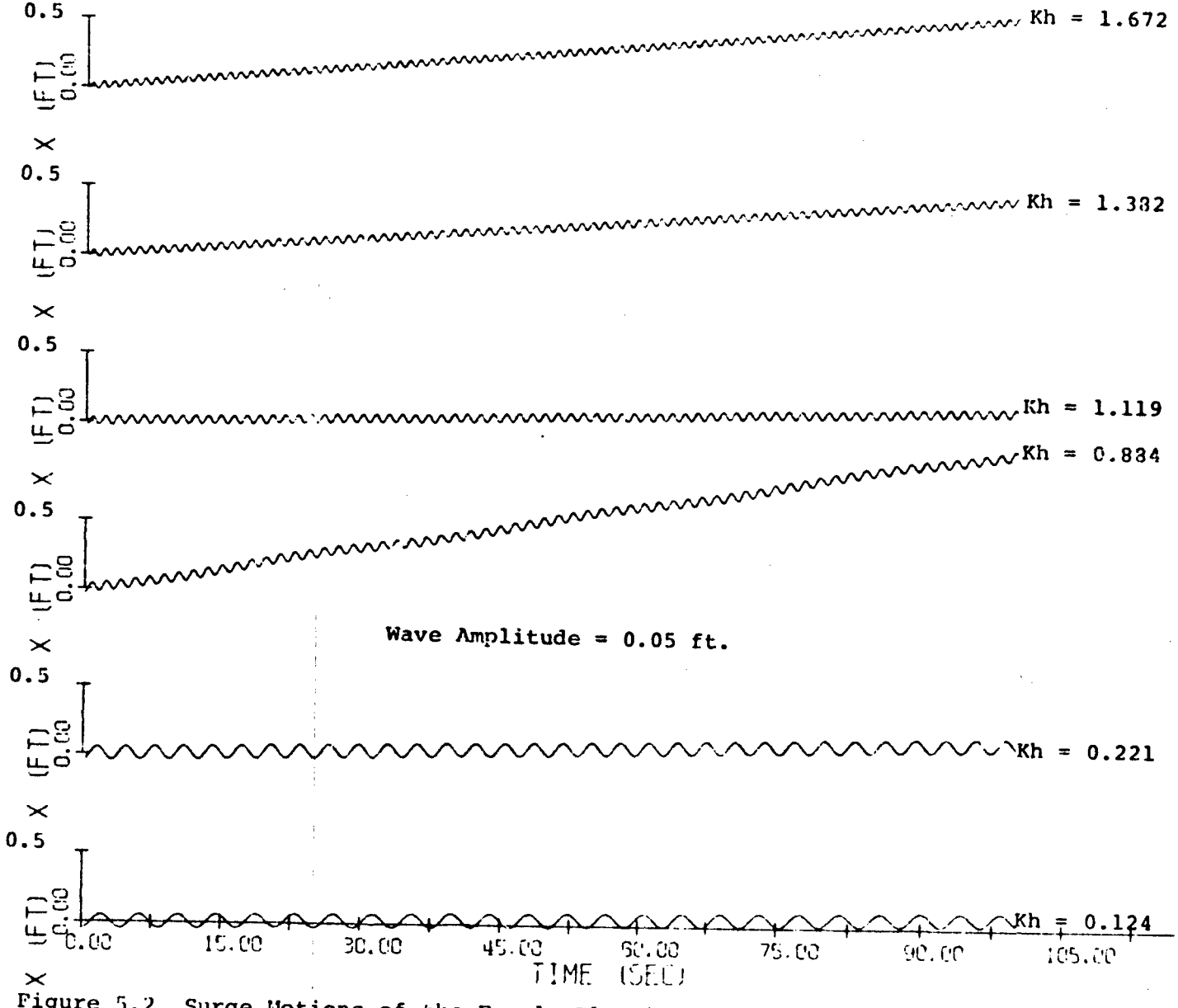


Figure 5.2 Surge Motions of the Freely Floating Cylindrical Spar Buoy

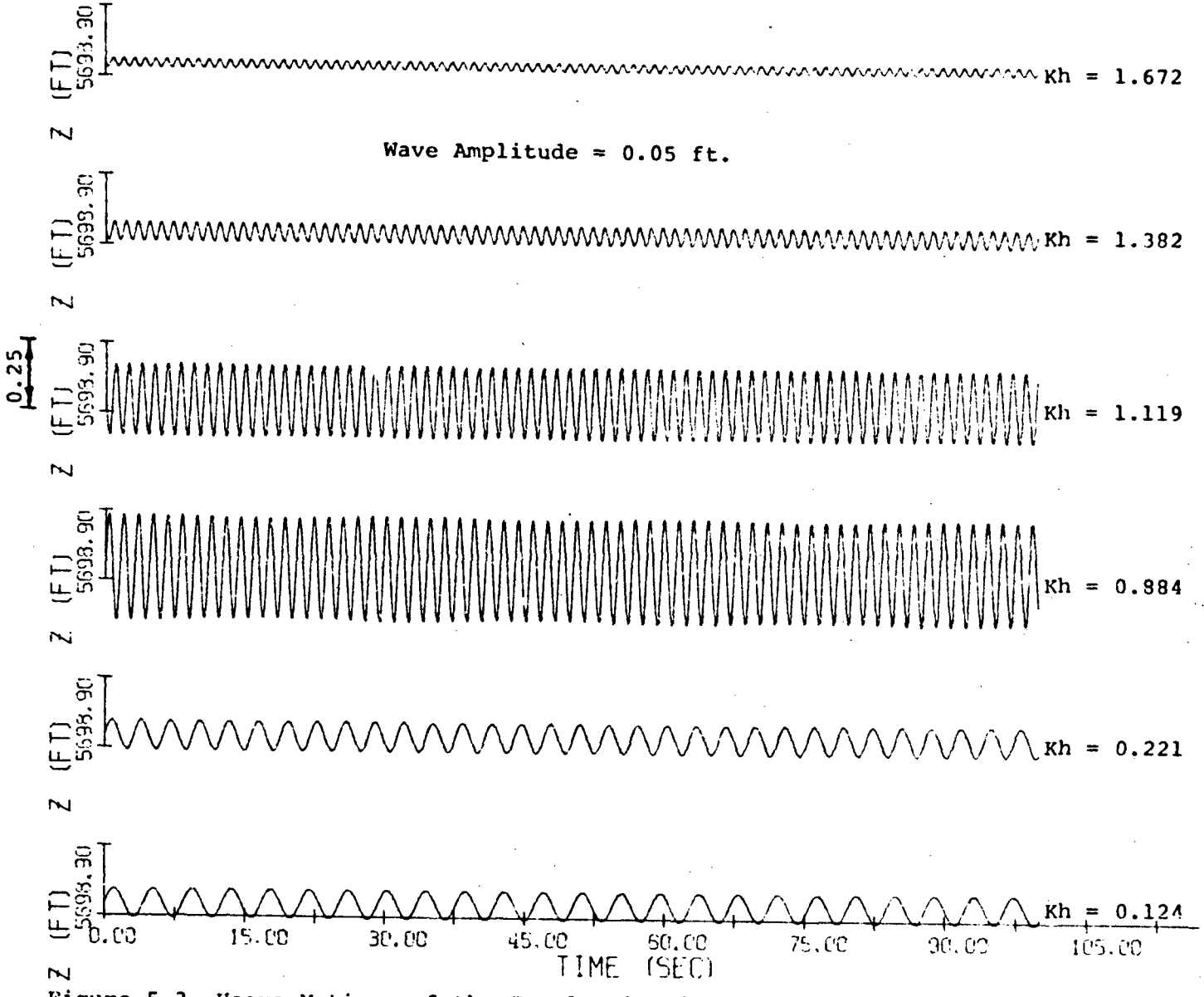


Figure 5.3 Heave Motions of the Freely Floating Cylindrical Spar Buoy

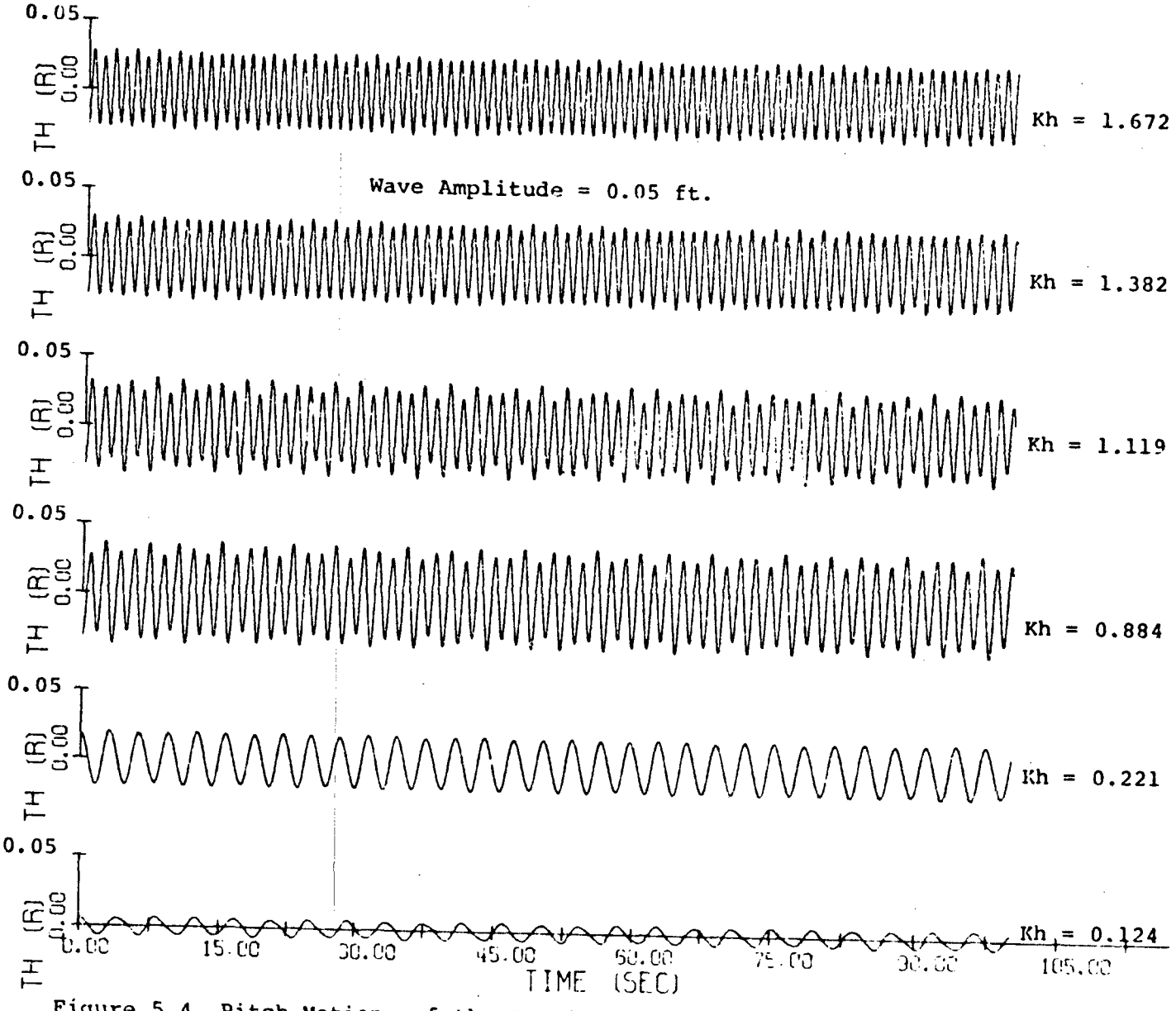


Figure 5.4 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

amplitude 0.05 ft. and different frequencies. Table 5.1 is the input data used in conjunction with the computer program SD2.FORT to obtain these simulations. Noteworthy is the fact that the viscous (CDL) and pressure (CDP) drag coefficients in this study equal zero. Surge drift evident in Fig. 5.2 is due to the bias or rectification of wave exciting forces, which would be present for example if an oscillatory hydrodynamic force were to act on a heaving body whose motions were at varying with this force. Simulations for the same data as Table 5.1 and viscous and pressure drag coefficients equal to 1.0 are shown in Figures 5.5, 5.6, and 5.7. Comparison between the surge motions of Figures 5.2 and 5.5 shows the effect of viscous drag on surge drift. Surge drift due to viscous drag occurs as the submerged area of the spar and its inclination vary over a wave period, and is a function of their phase relationships. This drift is most predominant near resonance. In this case heave resonance occurs at Kh = 1.0, and pitch resonance occurs at Kh = 0.4. Heave motions (Figures 5.3 and 5.6) are not effected much by viscous drag because the tangential drag due to CDL and CDP is comparitively small. Comparison of Figures 5.4 and 5.7 shows that the system frequency present in the top four curves of Figure 5.4, due to approximate initial conditions, is damped out in Figure 5.7 due to viscous drag damping. Otherwise, the amplitudes do not show much change.

```
 SD2C.DATA
00010      1     1     0    25     6     0 5700.      0.020     100.0       0.1
00020 0.03125    0.03125    0.725      0.704      2.258      0.0        1.1
00021  1.0         0.0        0.0        1.0
00055  0.35        0.         0.         0.         0.         0.
00090  0.          0.         0.         0.         0.         -18.
00105 0.          0.         0.         0.
00110 0.0
00120  1.50        0.05
00130  2.00        0.05
00140  4.00        0.05
00150  4.50        0.05
00160  5.00        0.05
00170  5.50        0.05
READY
```

Table 5.1 Input Data for the 2-D Analysis of the Cylindrical Spar Buoy

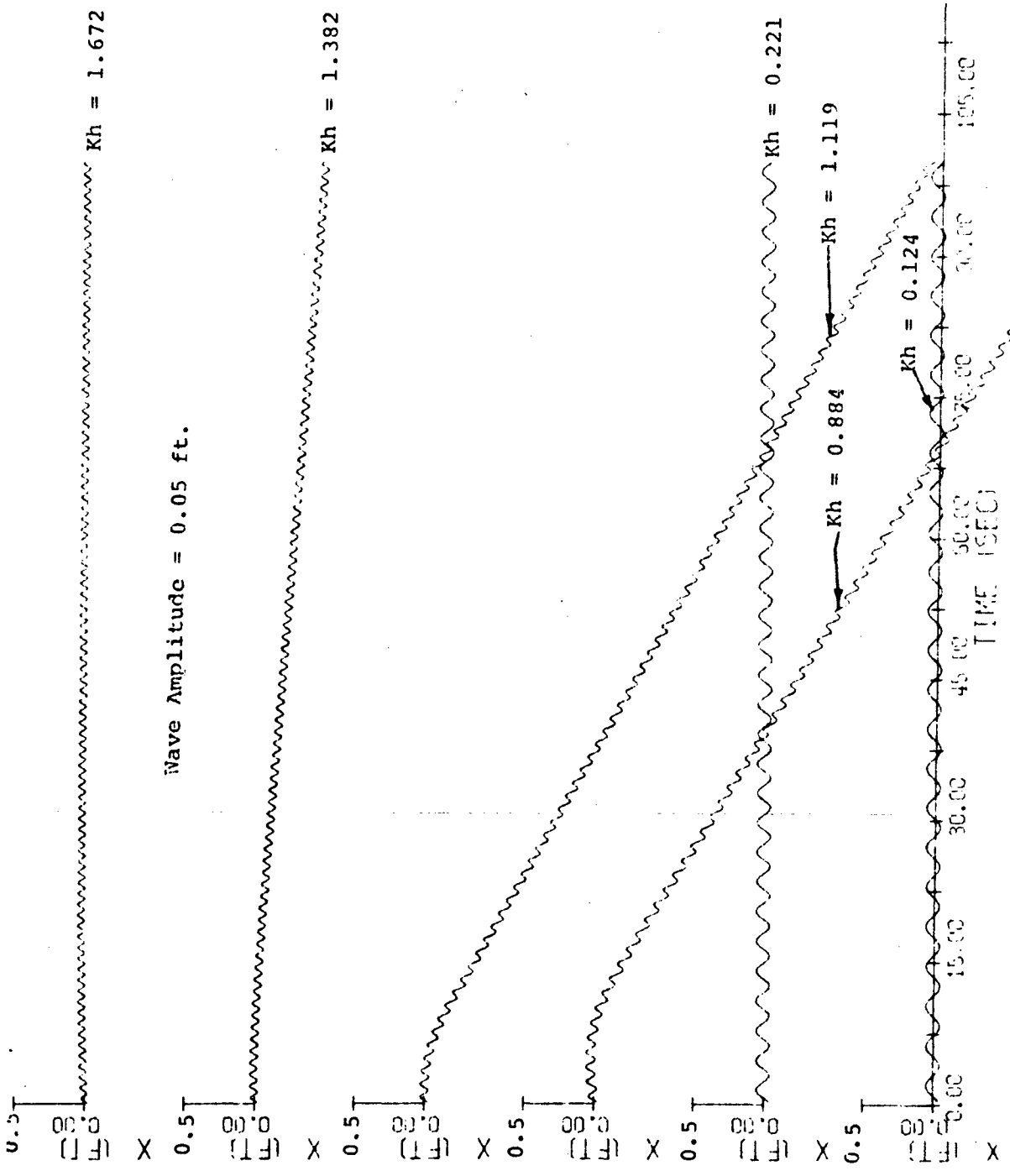


Figure 5.5 Surge Motions of the Freely Floating Cylindrical Spar Buoy

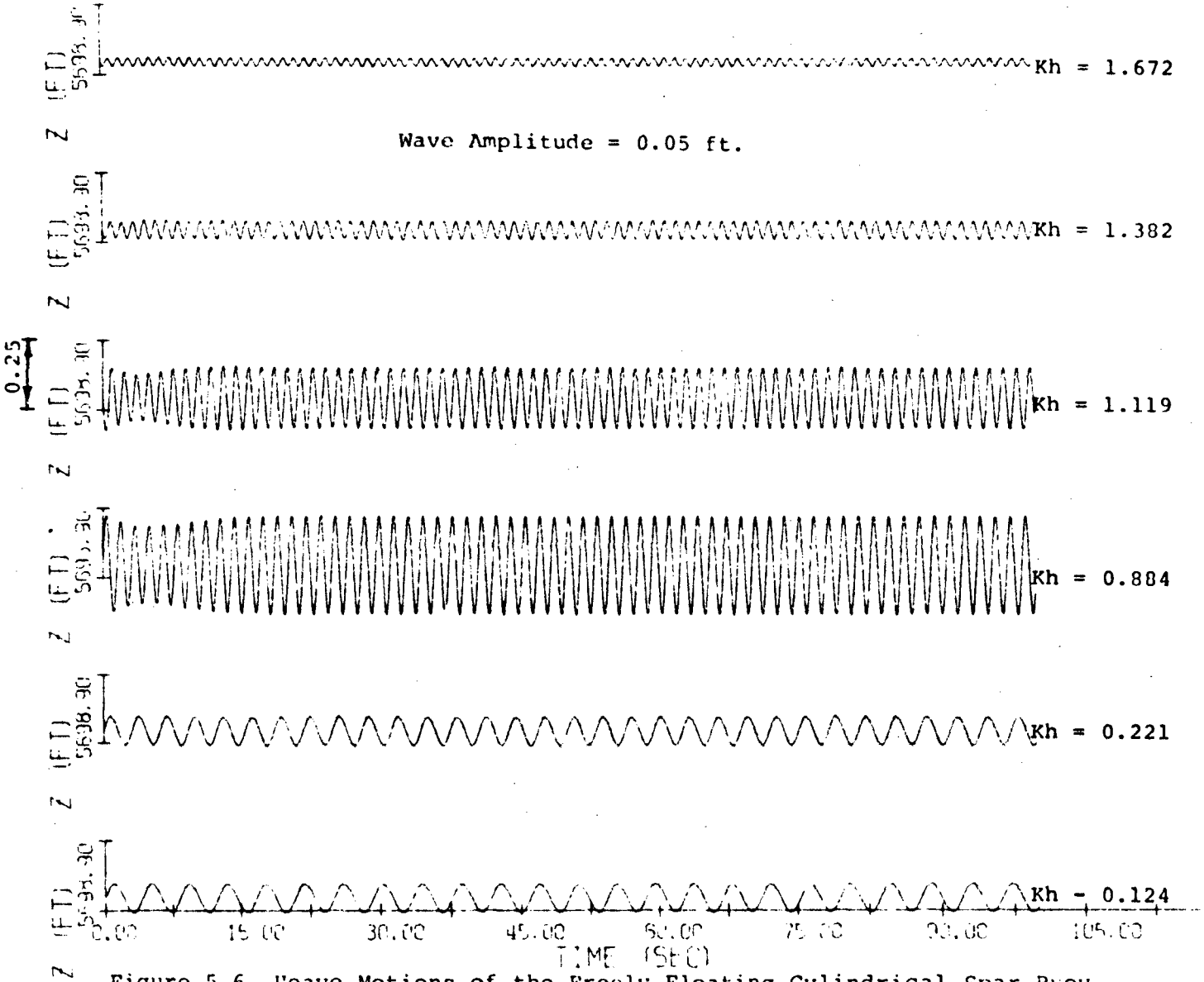


Figure 5.6 Heave Motions of the Freely Floating Cylindrical Spar Buoy

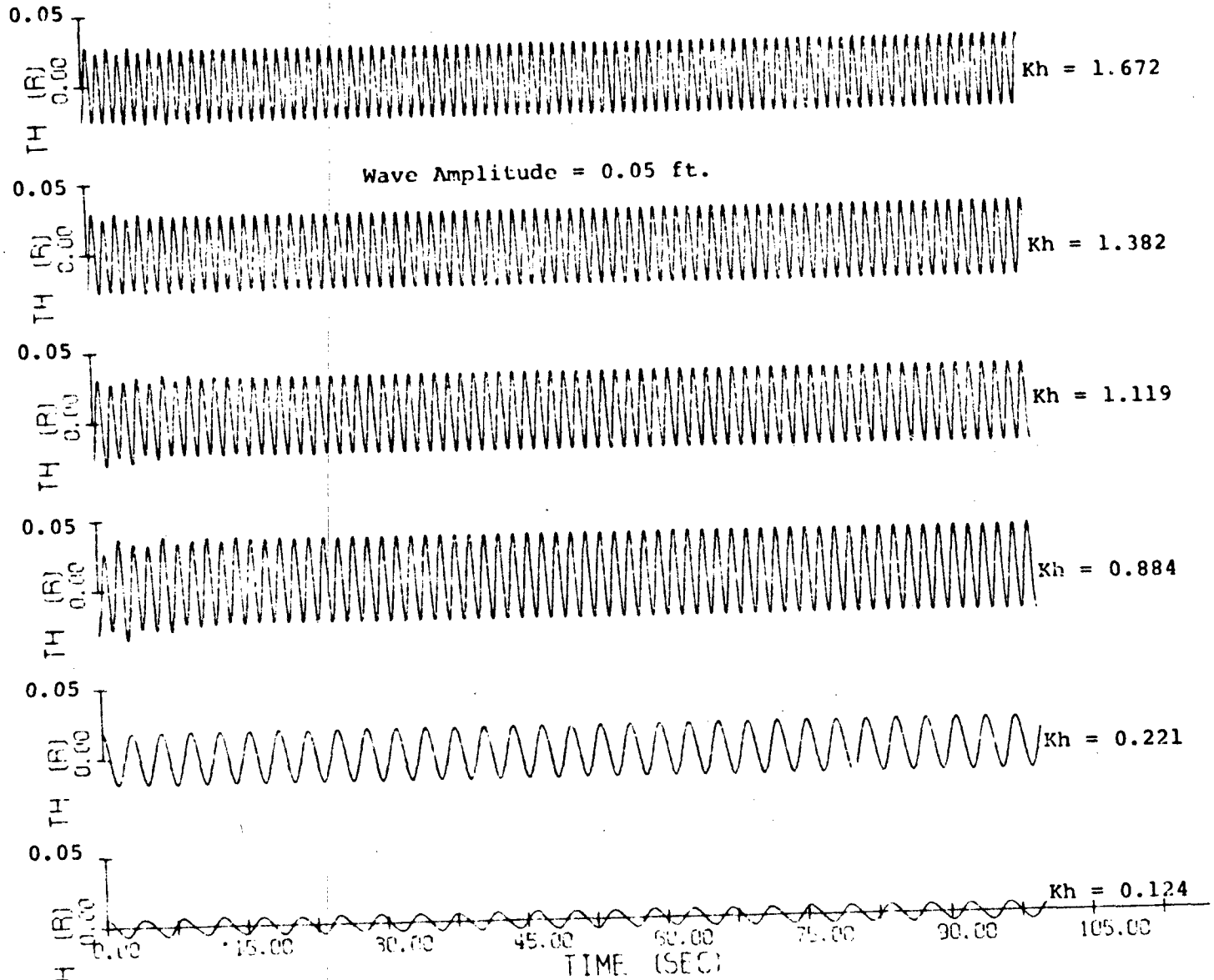


Figure 5.7 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

Response amplitudes of surge, heave, and pitch; and surge drift as calculated from the steady state portions of the times histories presented in Figures 5.2 through 5.7, and other similar simulations are tabulated in Table 5.2. In this table column 1 tabulates the frequencies (ω) of waves studied. From these frequencies Kh is calculated and tabulated in column 2. Columns 3 through 6 are four columns of surge amplitude divided by ξ_o. The first of these four columns presents the results of the undamped linear analytical solution as derived in Section 3.2. The second presents the computer simulation results with ξ_o equal to 0.05 ft. and no viscous drag. Third and fourth present the computer simulation results with viscous drag and ξ_o equal to 0.05 ft. and 0.25 ft. respectively. The simulated plots are shown for two wave amplitudes to emphasize the nonlinear response obtained by the nonlinear theory. Similarly columns 7 through 10 tabulate four columns of heave amplitude divided by ξ_o, and columns 11 through 14 tabulate four columns of pitch amplitude divided by $K\xi_o$. $K\xi_o$ is the maximum wave slope. Analytical solution for surge drift was not found hence only three columns 15 through 17 are given for surge drift divided by $K\omega\xi_o^2$. $K\omega\xi_o^2$ is the mean "stokes drift" velocity at the mean free surface. Data points of Table 5.2 are plotted in Figures 5.8 through 5.11

ω	Kh	Surge Amplitude/ζ_o				Heave Amplitude/ζ_o				Pitch Amplitude/$K\zeta_o$				Surge Drift/$Kw\zeta_o^2$		
		1	2	3	4	1	2	3	4	1	2	3	4	2	3	4
1.50	0.12	-0.94	0.95	0.95	0.95	1.01	1.01	1.01	1.01	1.71	1.79	1.72	1.72	2.91	0.9[illegible]	0.94
2.52	0.35	-0.84		0.86	0.75	1.09		1.13	1.23	18.19		7.94	5.42		-6.37	-1.10
2.69	0.40	-0.82		0.72		1.12		1.16		-26.52		8.22			-6.31	
2.85	0.45	-0.81		0.68	0.70	1.17		1.17	1.22	-7.97		6.06	4.01		-4.11	-0.48
3.30	0.60	-0.75		0.67		1.39		1.39		-2.67		2.65			-0.61	
3.56	0.70	-0.72		0.66	0.65	1.69		1.68	1.62	-1.91		1.89	1.85		-0.43	0.13
4.00	0.88	-0.66	0.61	0.64	0.85	3.79	3.75	3.53	1.97	-1.28	1.26	1.22		2.36	-7.91	0.19
4.10	0.93	-0.65		0.53		6.18		4.23		-1.19		1.01			-11.88	
4.20	0.98	-0.64		0.53		21.35		4.30		-1.11		0.85			-12.45	
	1.00															
4.30	1.02	-0.63		0.51		-12.13		3.76		-1.04		0.86			-10.53	
4.40	1.07	-0.61		0.53		-4.39		2.96		-0.98		0.90			-7.58	
4.50	1.12	-0.60	0.54	0.57	0.62	-2.56	2.57	2.15	1.34	-0.92	0.86	0.89	0.45	0.20	-5.63	-0.38
4.67	1.21	-0.58		0.52		-1.40		1.35		-0.83		0.82			-2.47	
5.00	1.38	-0.54	0.50	0.43	0.50	-0.64	0.64	0.64	0.60	-0.70	0.70	0.70	0.66	0.47	-0.47	-0.07
5.50	1.67	-0.49	0.44	0.44	0.45	-0.27	0.27	0.28	0.28	-0.55	0.56	0.55	0.55	0.49	0.02	0.10
6.72	2.50	-0.37		0.32	0.33	-0.05		0.05	0.06	-0.34		0.34	0.35		0.15	0.16

TABLE 5.2 Dynamic Response (Surge, Heave and Pitch Amplitude; and Surge Drift) of a Cylindrical Spar Buoy to Waves

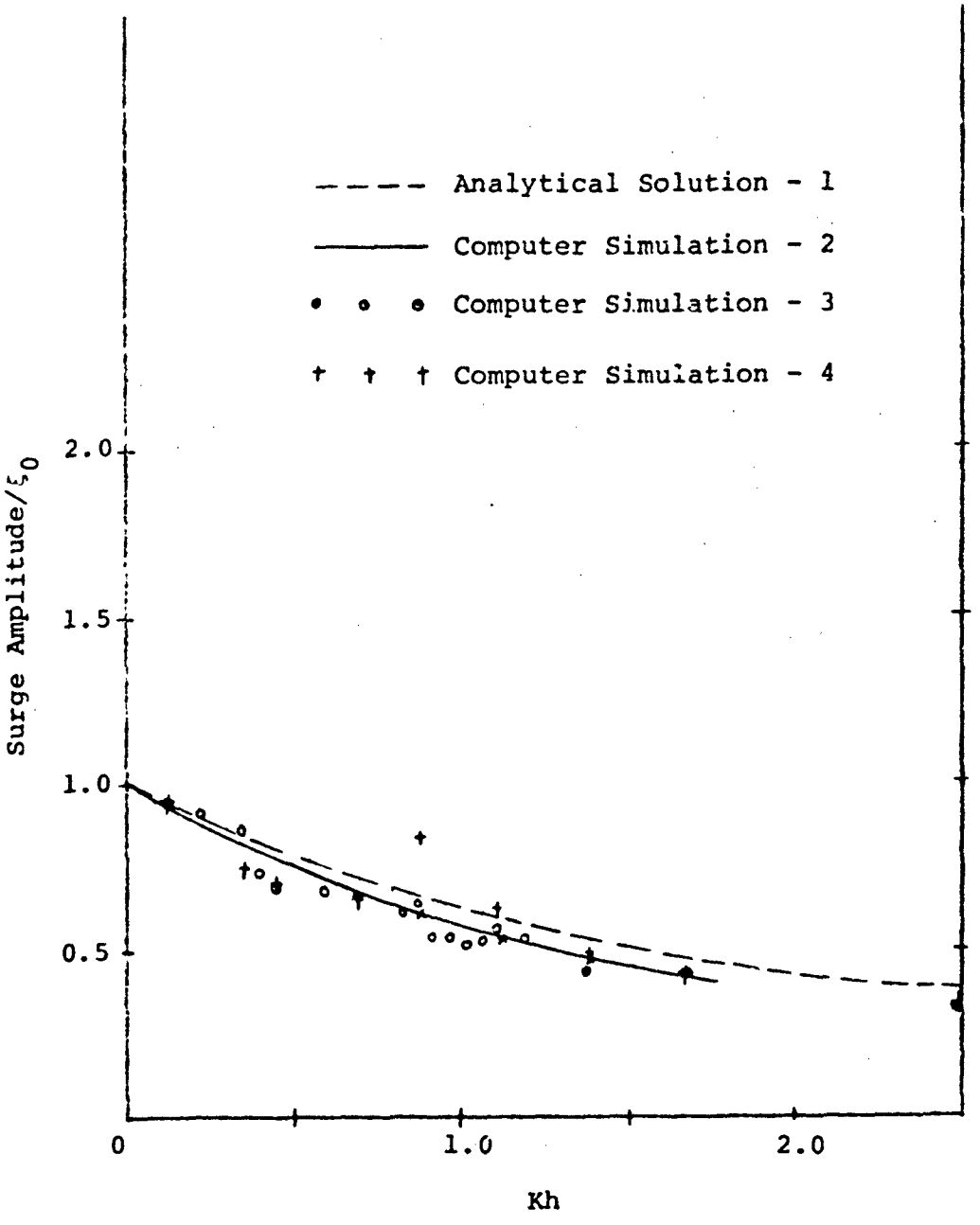


Figure 5.8 Surge Magnification Vs Kh

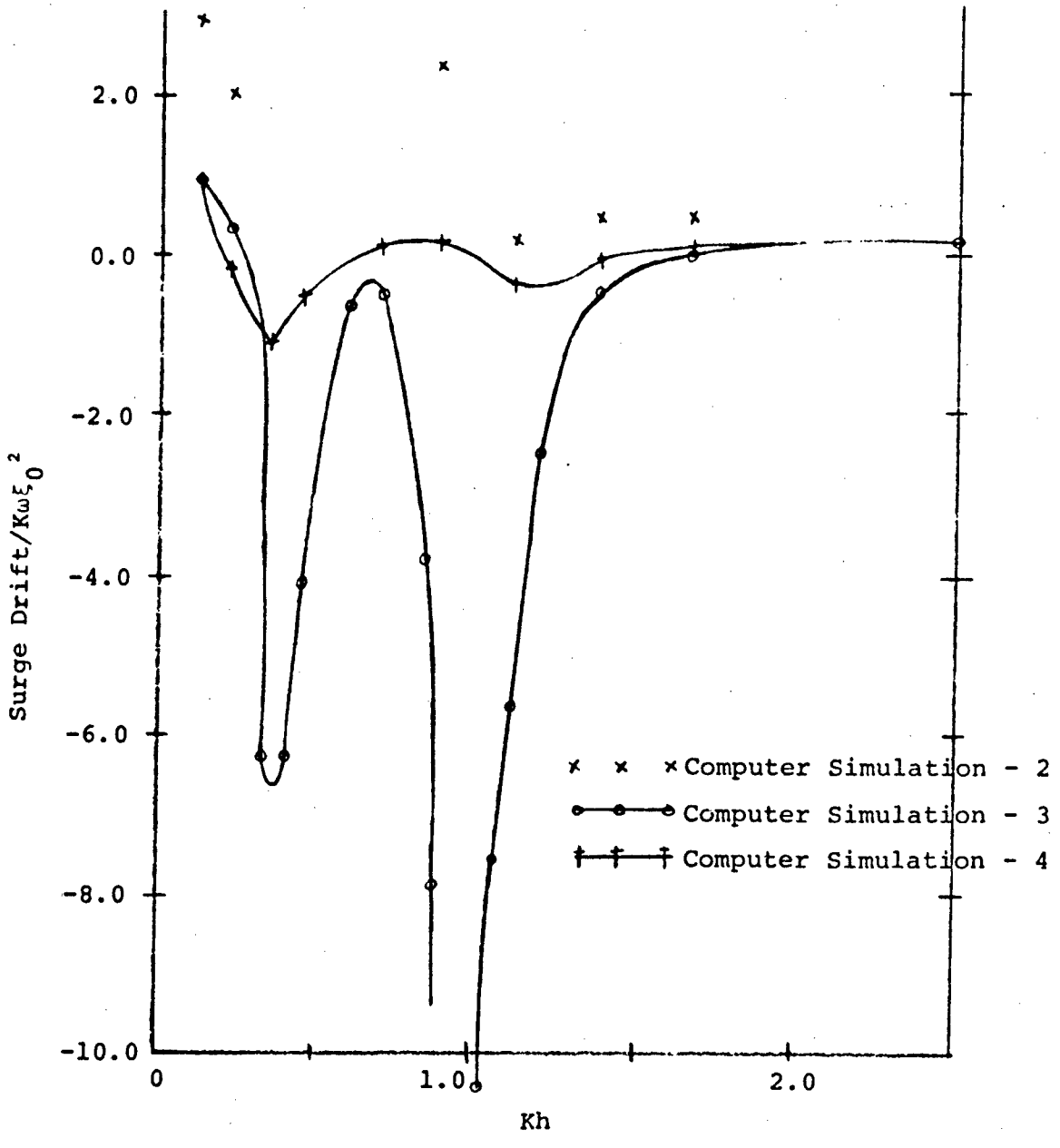


Figure 5.9 Surge Drift Vs Kh

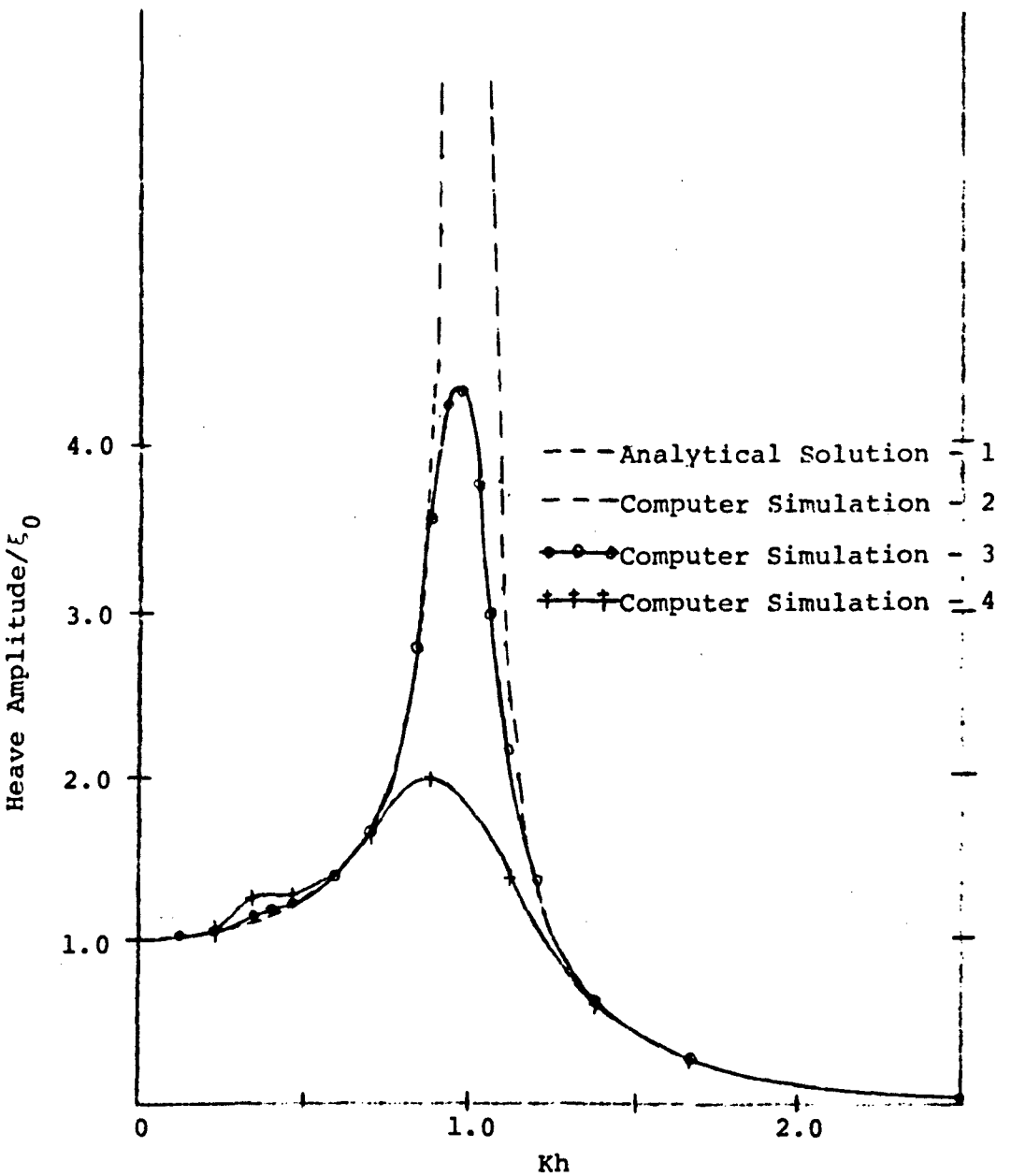


Figure 5.10 Heave Magnification Vs Kh

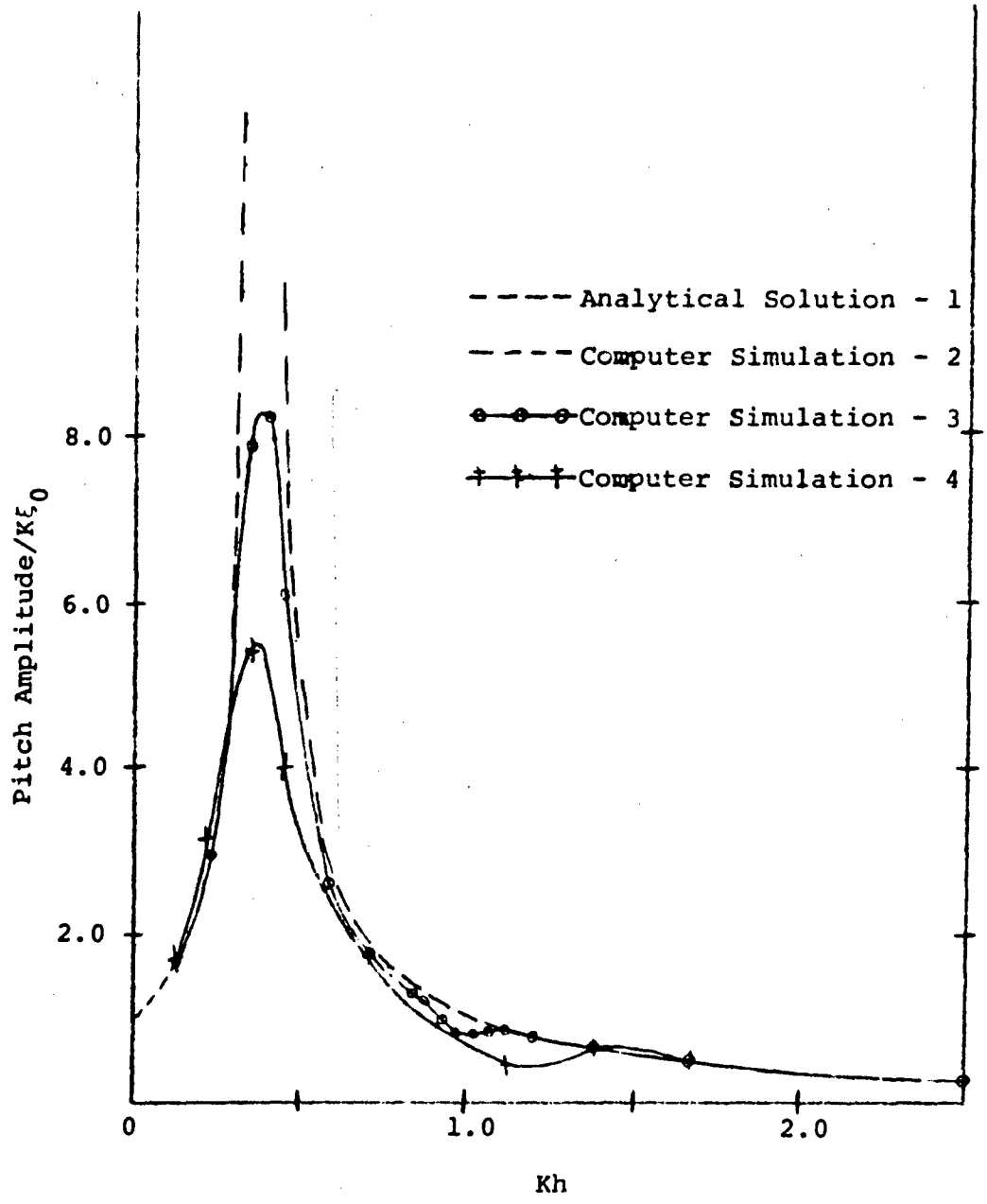


Figure 5.11 Pitch Magnification Vs Kh

to present plots of RAO vs Kh. Figure 5.8 shows the buoy surge per foot of wave amplitude as a function of Kh. For zero frequency this ratio is one and for increasing frequencies it decreases to zero. Figure 5.9 shows the buoy drift (in surge direction) per $K\omega\xi_o^2$ as a function of Kh. Figure 5.10 shows the buoy heave per foot of wave amplitude as a function of Kh. This ratio is one for zero frequency and exhibits resonance at Kh of approximately 1.0. The ratio decreases to zero with increasing frequencies. Figure 5.11 shows the buoy pitch per $K\xi_o$ as a function of Kh. This ratio is one at zero frequency and, after exhibiting resonance phenomenon at arcund 0.4 Kh, the ratio goes to zero with increasing frequencies. Figures 5.8 through 5.11 are further discussed in Section 6.

Three Dimensional Simulations

Figures 5.12 through 5.17 present the surge, sway, heave, roll, pitch and yaw responses of the buoy to surface waves of the same amplitude and frequencies as in Figures 5.5 and 5.7. Table 5.3 is the input data used in conjunction with the computer program SD3.FORT to obtain these simulations. Here the angle β, giving the wave propagation direction is zero. These simulations should give the same response as Figures 5.5 through 5.7. Comparing Figures 5.5 and 5.12, we see a lot of difference in the surge drifts. Figure 5.12 looks more like Figure 5.2 which

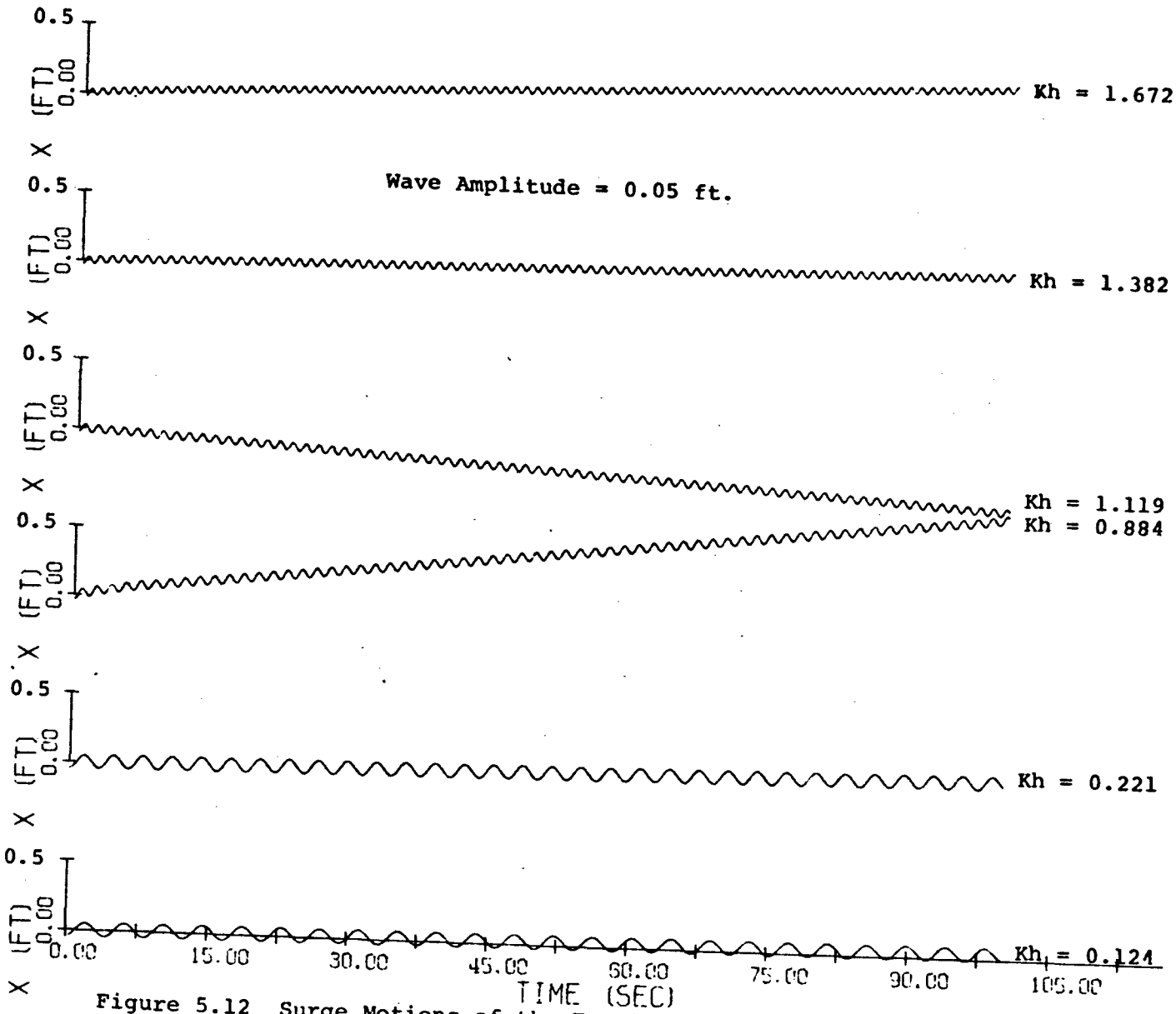


Figure 5.12 Surge Motions of the Freely Floating Cylindrical Spar Buoy

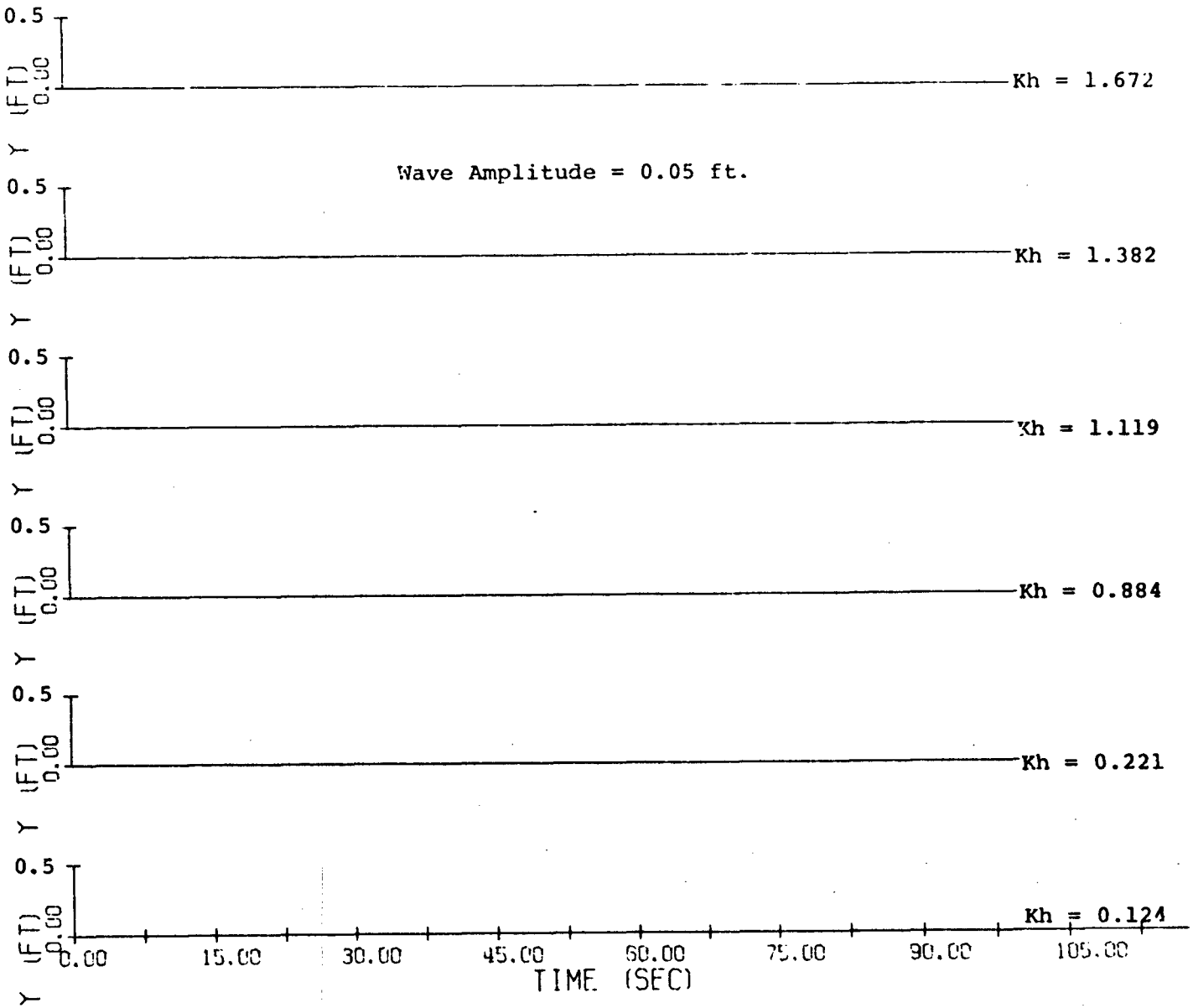


Figure 5.13 Sway Motions of the Freely Floating Cylindrical Spar Buoy

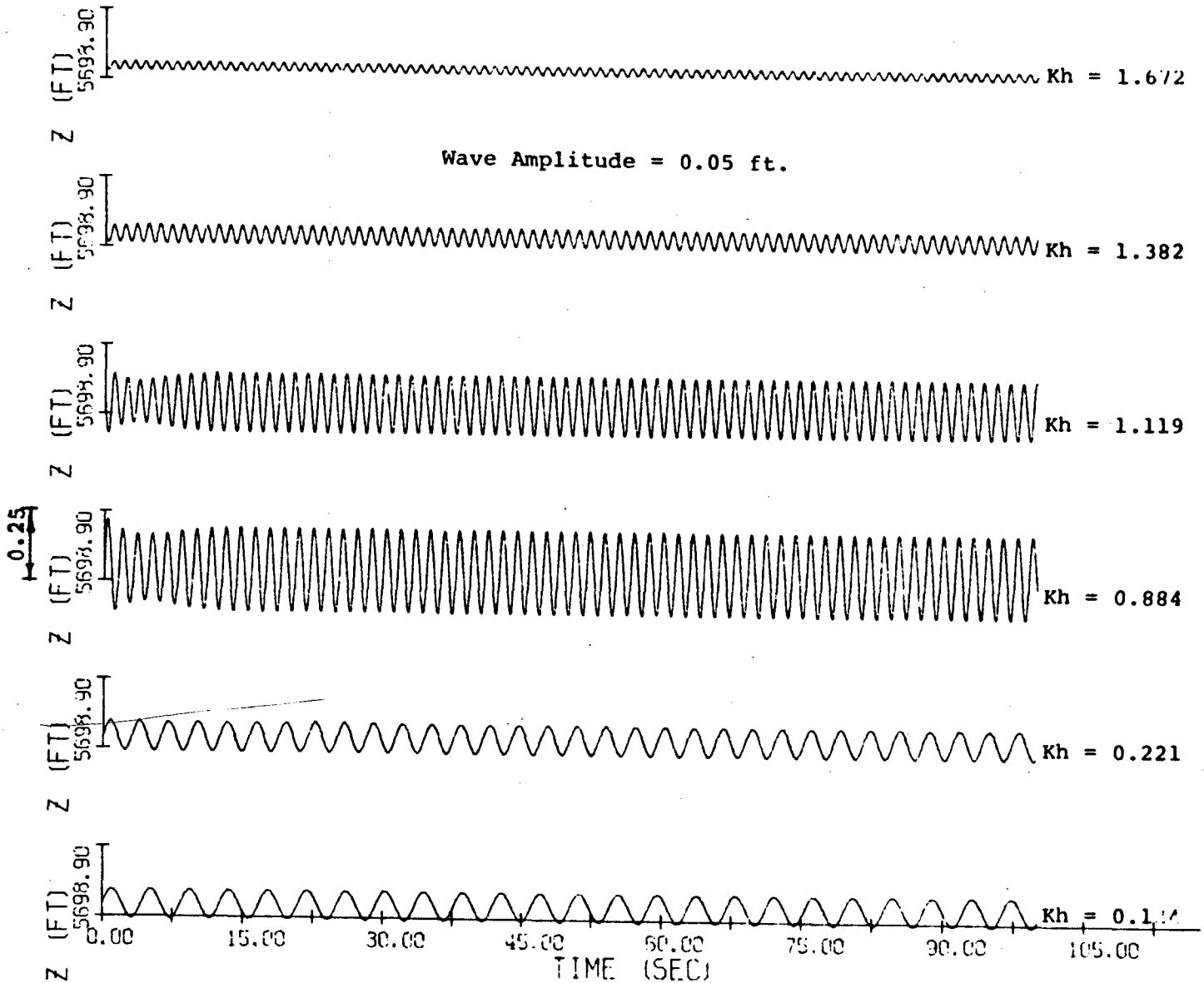


Figure 5.14 Heave Motions of the Freely Floating Cylindrical Spar Buoy

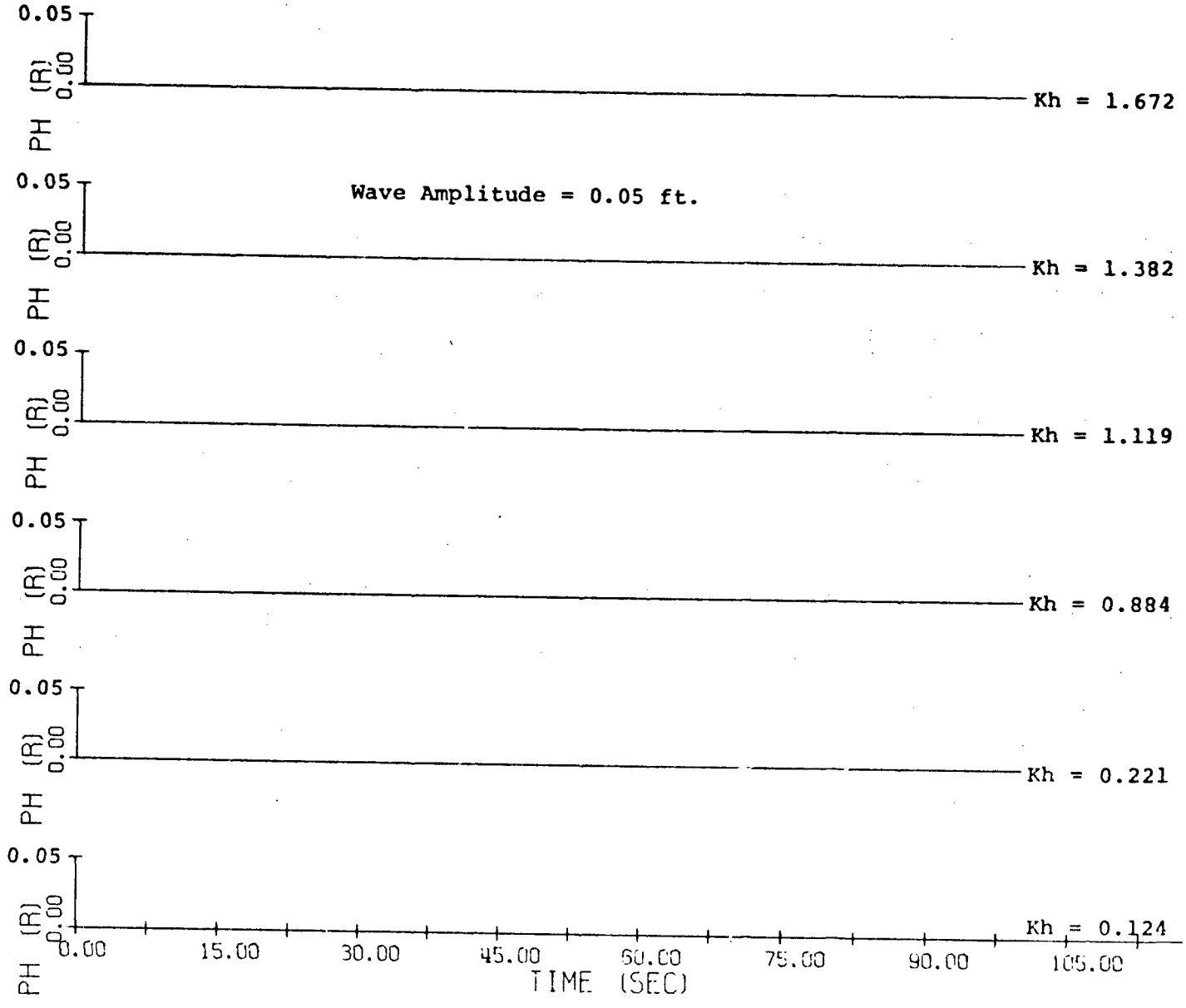


Figure 5.15 Roll Motions of the Freely Floating Cylindrical Spar Buoy

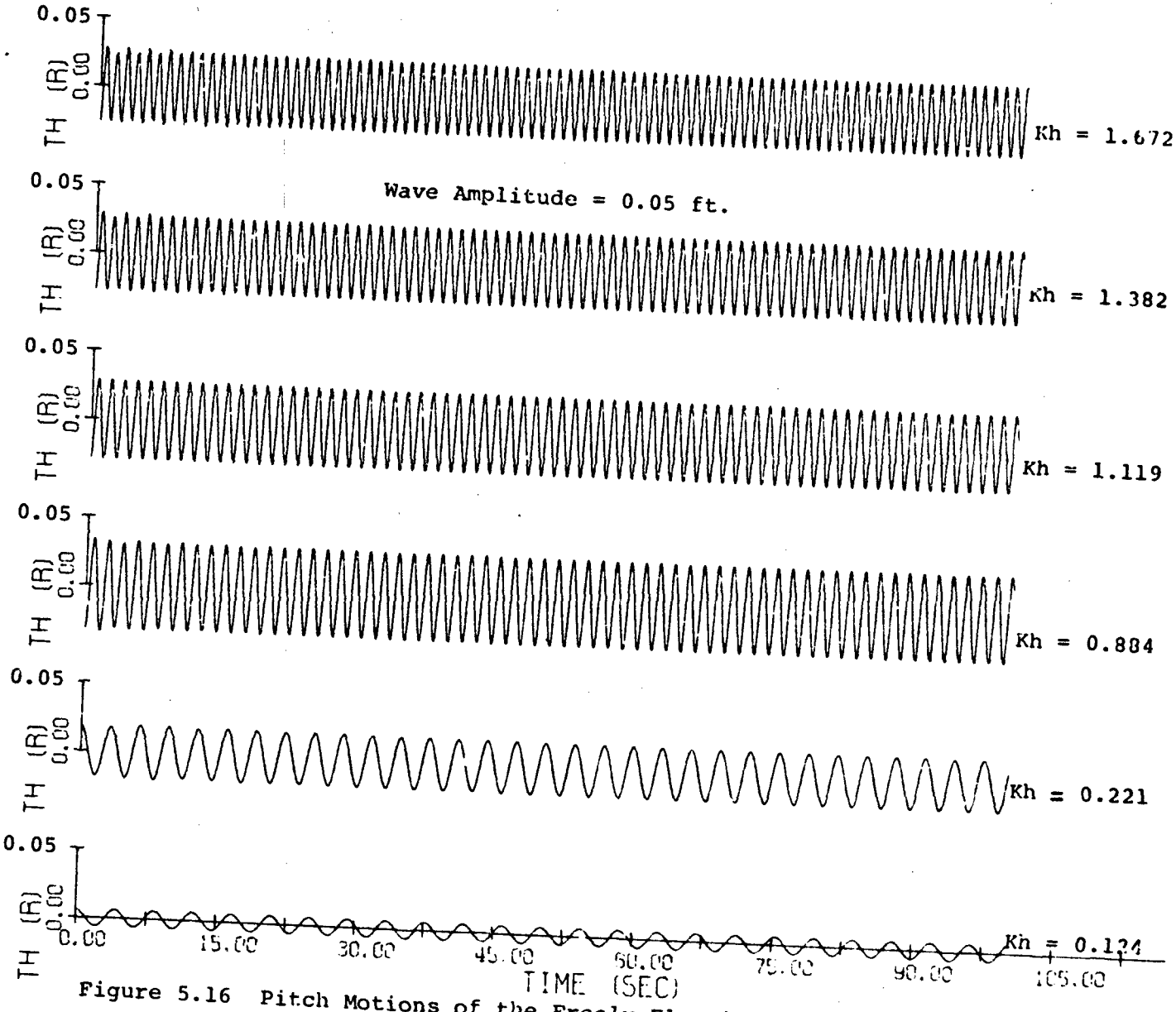


Figure 5.16 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

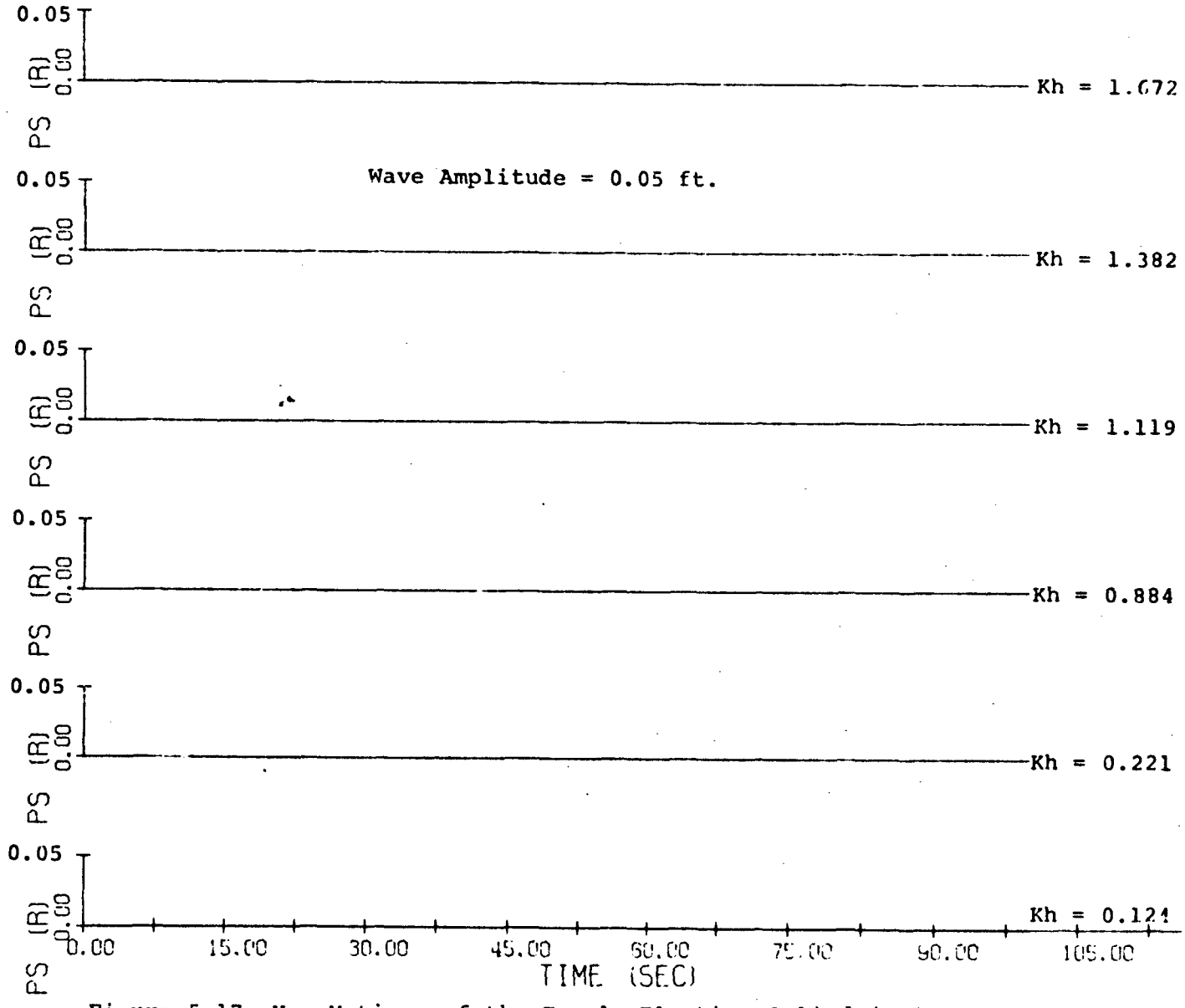


Figure 5.17 Yaw Motions of the Freely Floating Cylindrical Spar Buoy

```
 SCBUOY.DATA
00010      1       1    0   25    6    0 5700.     0.020     100.0     0.1
00020 0.03125    0.03125    0.725      0.704       2.258     1.0       1.1
00021  1.         1.0
00055  0.35       0.         0.         0.         0.        0.
00090  0.         0.         0.         0.         0.        -18.
00105  0.         0.         0.         0.         0.        0.
00110  0.         0.         0.
00120  1.50       0.05       0.0
00130  2.00       0.05       0.0
00140  4.00       0.05       0.0
00150  4.50       0.05       0.0
00160  5.00       0.05       0.0
00170  5.50       0.05       0.0
READY
```

Table 5.3 Input Data for the 3-D Analysis of the Cylindrical Spar Buoy

was obtained by neglecting viscous drag. Hence we see that the viscous drag effects on surge drift which show up in the two-dimensional non-linear simulations (Figure 5.5), do not show up in the three-dimensional simulation (Figure 5.12) where assumptions of small motions are made. This assumption may be in error the most near the resonance frequency (Heave resonance; Kh = 1.0. Pitch resonance; Kh = 0.4), where the two Figures 5.5 and 5.12 differ the most. Comparison of Figures 5.14 and 5.16 to Figures 5.6 and 5.7 shows only slight differences. Figures 5.13, 5.15, and 5.17 show no response as expected. Simulations for the same data as Table 5.3 but $\beta = \pi/4$ are presented in Figures 5.18 through 5.23. These figures show the resolution of surge in Figure 5.12 to surge and sway in Figures 5.18 and 5.19; and the resolution of pitch in Figure 5.16 to roll and pitch in Figures 5.21 and 5.22. Yaw response remains the same as expected.

5.1.2 Tuned Spar

The spar buoy used in these simulations along with its dimensions and characteristics is shown in Figure 5.24.

Two-Dimensional Simulations

Figures 5.25, 5.26, and 5.27 show the typical surge, heave, and pitch response of the buoy to surface waves of amplitude 0.5 ft. and different frequencies. Table 5.4 is the input data used in conjunction with the computer program SD2.FORT to obtain these simulations. Here again

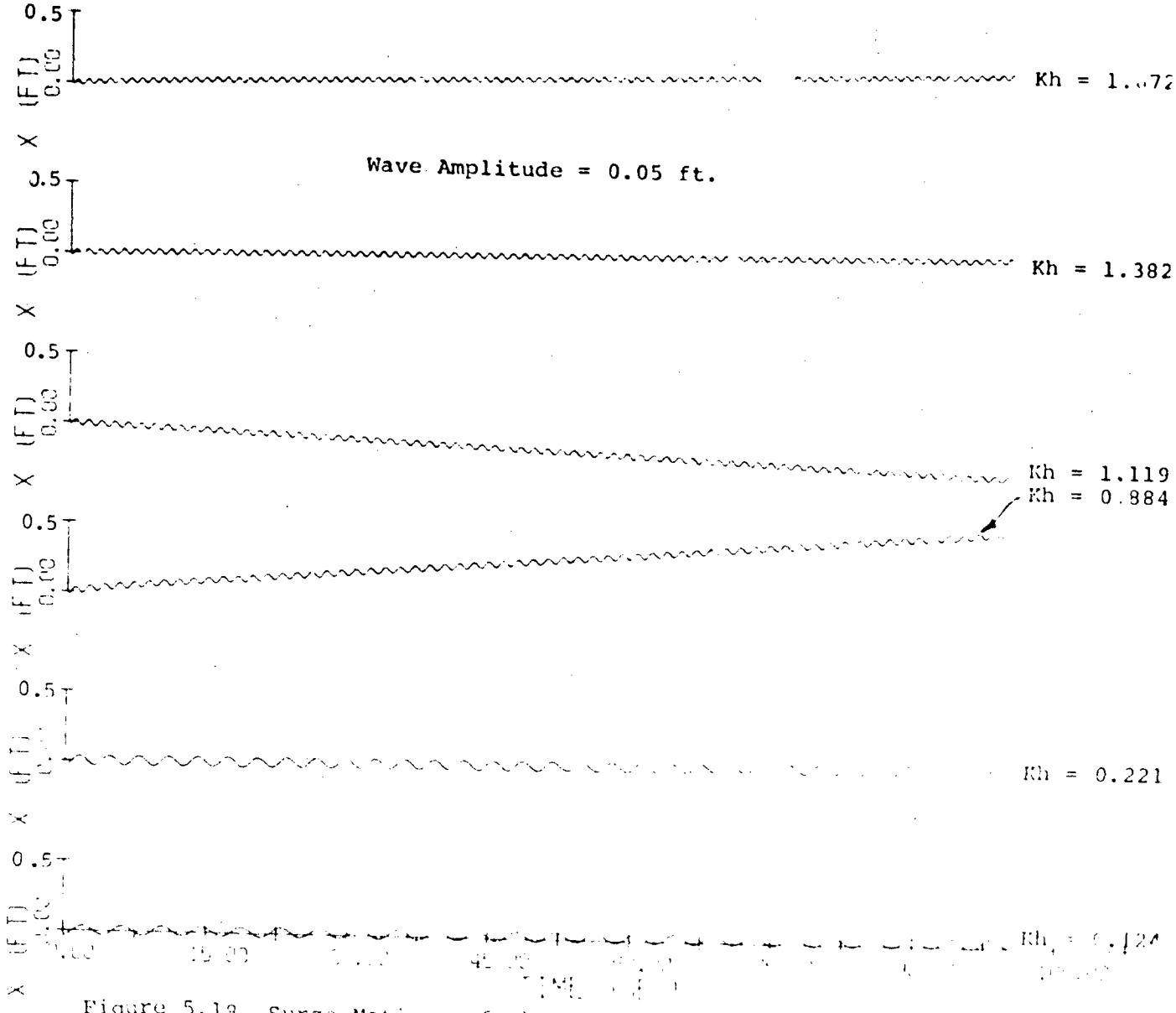


Figure 5.13 Surge Motions of the Freely Floating Cylindrical Spar Buoy

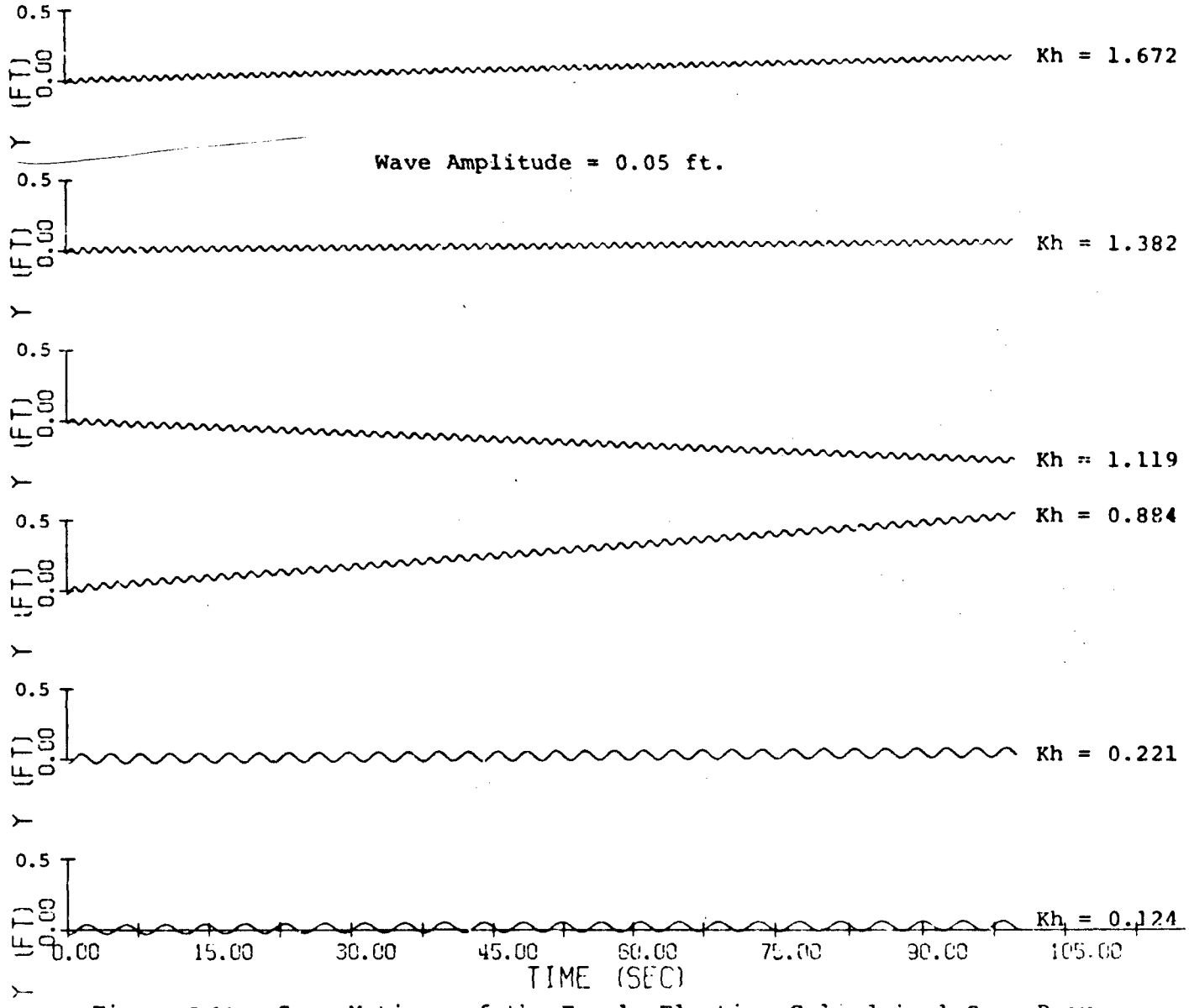


Figure 5.19 Sway Motions of the Freely Floating Cylindrical Spar Buoy

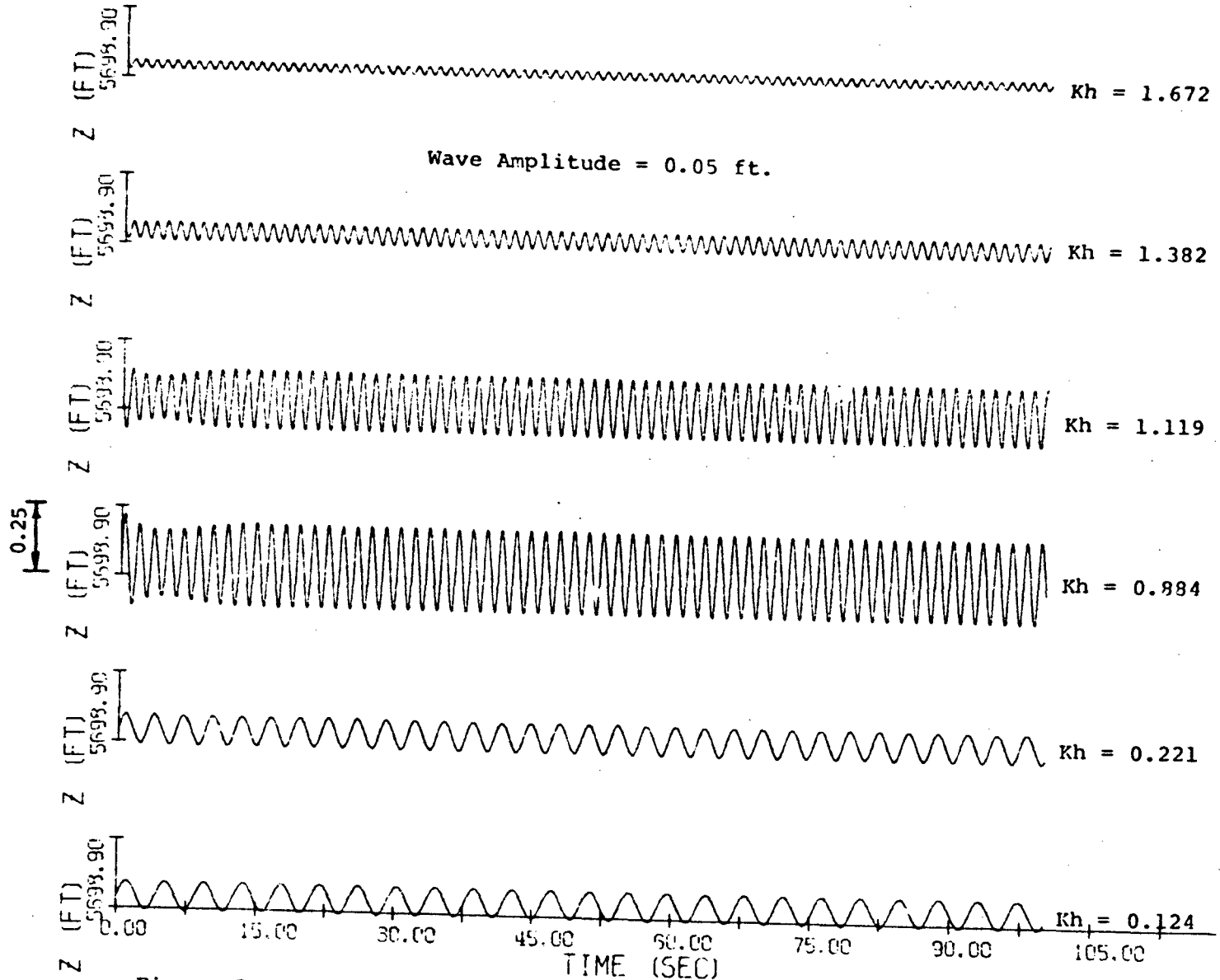


Figure 5.20 Heave Motions of the Freely Floating Cylindrical Spar Buoy

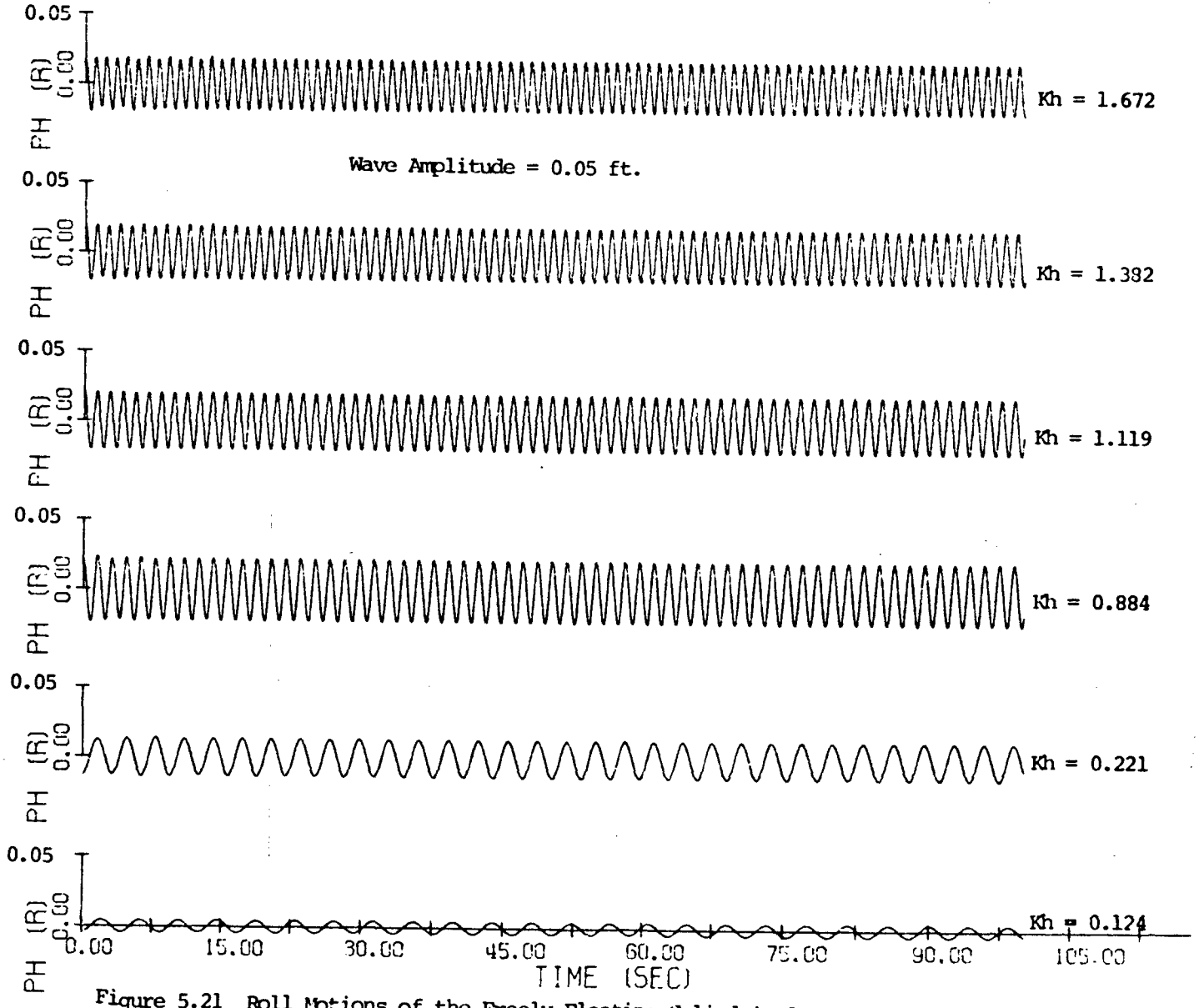


Figure 5.21 Roll Motions of the Freely Floating Cylindrical Spar Buoy

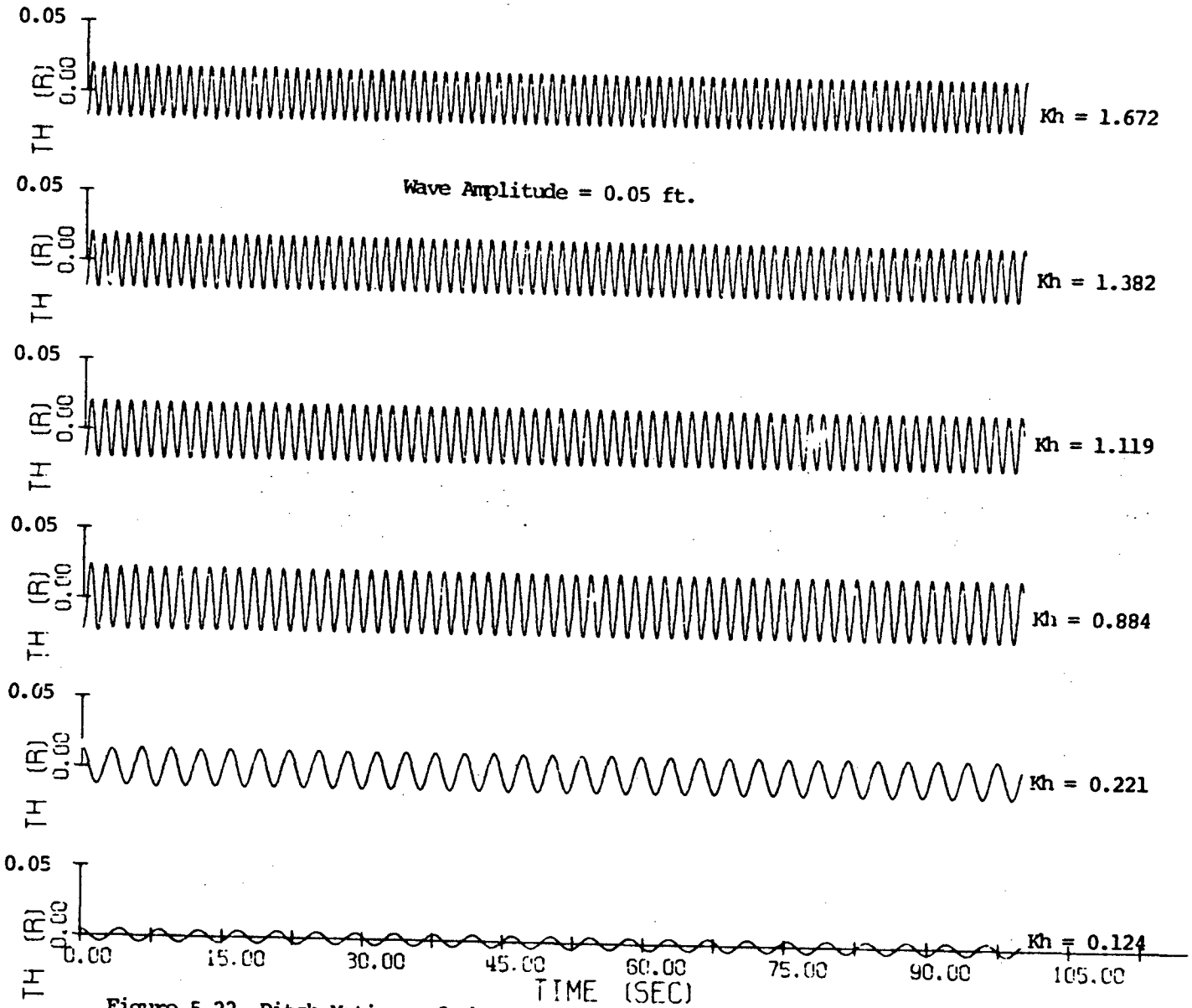


Figure 5.22 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

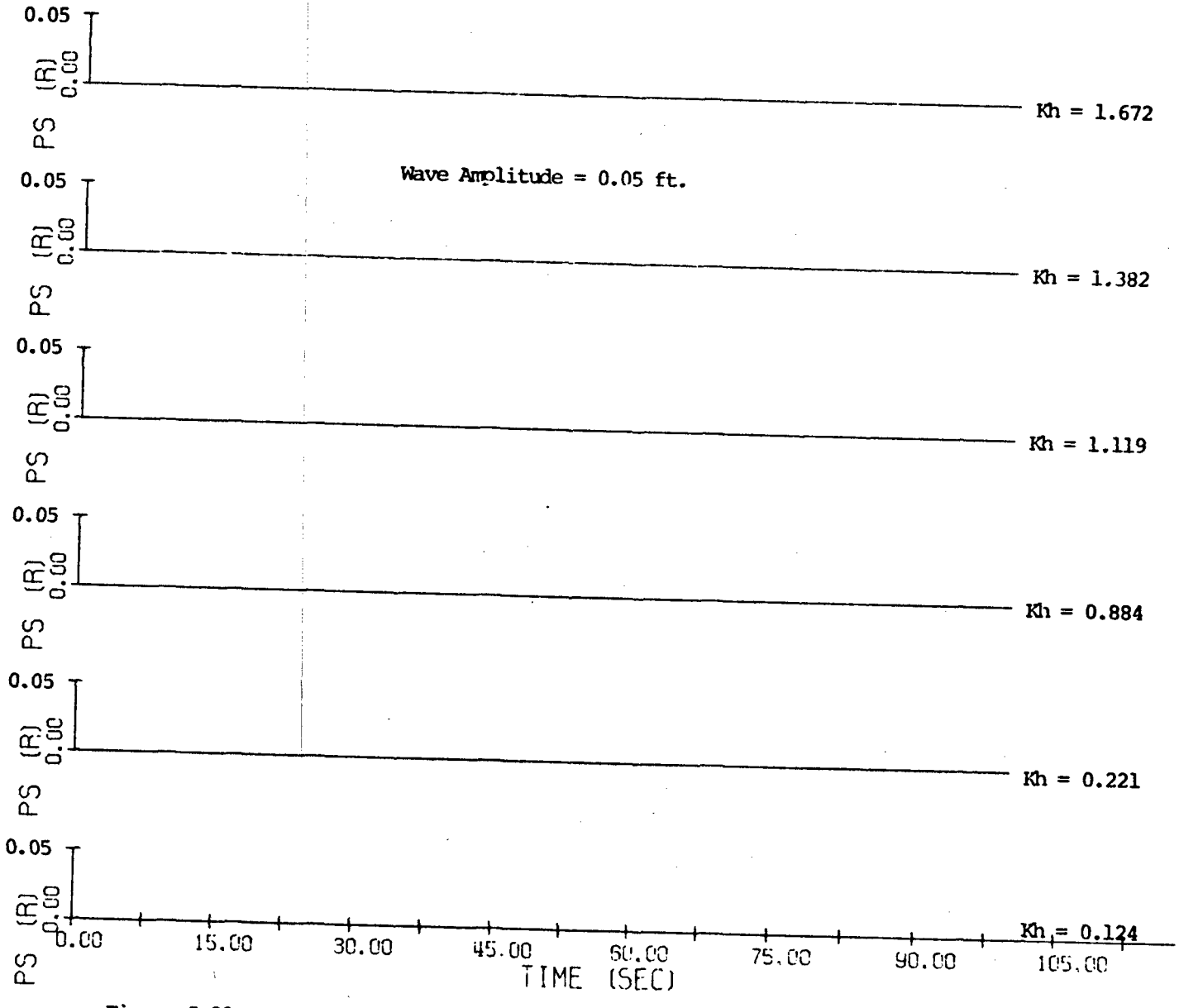
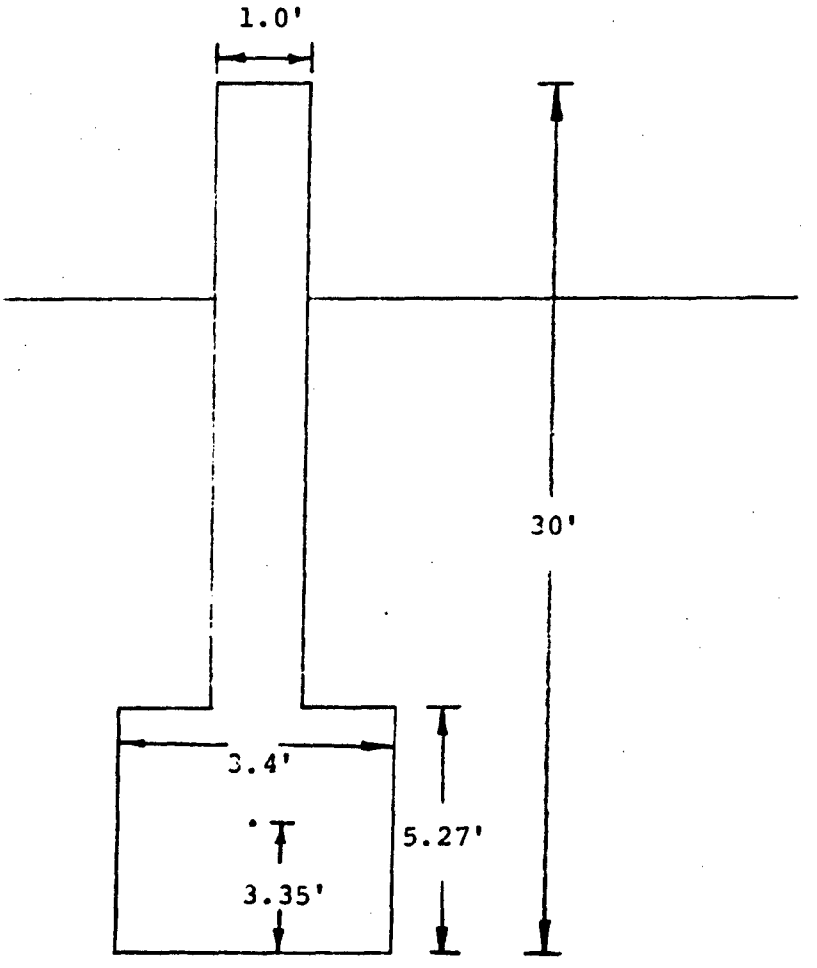


Figure 5.23 Yaw Motions of the Freely Floating Cylindrical Spar Buoy



Buoy Radius of Gyration = 5.11'

Total Weight of the Buoy = 3745.62 pounds

Figure 5.24 Tuned Spar Buoy Description

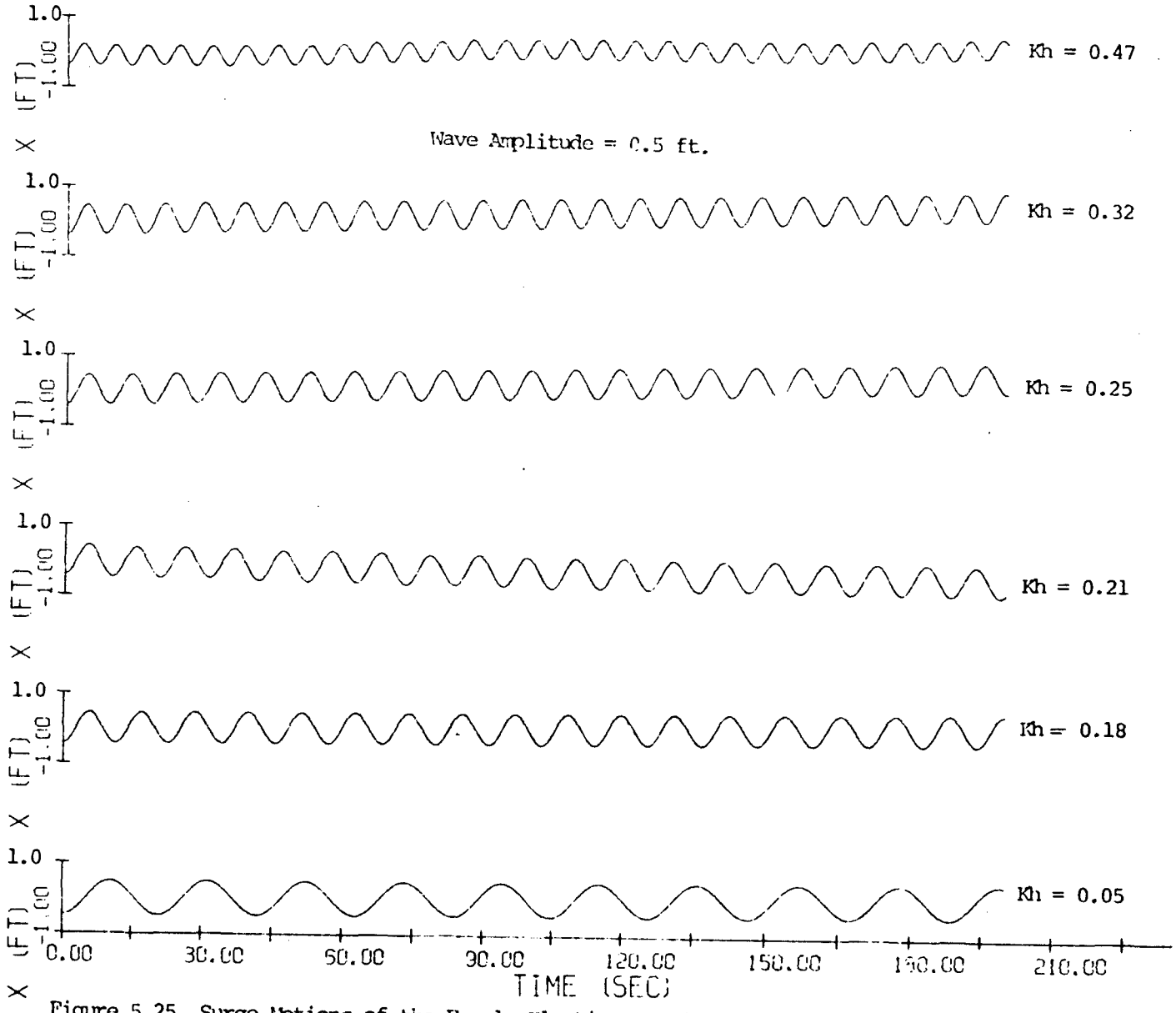


Figure 5.25 Surge Motions of the Freely Floating Tuned Spar Buoy

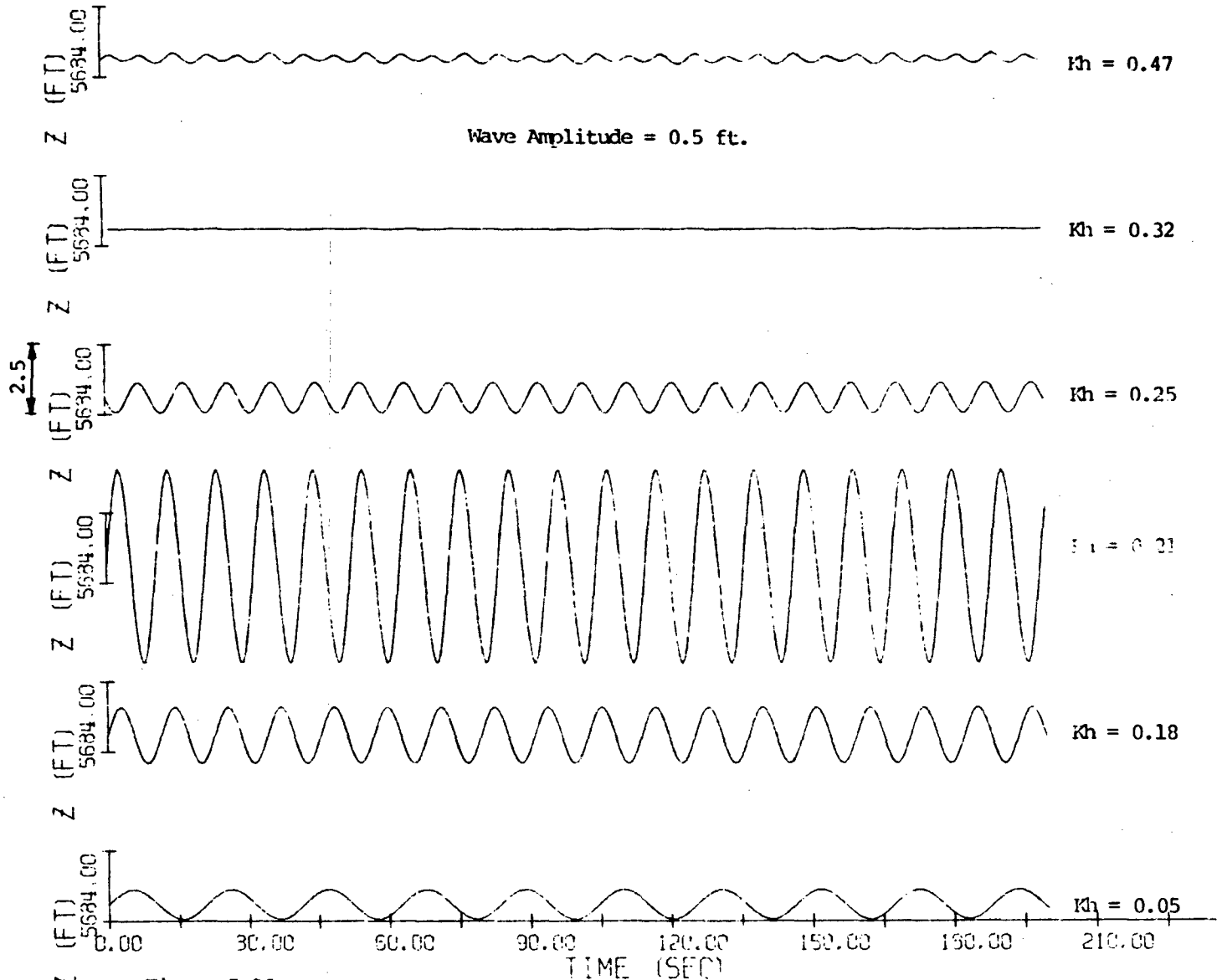


Figure 5.26 Heave Motions of the Freely Floating Tuned Spar Buoy

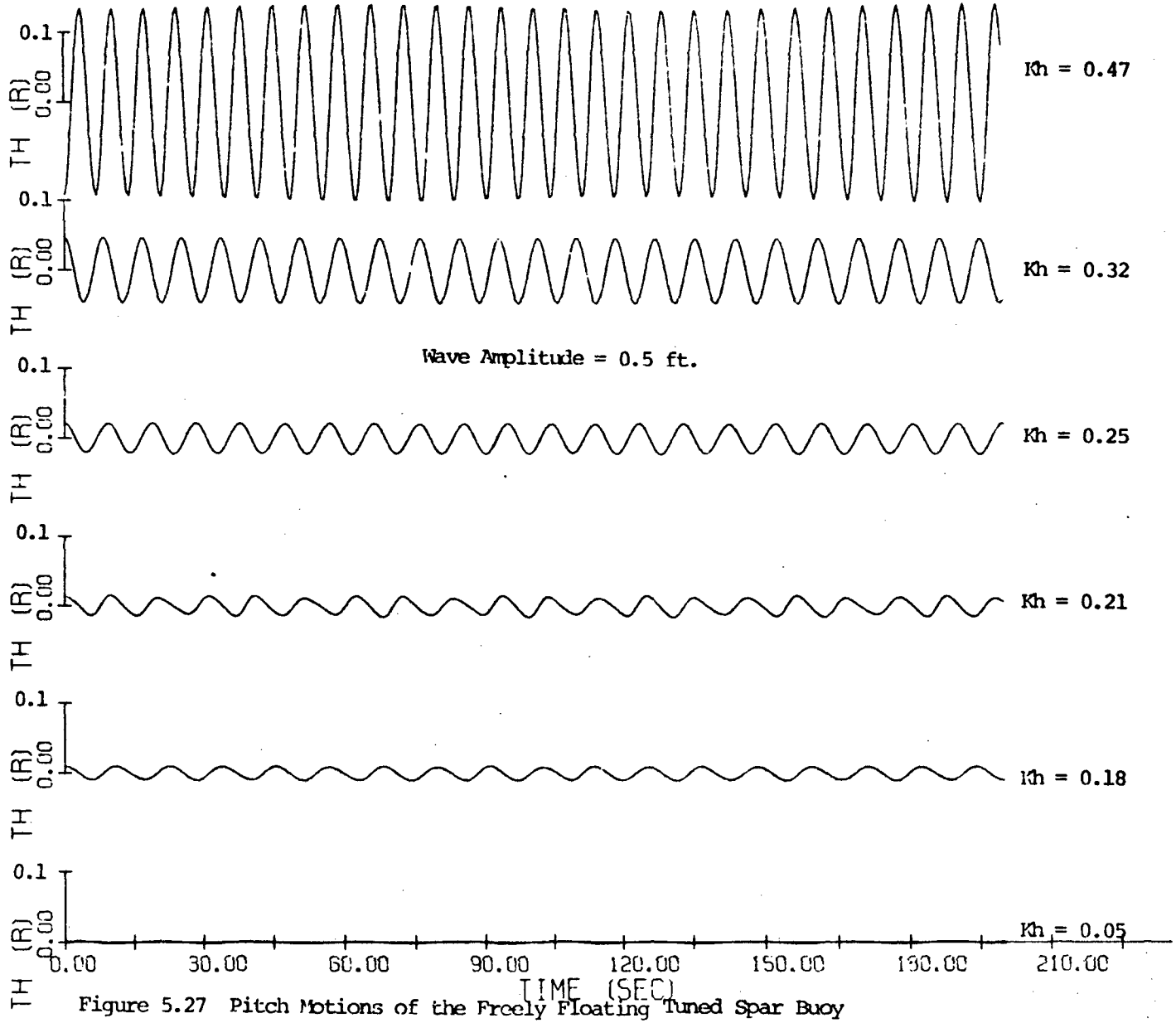


Figure 5.27 Pitch Motions of the Freely Floating Tuned Spar Buoy

```
 SD2T.DATA
00010      1     1    0     25     6     0 5700.      0.020      200.0      0.5
00020   1.7          0.5          3.35       5.11      30.        0.0        5.27
00021   1.0          0.0          0.0        1.0
00055   3745.62      0.0          0.0        0.0       0.0        0.0
00090   0.0          0.0          0.0        0.0       0.0        -18.
00105   0.0          0.0          0.0        0.0
00110   0.0
00120    0.30         0.5
00130    0.55         0.5
00140    0.60         0.5
00150    0.66         0.5
00160    0.74         0.5
00170    0.90         0.5
READY
```

Table 5.4 Input Data for the 2-D Analysis of the Tuned Spar Buoy

viscous and pressure drag coefficients equal zero. Figure 5.28 through 5.30 use the same data as Table 5.4 except that viscous and pressure drag coefficients are equal to 1.0. Once again comparisons between the two sets of figures (5.25 through 5.30) show similar differences as explained for the cylindrical spar buoy. In this case the heave resonance occurs at Kh = 0.22 and the pitch resonance at Kh = 0.4. Also in this case the viscous damping in heave and pitch responses is more pronounced.

Response amplitudes of surge, heave, and pitch; and surge drift as calculated from the steady-state portions of the time histories presented in Figures 5.25 through 5.30, and other similar simulations are tabulated in Table 5.5 which is similar to Table 5.2. In this case the wave amplitudes ξ_o studied are 0.5 ft. and 2.5 ft. respectively. Data points of Table 5.5 are plotted in Figures 5.31 through 5.34 to presents plots of RAO vs Kh. These plots are similar to Figures 5.8 through 5.11. Figure 5.33b is an expanded view of Figure 5.33a, near the heave resonance frequency. The ratio between buoy heave and the wave amplitude is one at zero frequency and exhibits resoance at Kh of approximately 0.2. The ratio decreases to zero at Kh equal to 0.32, after which it increases a little before decreasing to zero with increasing frequencies. Such behavior, where the heave response is minimized at a given frequency is the main function of a tuned (as opposed to cylindrical) spar buoy.

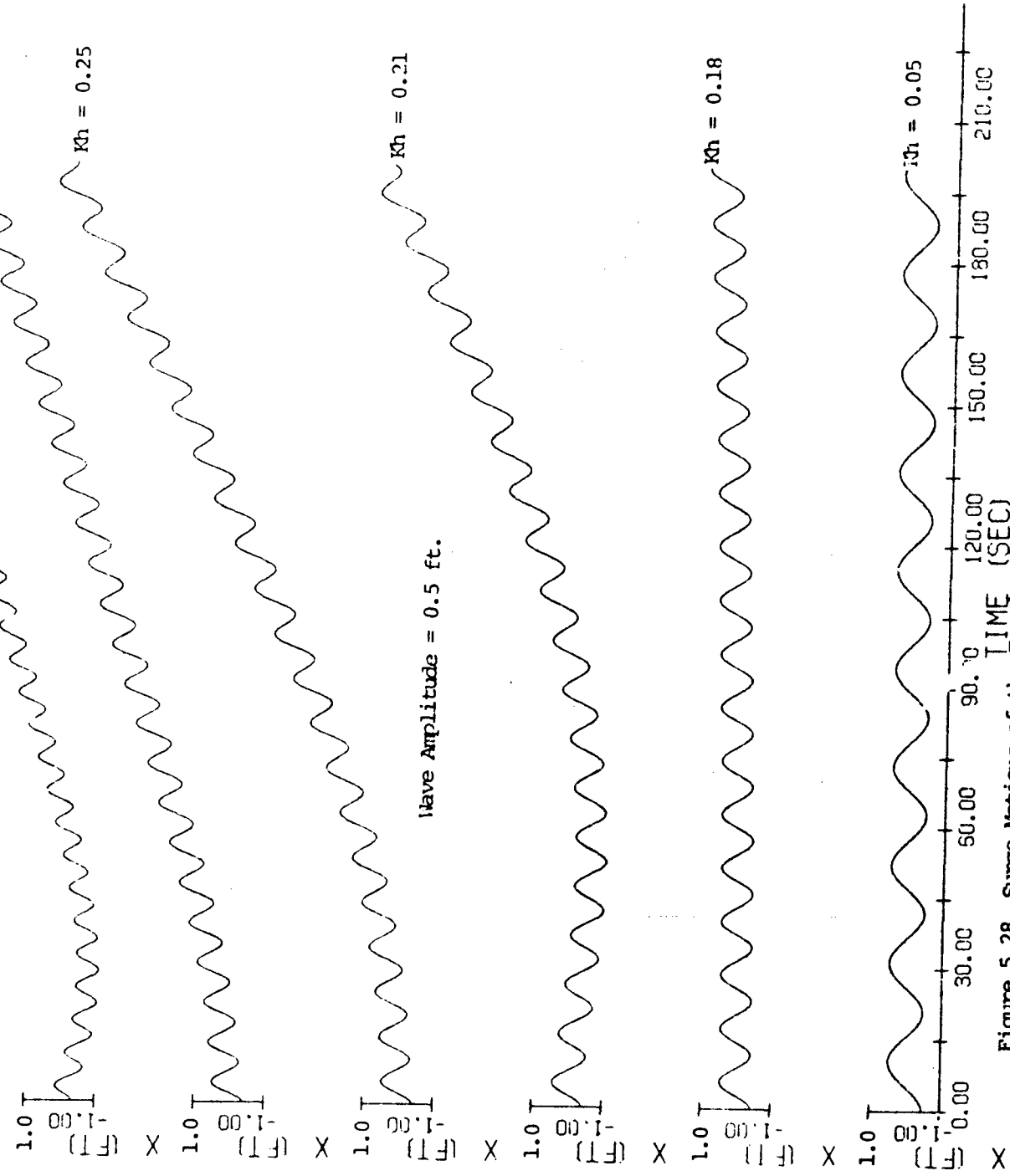


Figure 5.28 Surge Motions of the Freely Floating Tuned Spar Buoy

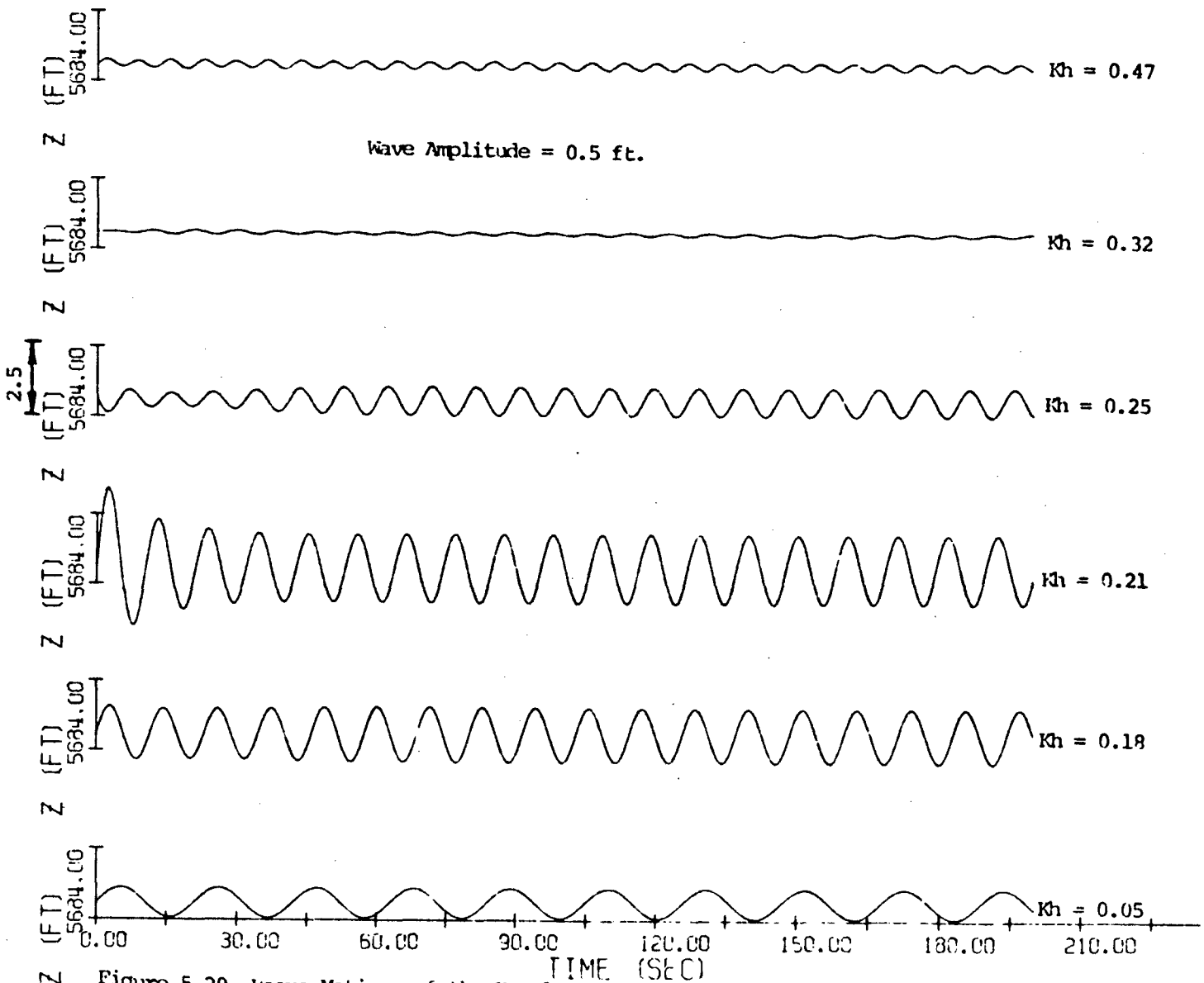


Figure 5.29 Heave Motions of the Freely Floating Tuned Spar Buoy

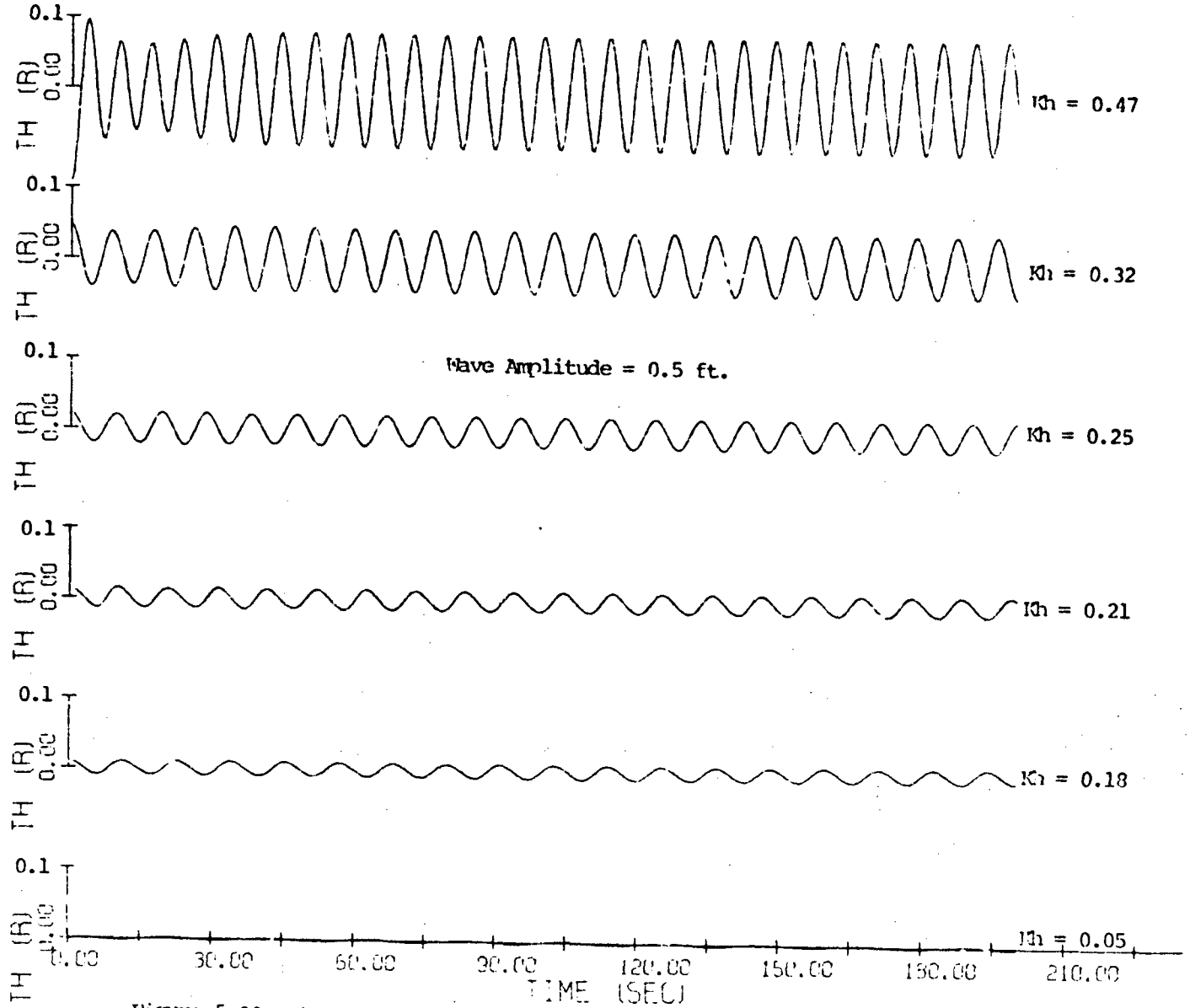


Figure 5.30 Pitch Motions of the Freely Floating Tuned Spar Buoy

ω	Kh	Surge Amplitude/ζ_o				Heave Amplitude/ζ_o				Pitch Amplitude/$k\zeta_o$				Surge Drift/$kw\zeta_o^2$		
		1	2	3	4	1	2	3	4	1	2	3	4	2	3	4
0.30	0.05	-0.96	0.96	0.96	0.96	1.05	1.05	1.05	1.05	1.21	1.23	1.23	1.36	1.38	1.41	0.48
0.55	0.18	-0.87	0.88	0.88	0.88	1.99	1.99	1.94	1.54	2.04	2.16	2.04	2.02	0.51	4.97	2.80
0.57	0.19	-0.87		0.87	0.88	2.58		2.27	1.51	2.18		2.17	2.17		14.90	4.13
0.59	0.20	-0.86		0.85		4.20		2.53		2.34		2.33			33.40	
0.60	0.21	-0.85	0.86	0.82	0.85	6.87	6.87	2.50	1.46	2.43	2.77	2.53	2.44	-1.08	41.66	5.31
0.63	0.23	-0.84		0.78		-4.15		1.87		2.75		2.77			48.93	
0.66	0.25	-0.82	0.84	0.81	0.81	-1.07	1.07	1.01	0.86	3.19	3.26	3.26	2.94	1.49	31.62	5.50
0.70	0.29	-0.81		0.82		-0.29		0.34		4.03		3.96			17.24	
0.74	0.32	-0.79	0.82	0.81	0.77	-0.02	0.02	0.11	0.40	5.51	5.51	5.21	3.52	1.04	14.36	3.96
0.83	0.40	-0.74		0.67		0.21		0.24		35.79		7.81			15.61	
0.90	0.47	-0.70	0.55	0.60	0.60	0.27	0.36	0.28	0.31	-10.62	11.34	6.44	3.20	0.97	7.90	1.73
1.01	0.59	-0.64		0.58		0.30		0.30		-3.43		3.26			1.82	
1.17	0.80	-0.55		0.51	0.51	0.28		0.28	0.29	-1.69		1.67	1.55		0.50	0.42
1.61	1.50	-0.34		0.30	0.30	0.17		0.17	0.17	-0.62		0.62	0.65		0.20	0.17

Table 5.5 Dynamic Response (Surge, Heave and Pitch Amplitude; and Surge Drift) of a Tuned Spar Buoy to Waves

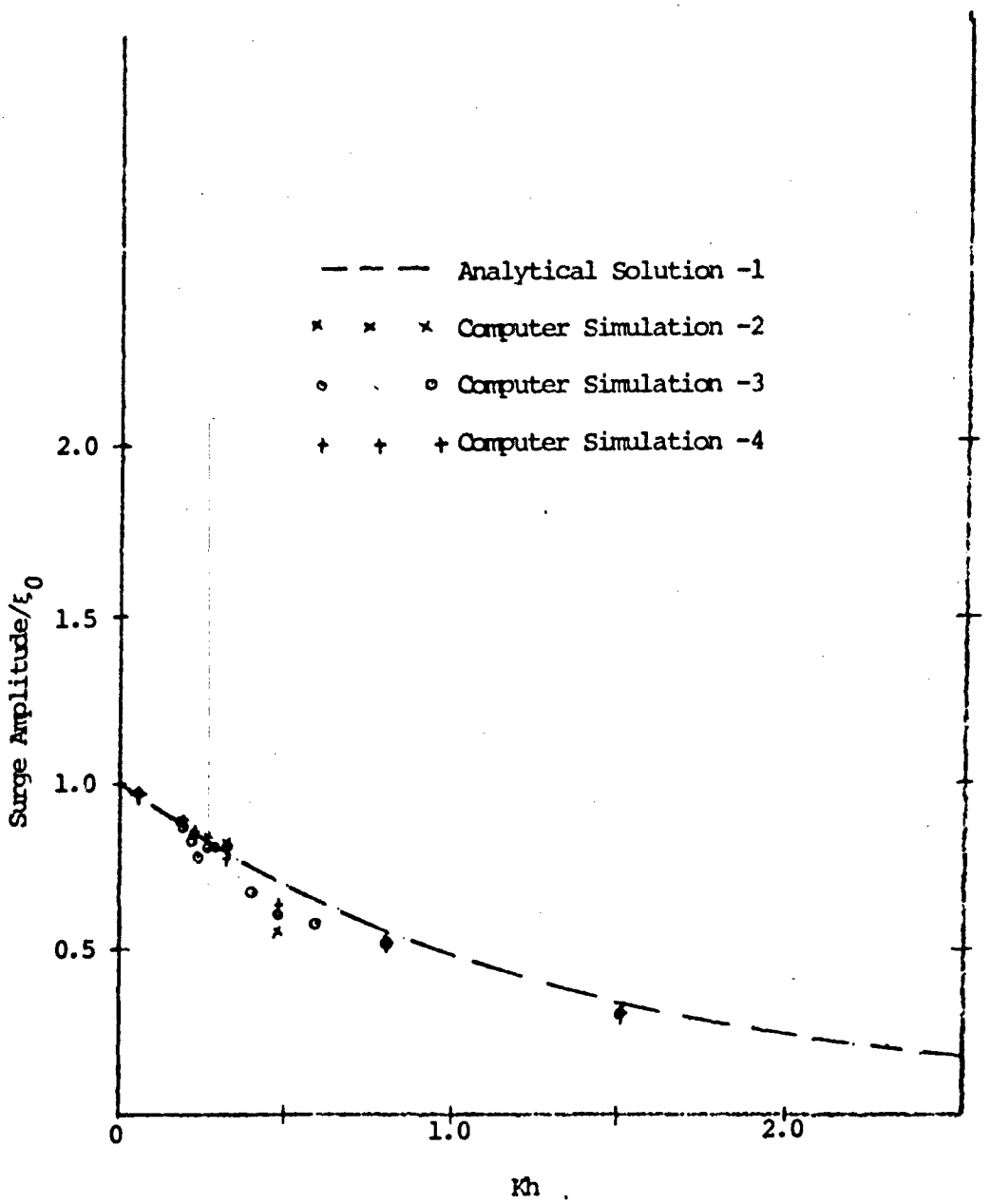


Figure 5.31 Surge Magnification Vs Kh

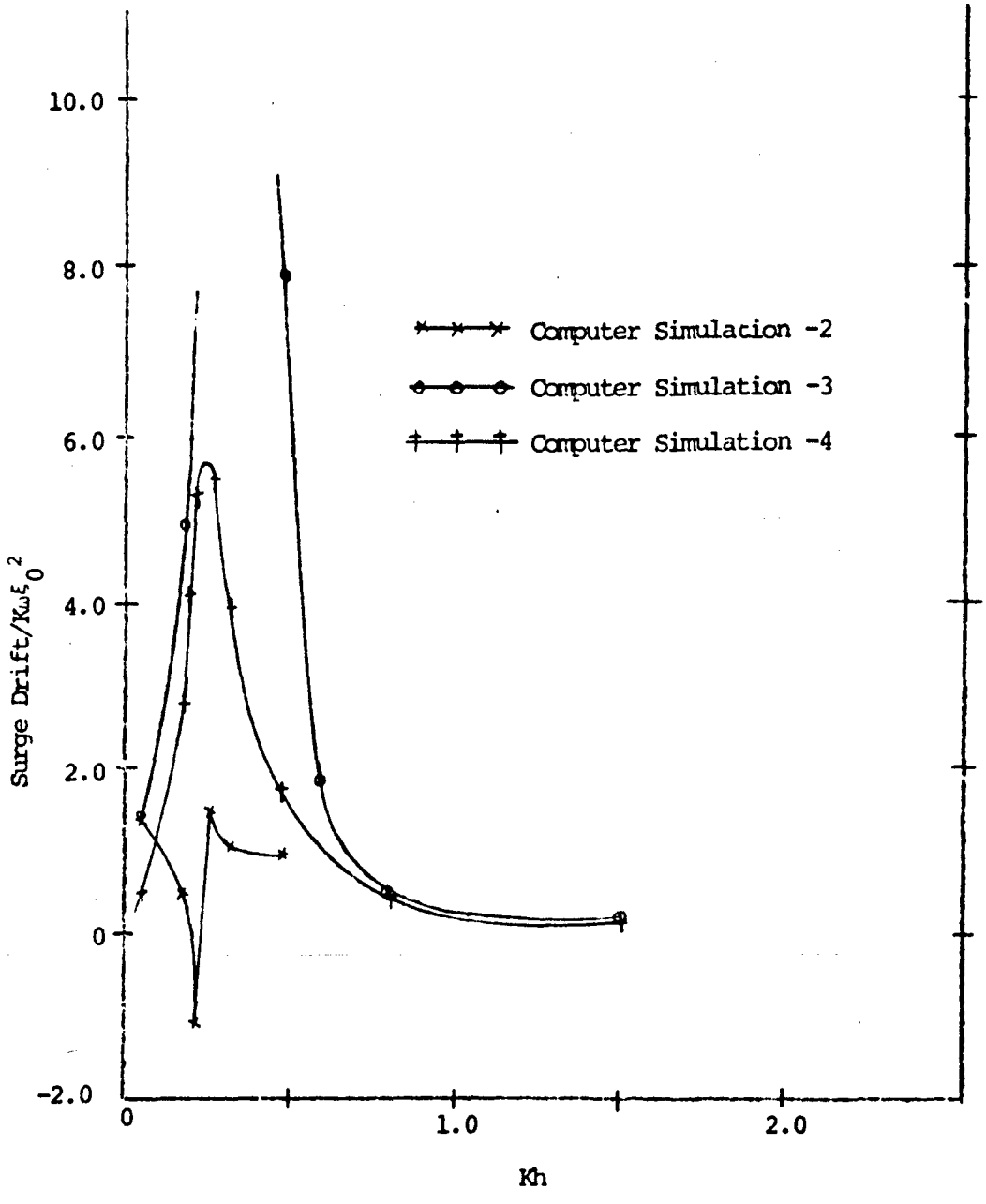


Figure 5.32 Surge Drift Vs Kh

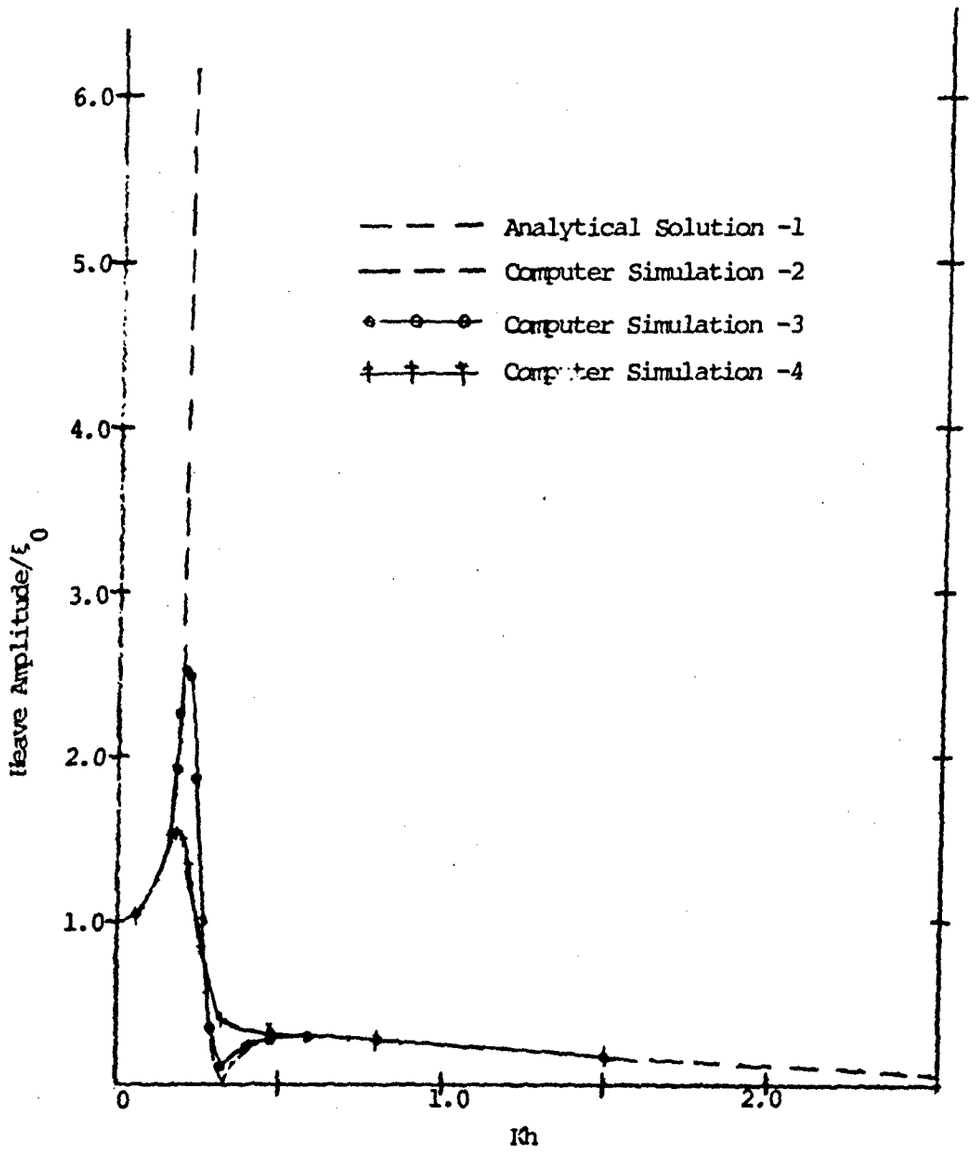


Figure 5.33a Heave Magnification Vs Kh

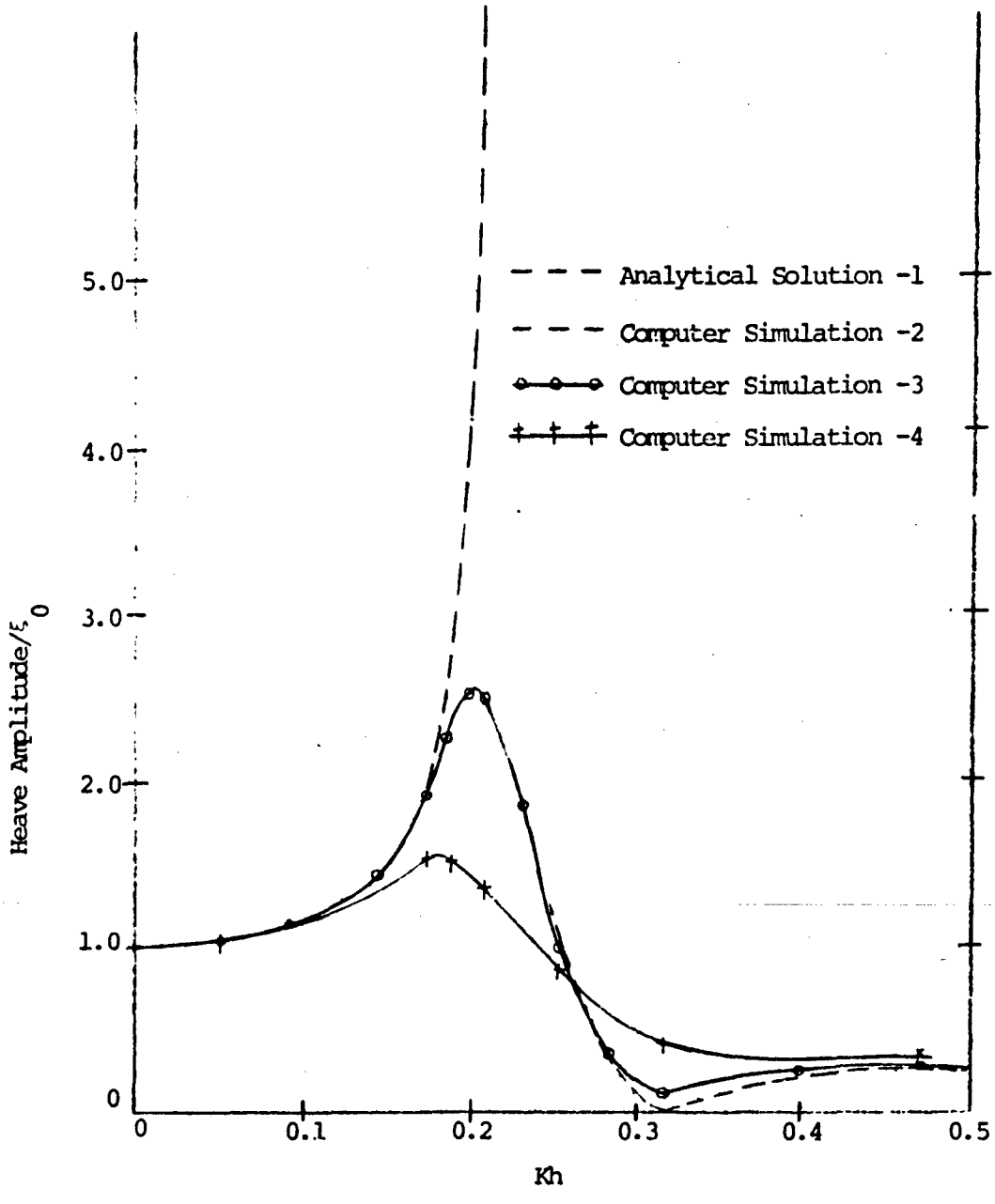


Figure 5.33b Expanded View of Figure 5.33a

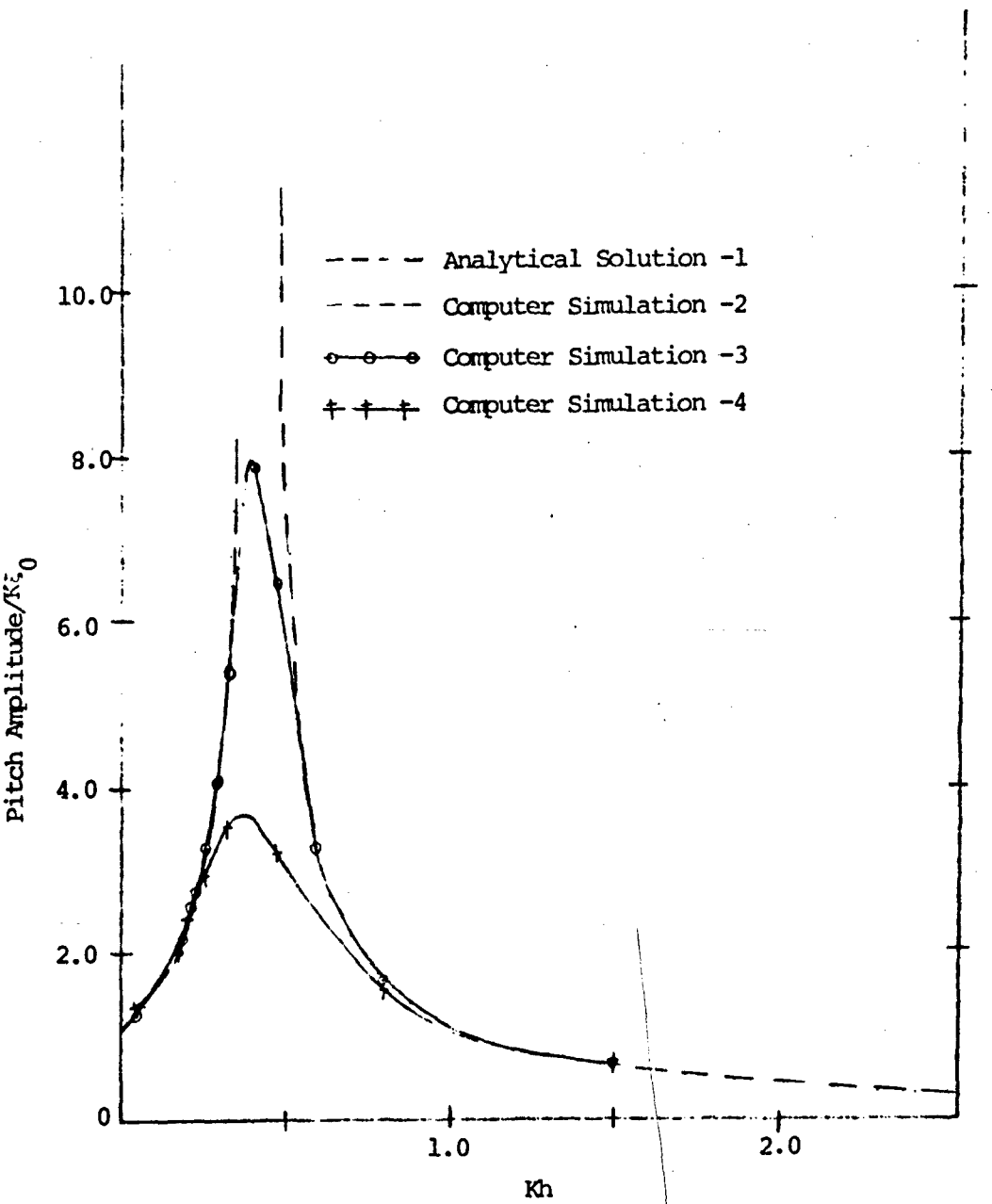


Figure 5.34 Pitch Magnification Vs Kh

Three-Dimensional Simulations

Figures 5.35 through 5.40 present the surge, sway, heave, roll, pitch, and yaw responses of the buoy to surface waves of the same amplitude and frequencies as in Figures 5.28 to 5.30. Table 5.6 is the input data used in conjunction with the computer program SD3.FORT to obtain these simulations. Here β equals zero. Differences between Figure 5.28 and 5.35 can be attributed directly to the assumptions of three-dimensional analysis. Simulations for the same data as Table 5.6 and $\beta = \pi/4$ are presented in Figures 5.41 through 5.46.

5.2 Subsurface Moored System

Three dimensional computer programs (S3S3.FORT, SSD3.FORT and SSD31.FORT) described in Section 4.2 are used for these simulations. Mooring system used as a case study for these simulations is shown in Figure 5.47. This mooring system was actually deployed during the MODE experiment and was called mooring No. 1 (station 481). Figures 5.48 and 5.49 display the three coordinates (x, y, z) of the top of this mooring line in response to the relative velocity data displayed in Figures 5.50 and 5.51. This simulation is a part of the study (CHHABRA, 1976), where the current record from the topmost vector-averaging

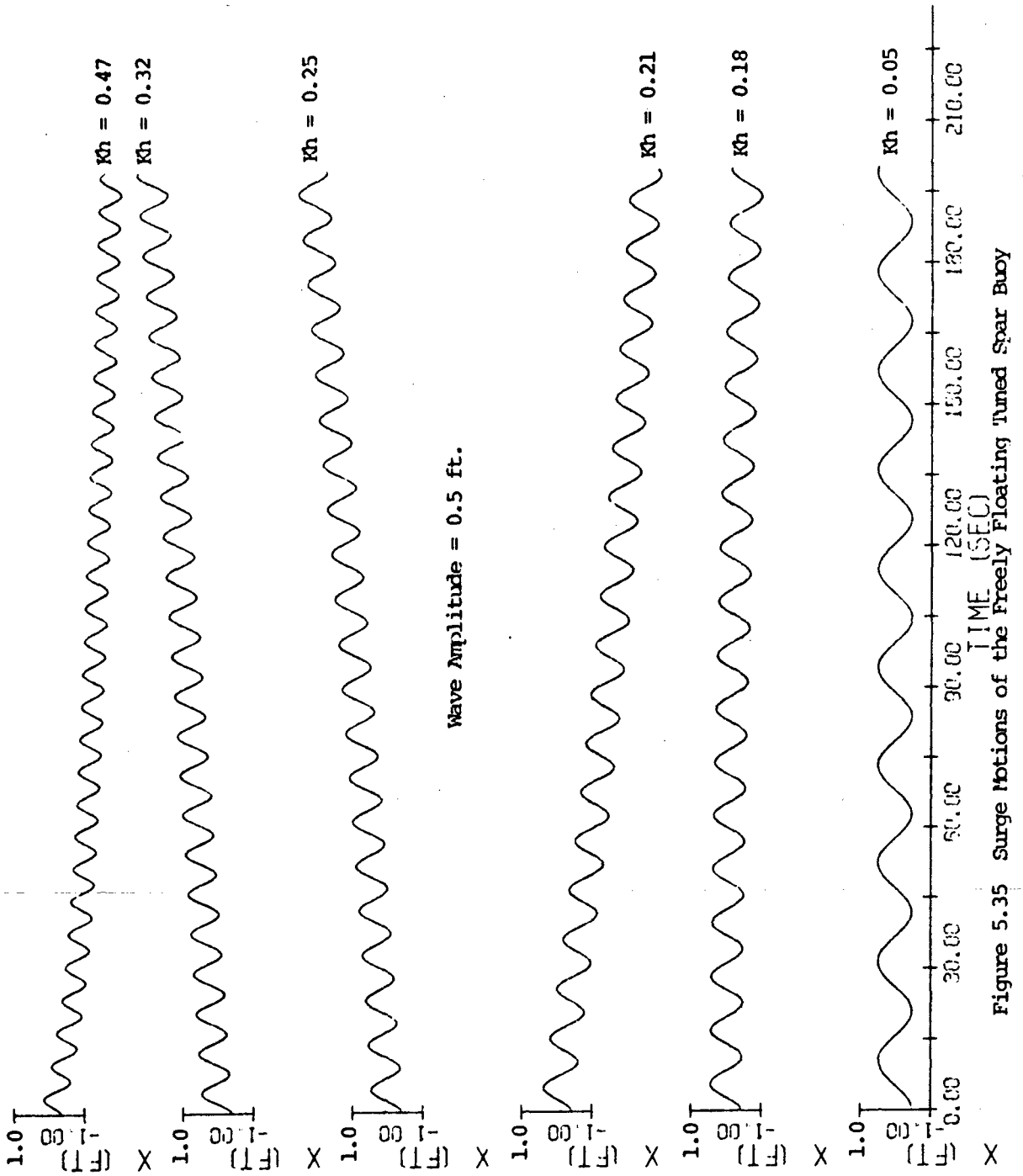


Figure 5.35 Surge Motions of the Freely Floating Tuned Spar Buoy

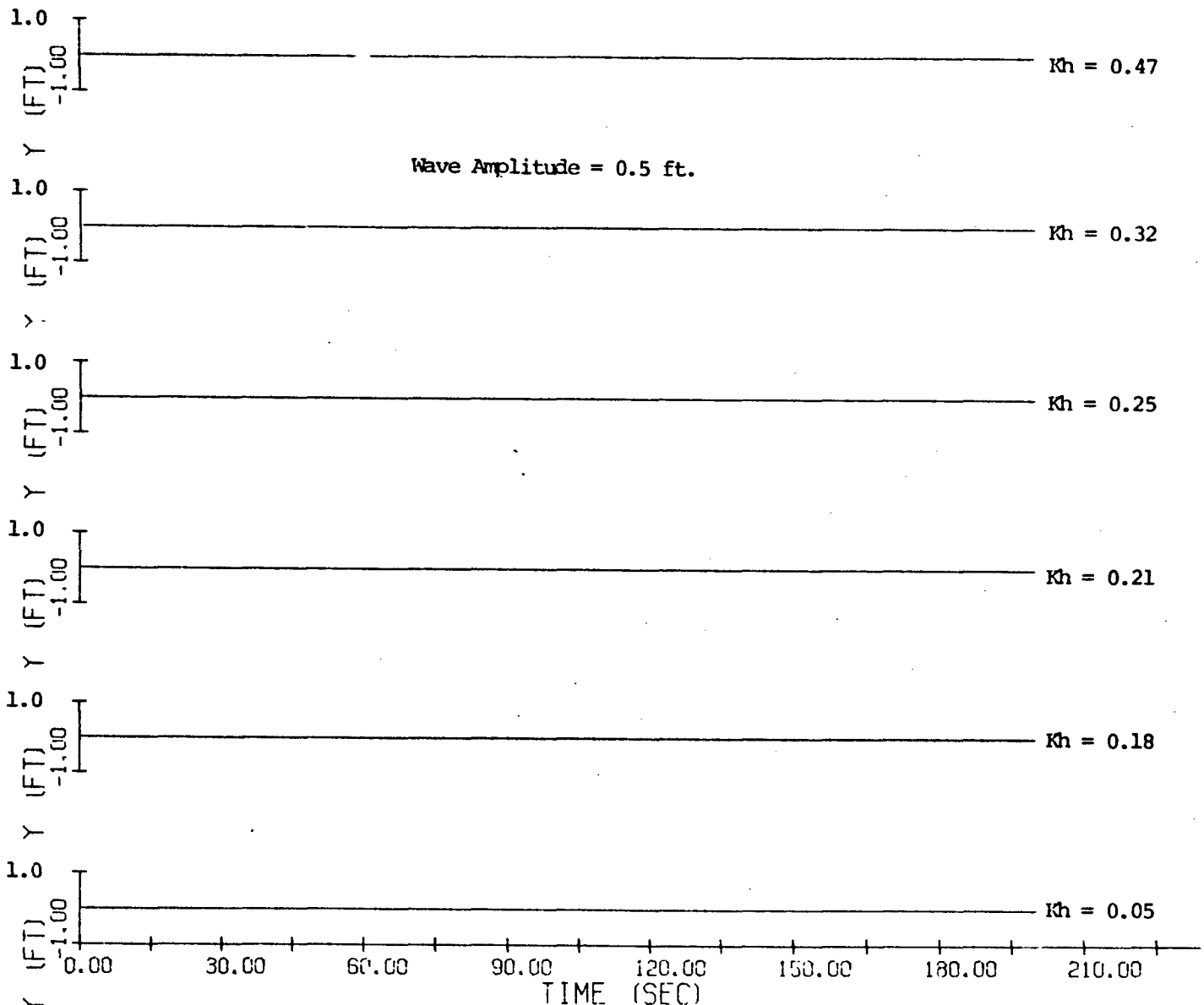


Figure 5.36 Sway Motions of the Freely Floating Tuned Spar Buoy

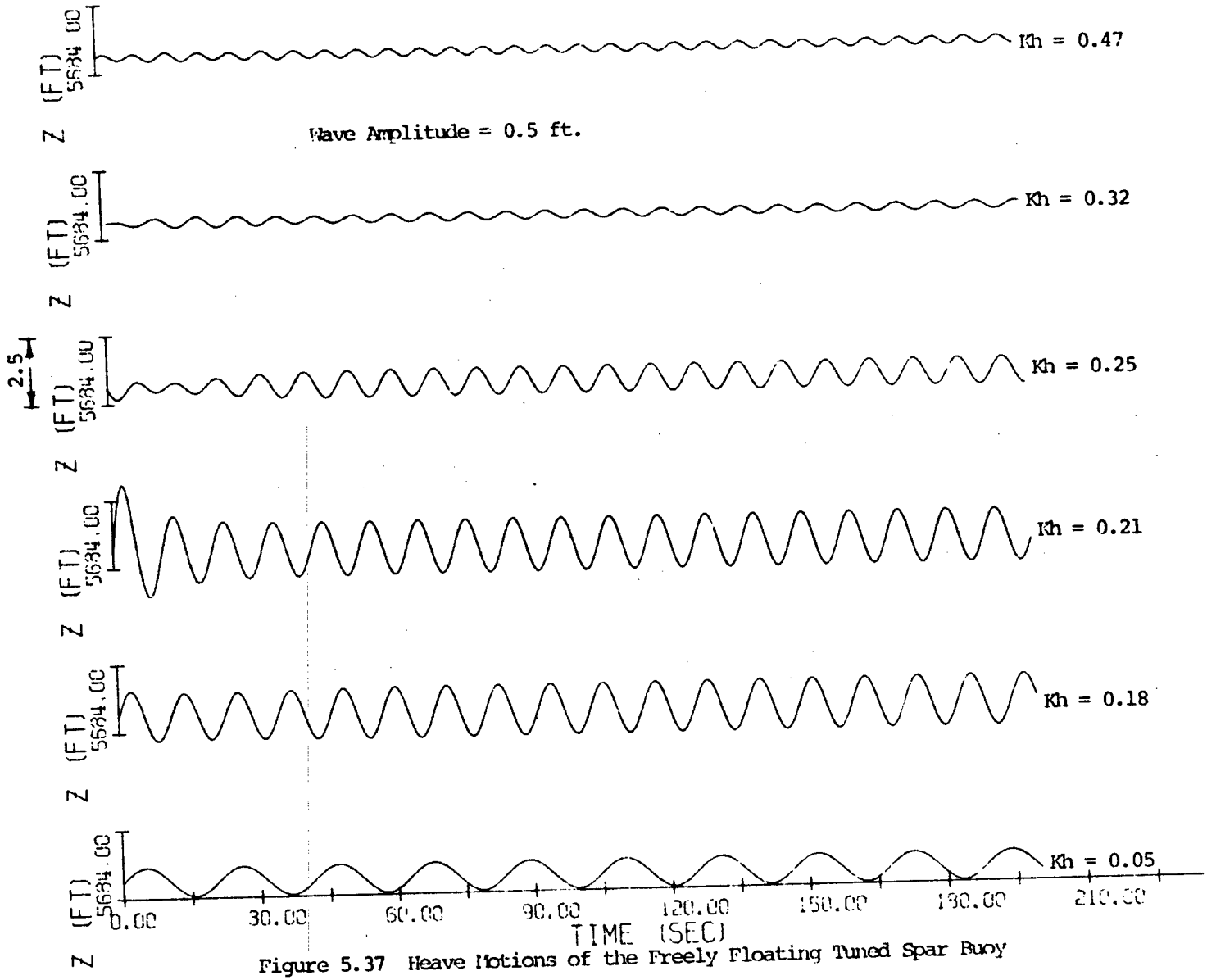


Figure 5.37 Heave Motions of the Freely Floating Tuned Spar Buoy

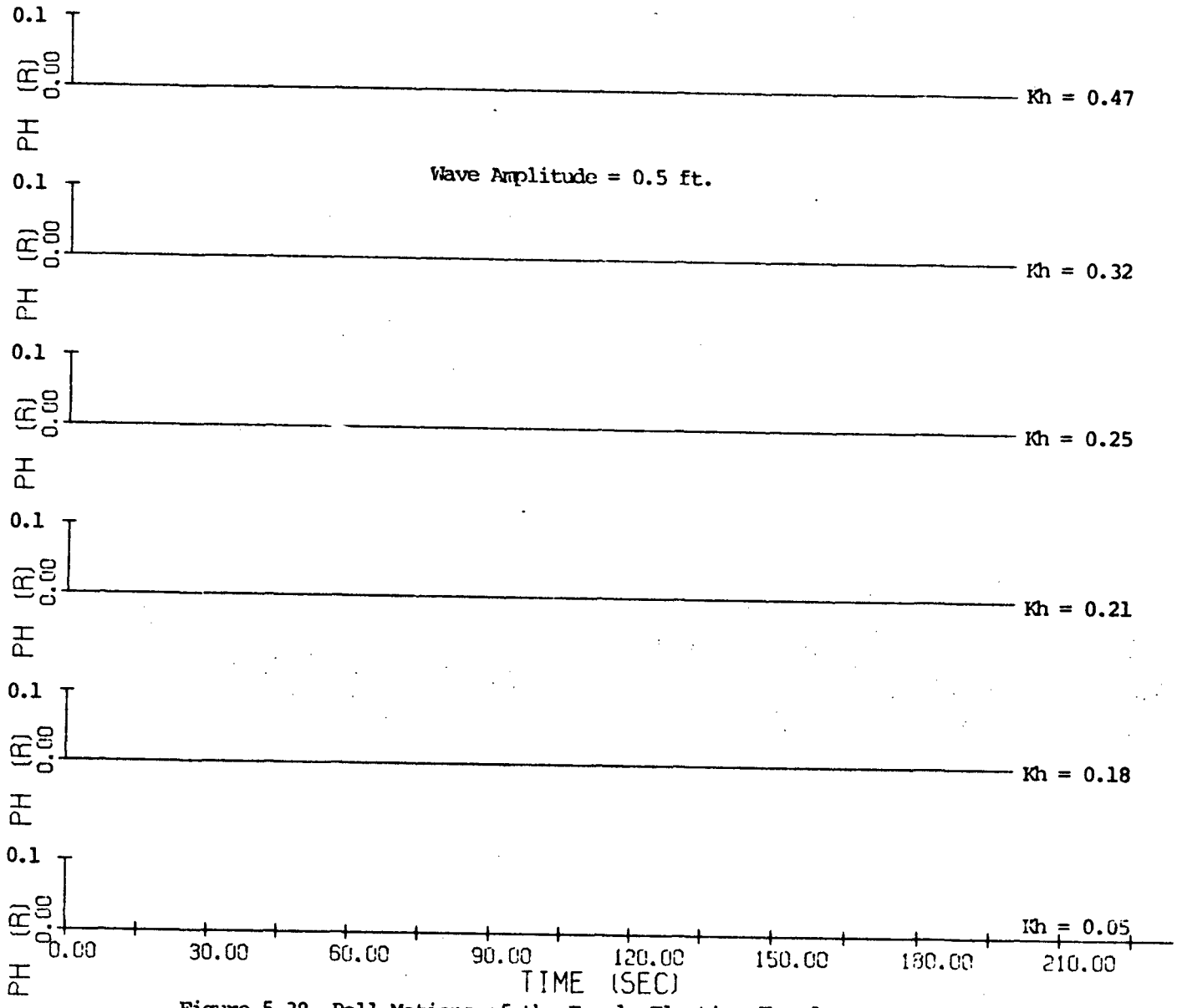


Figure 5.38 Roll Motions of the Freely Floating Tuned Spar Buoy

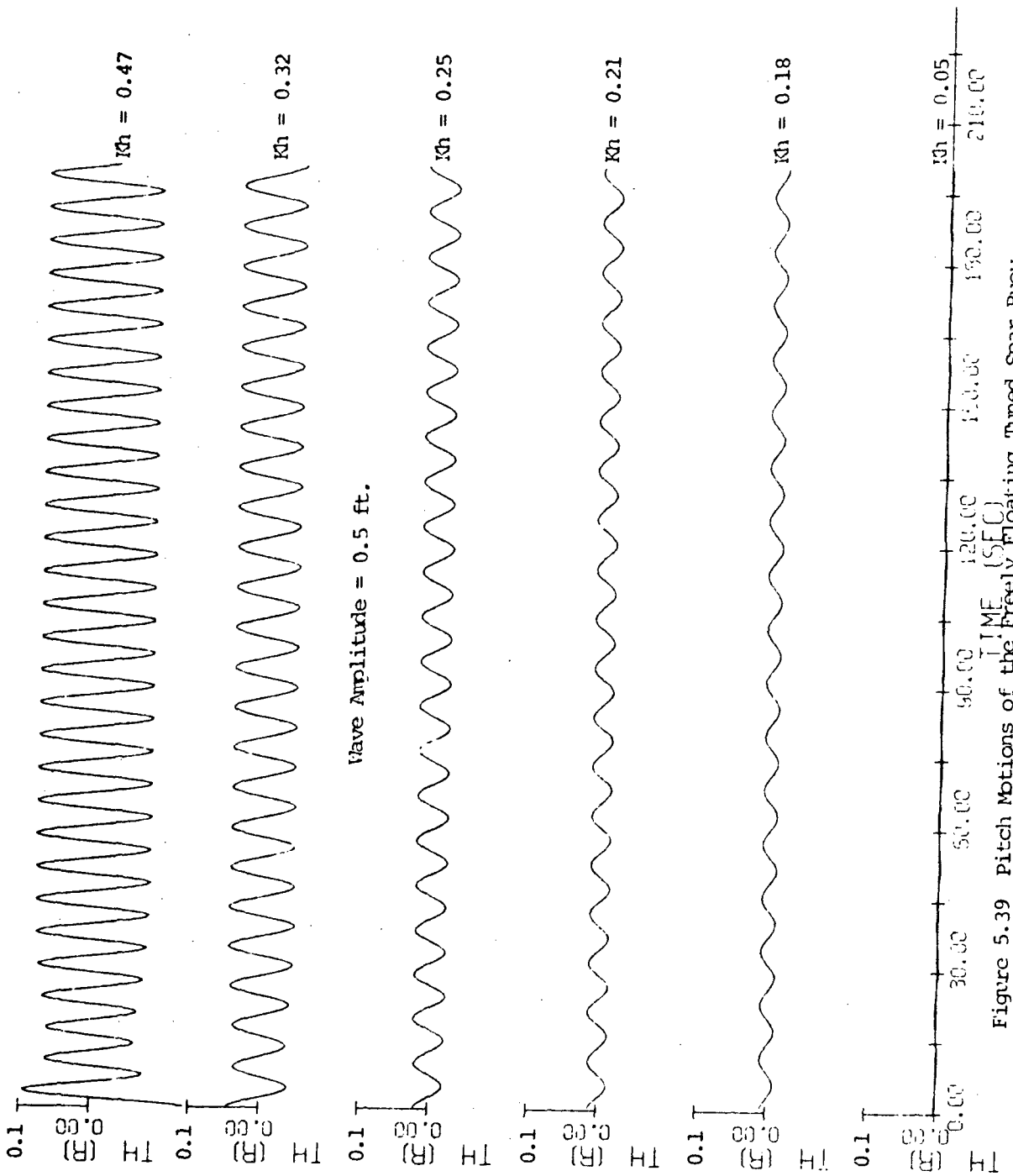


Figure 5.39 Pitch Motions of the Freely Floating Tuned Spar Buoy

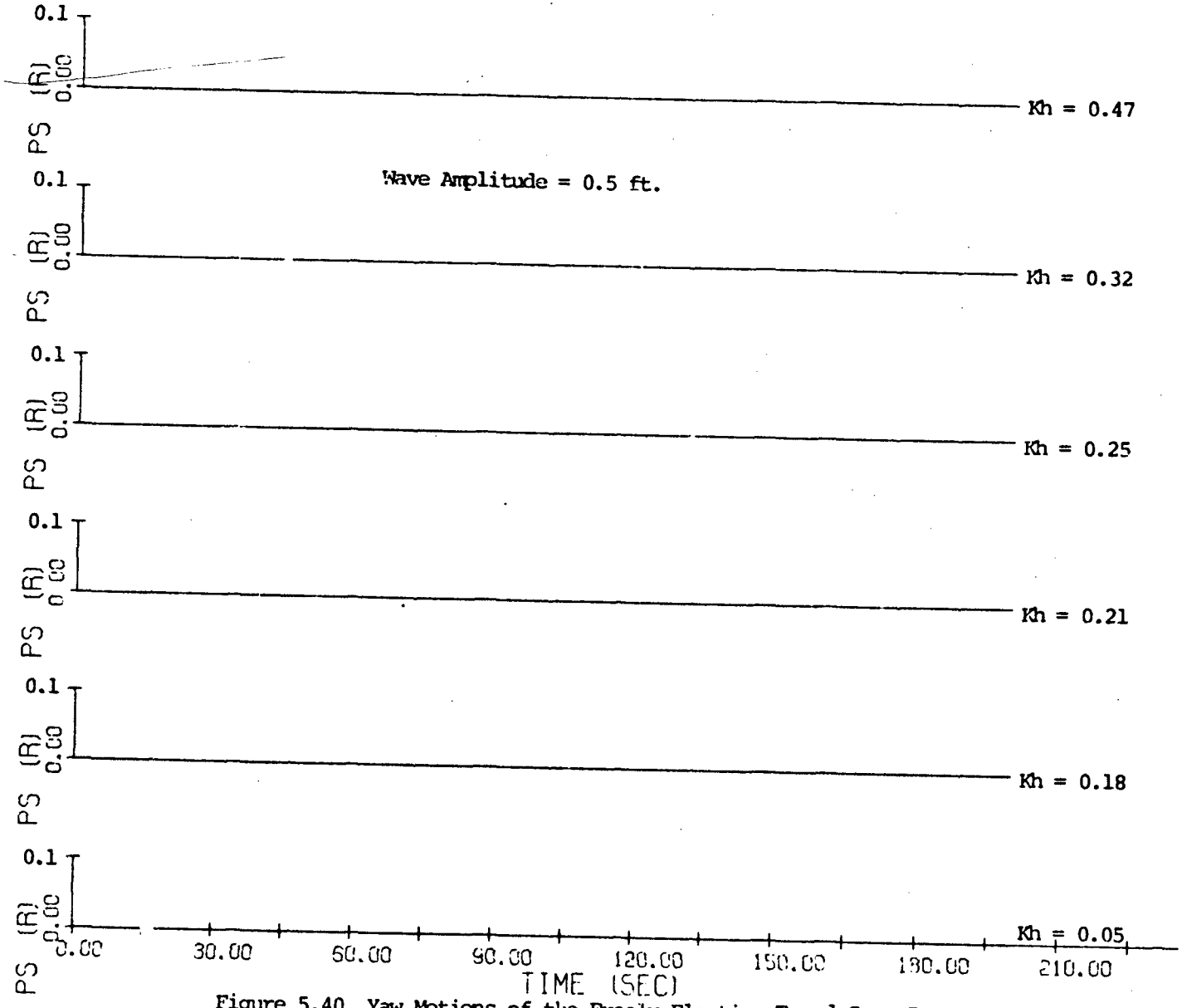


Figure 5.40 Yaw Motions of the Freely Floating Tuned Spar Buoy

```
 STBUOY.DATA
00010      1     1    0     25     6     0 5700.      0.020      200.0      0.5
00020  1.7           0.5          3.35       5.11       30.        1.0        5.27
00021  1.0           1.0
00055  3745.62       0.           0.         0.         0.         0.
00090  0.            0.           0.         0.         0.         0.
00105  0.            0.           0.         0.         0.        -18.
00110  0.            0.           0.         0.         0.         0.
00120  0.30          0.5          0.0
00130  0.55          0.5          0.0
00140  0.60          0.5          0.0
00150  0.66          0.5          0.0
00160  0.74          0.5          0.0
00170  0.90          0.5          0.0
READY
```

Table 5.6 Input Data for the 3-D Analysis of the Tuned Spar Buoy

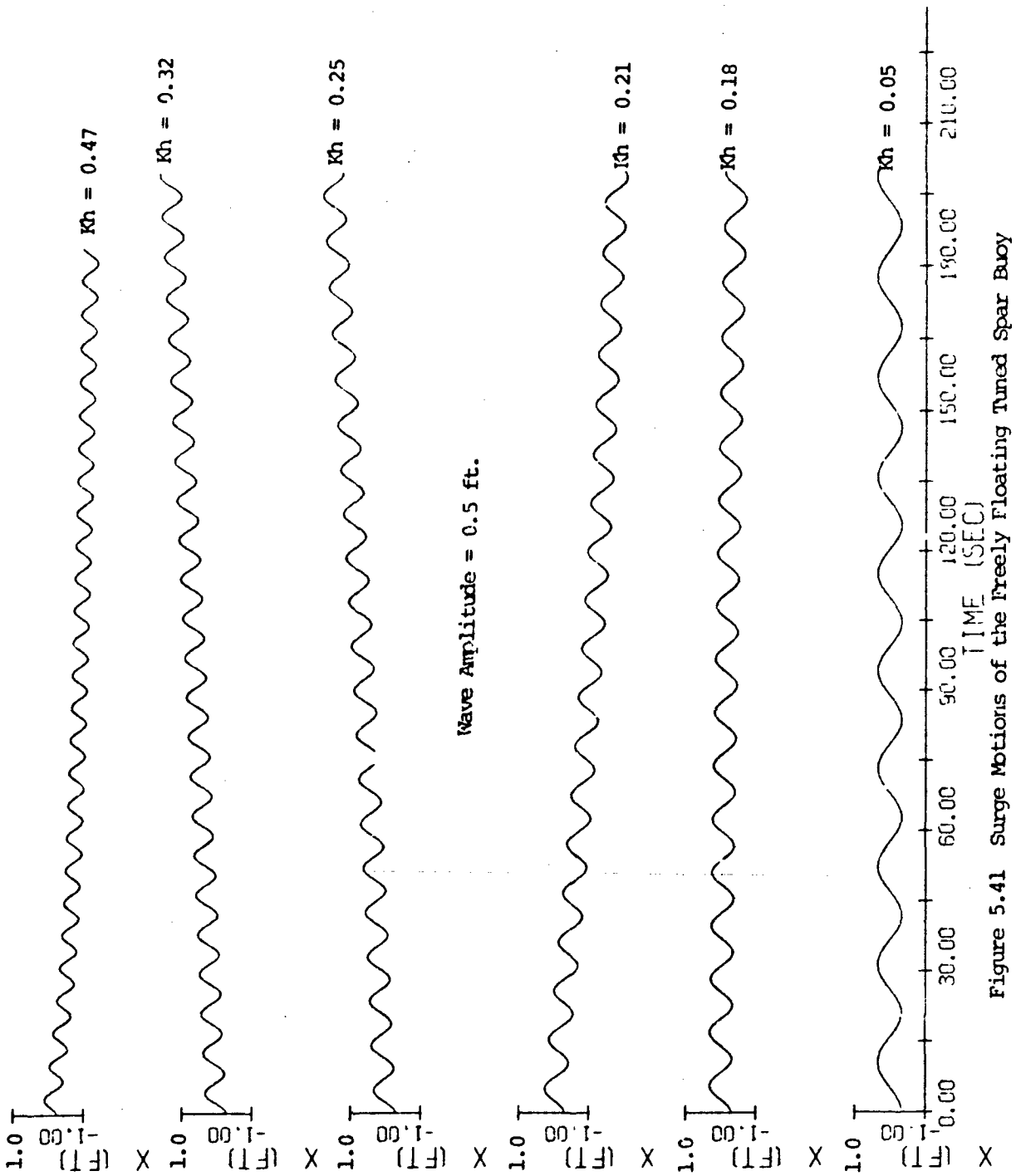


Figure 5.41 Surge Motions of the Freely Floating Tuned Spar Buoy

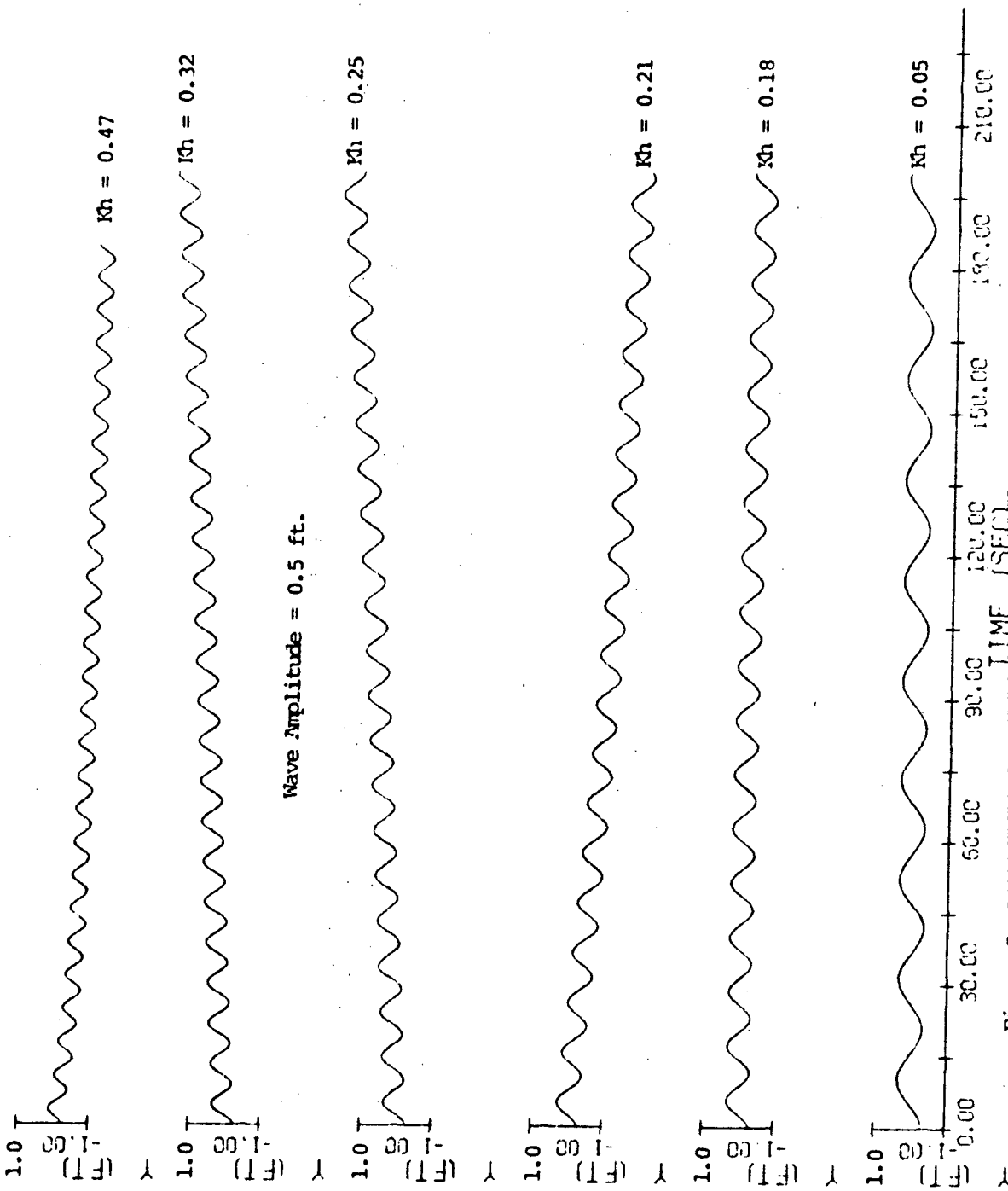


Figure 5.42 Sway Motions of the Freely Floating Tuned Spar Buoy

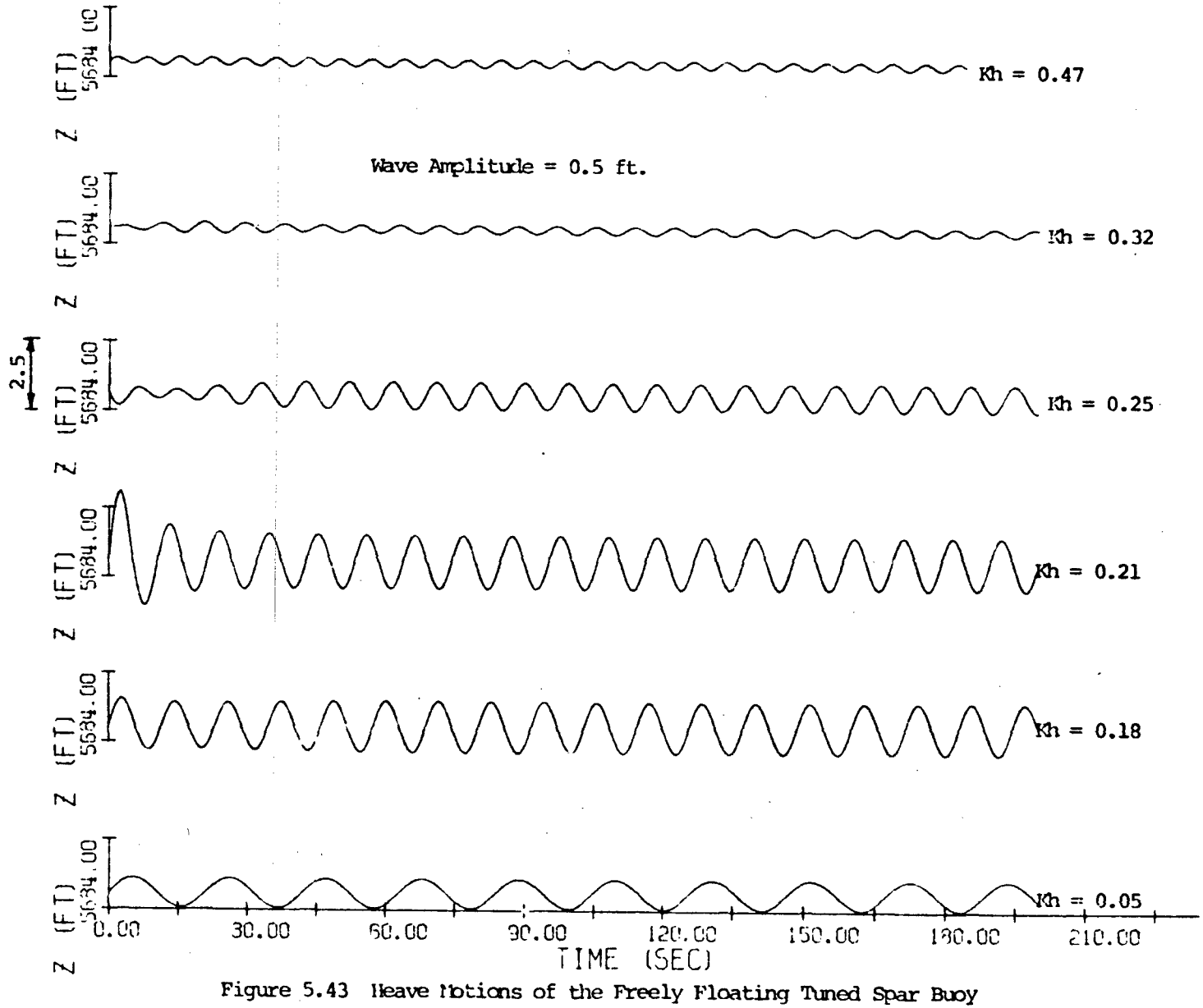


Figure 5.43 Heave Motions of the Freely Floating Tuned Spar Buoy

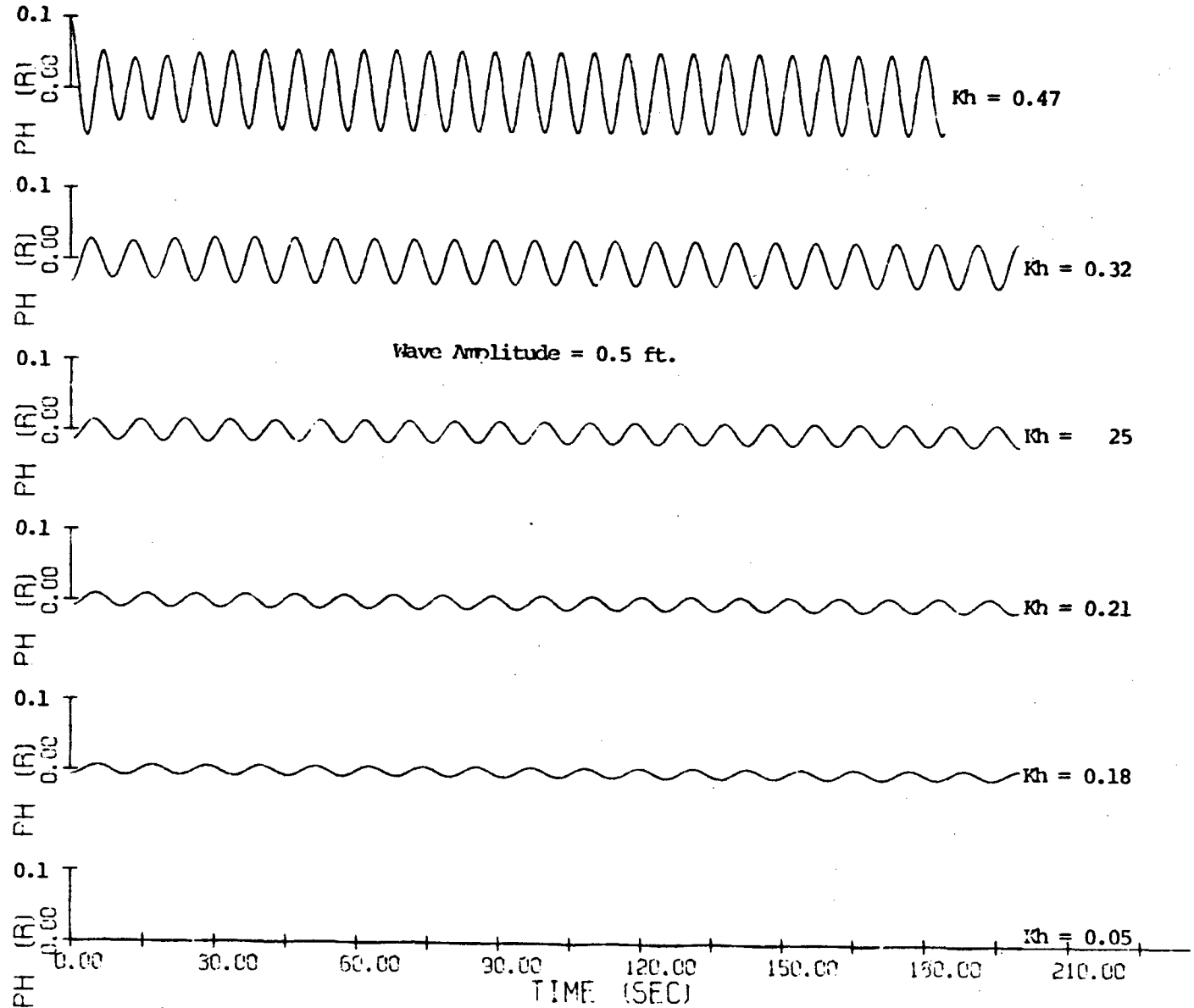


Figure 5.44 Roll Motions of the Freely Floating Tuned Spar Buoy

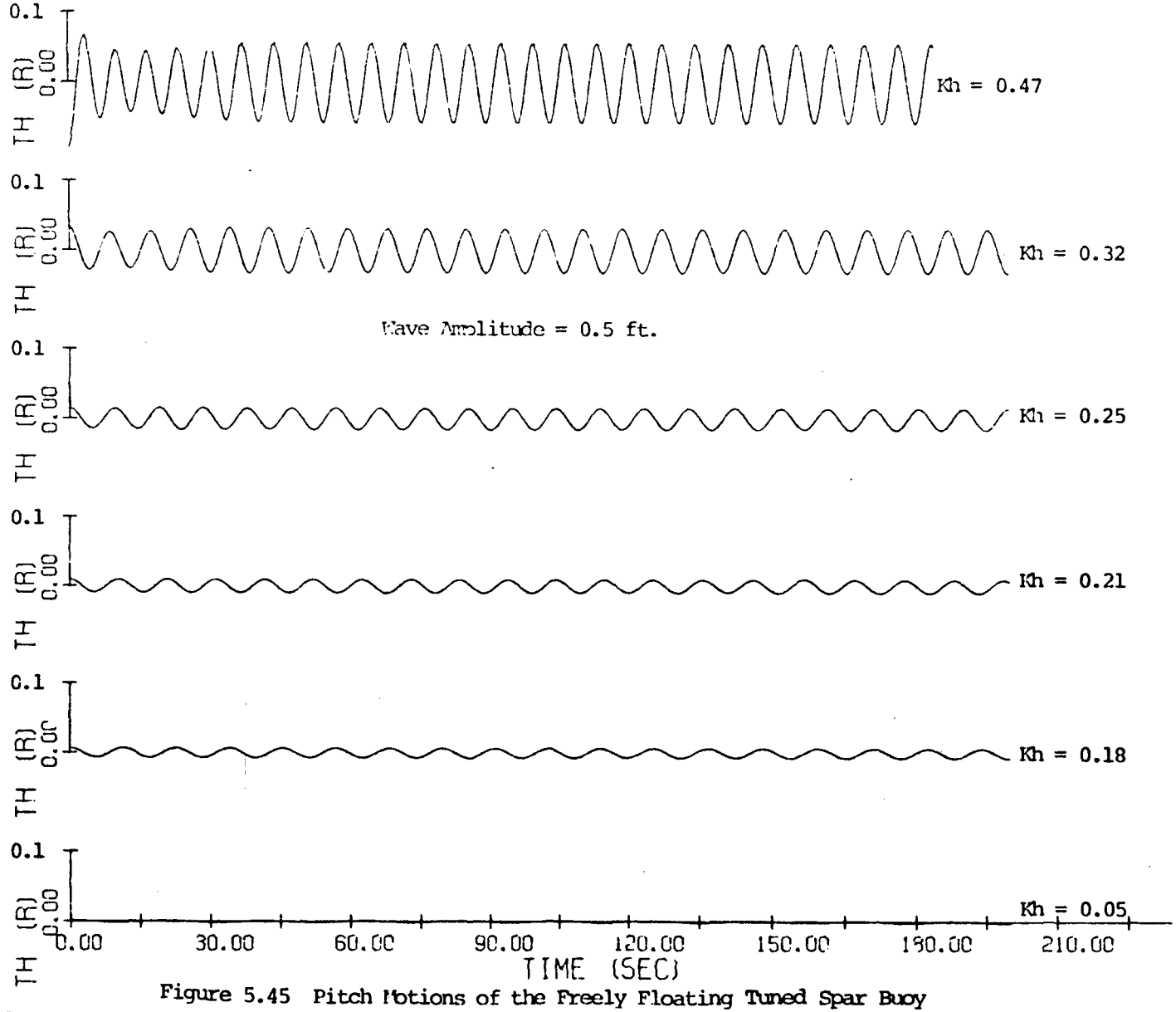


Figure 5.45 Pitch Motions of the Freely Floating Tuned Spar Buoy

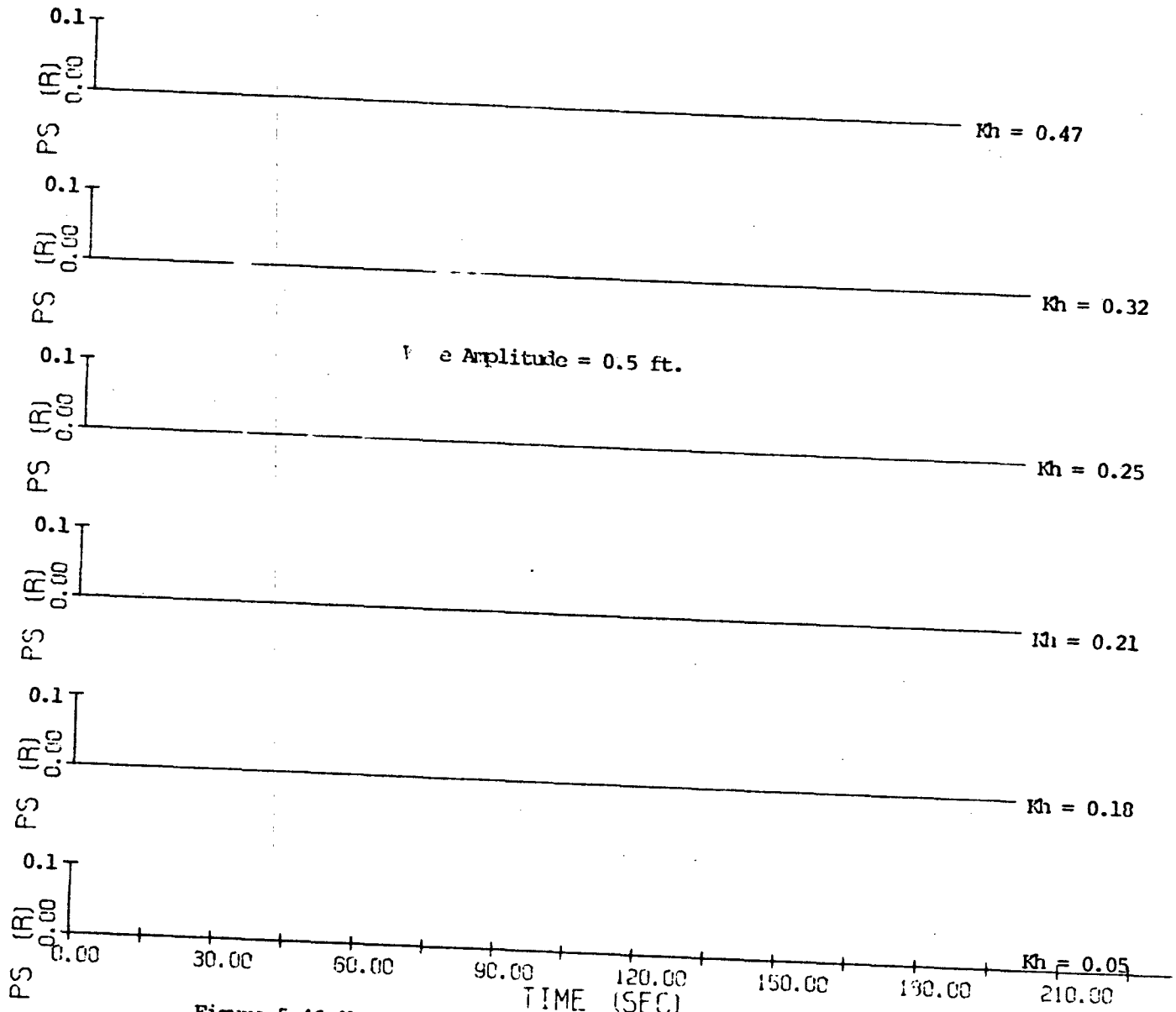


Figure 5.46 Yaw Motions of the Freely Floating Tuned Spar Buoy

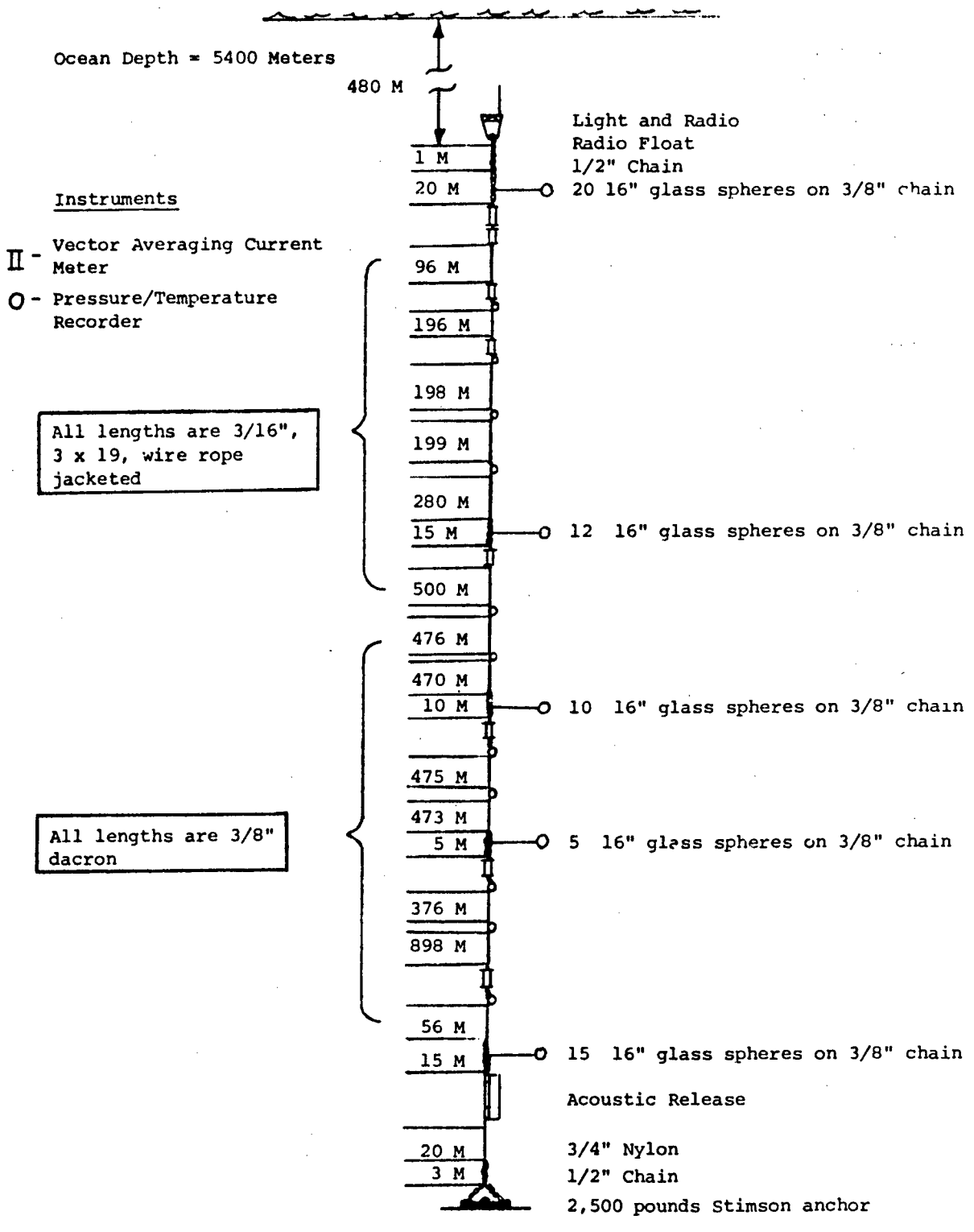


Figure 5.47 Subsurface Moored System

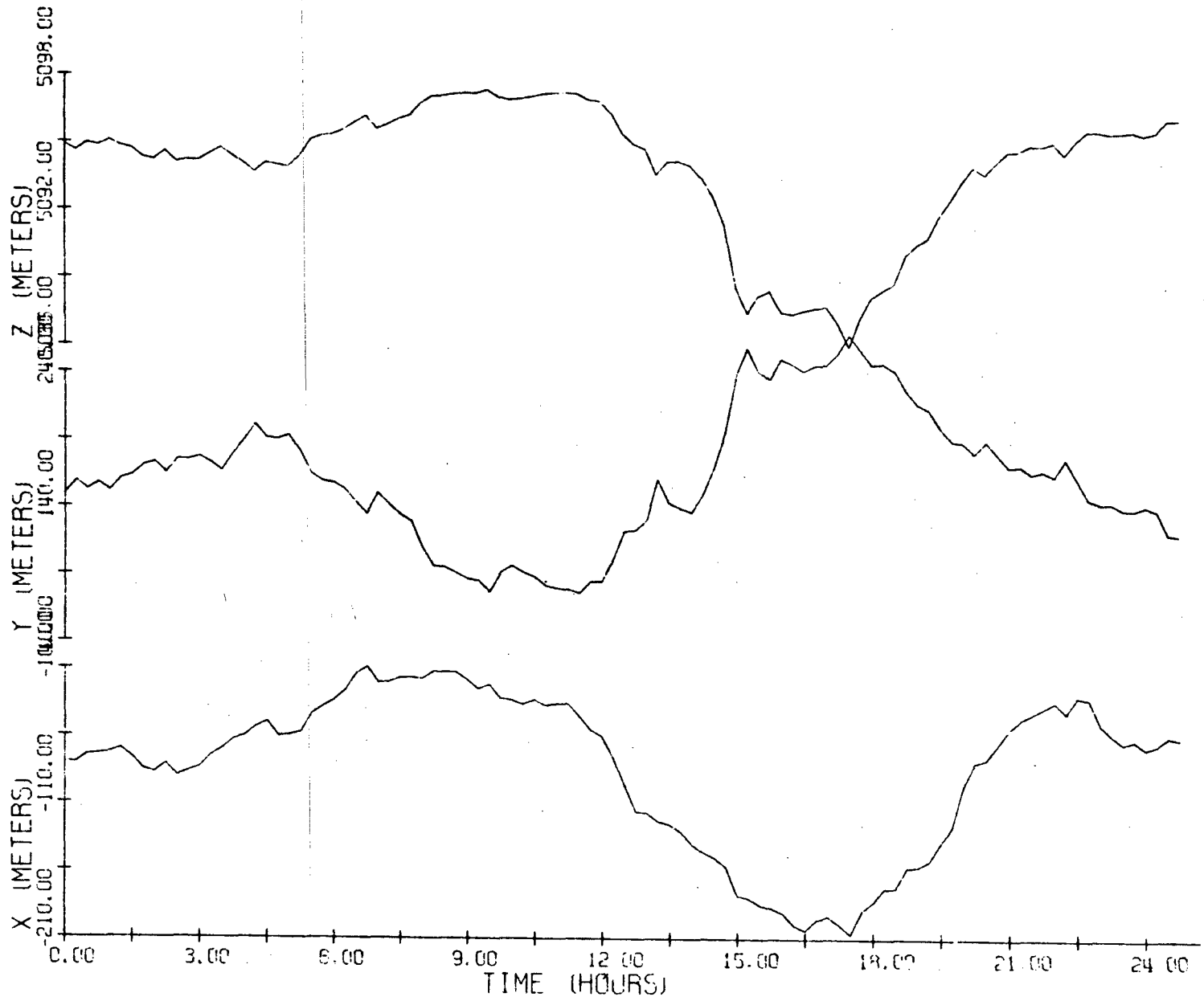


Figure 5.48 Motion of the Subsurface Moored System Top

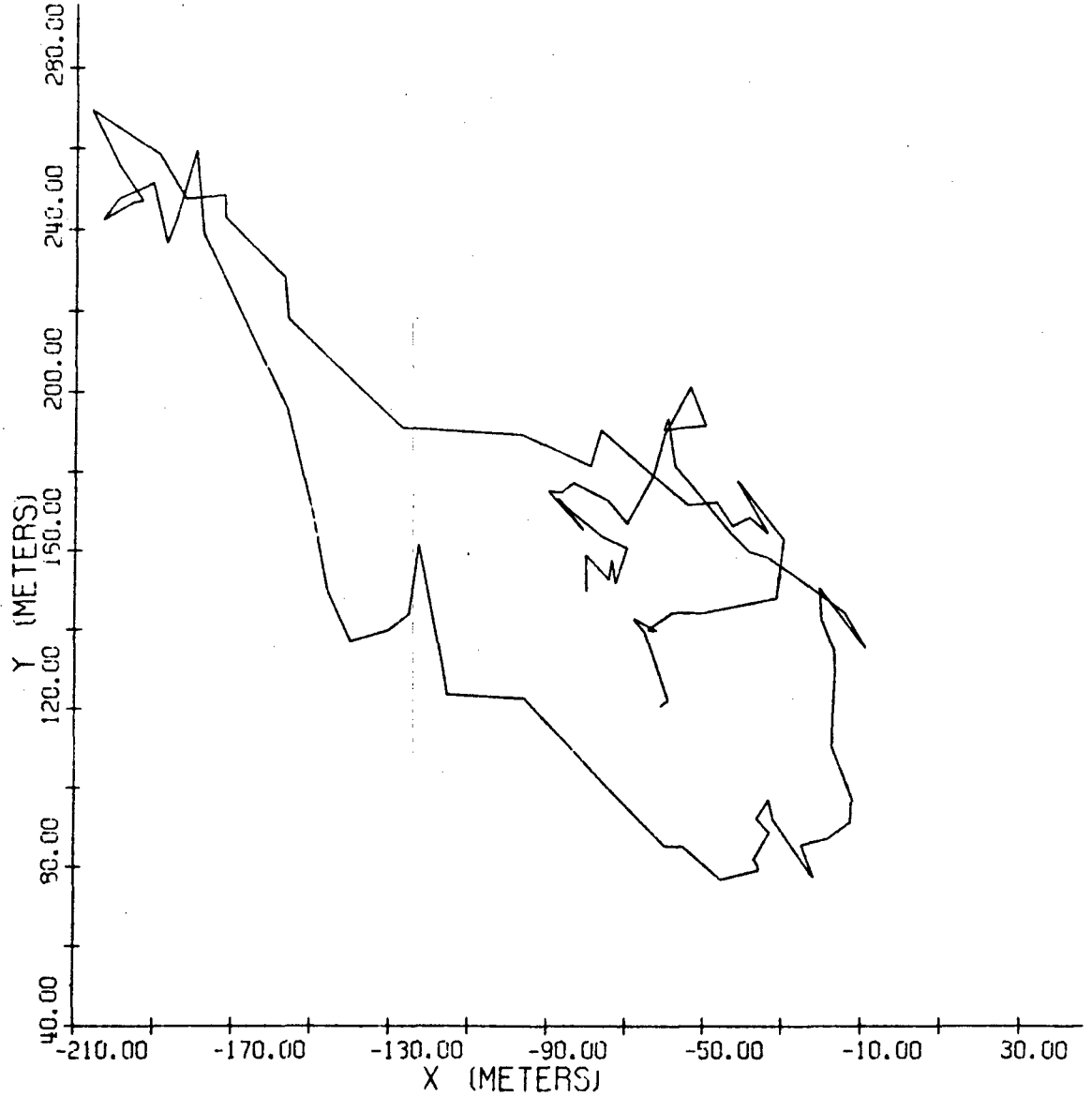


Figure 5.49 Trajectory of the Subsurface Moored System Top

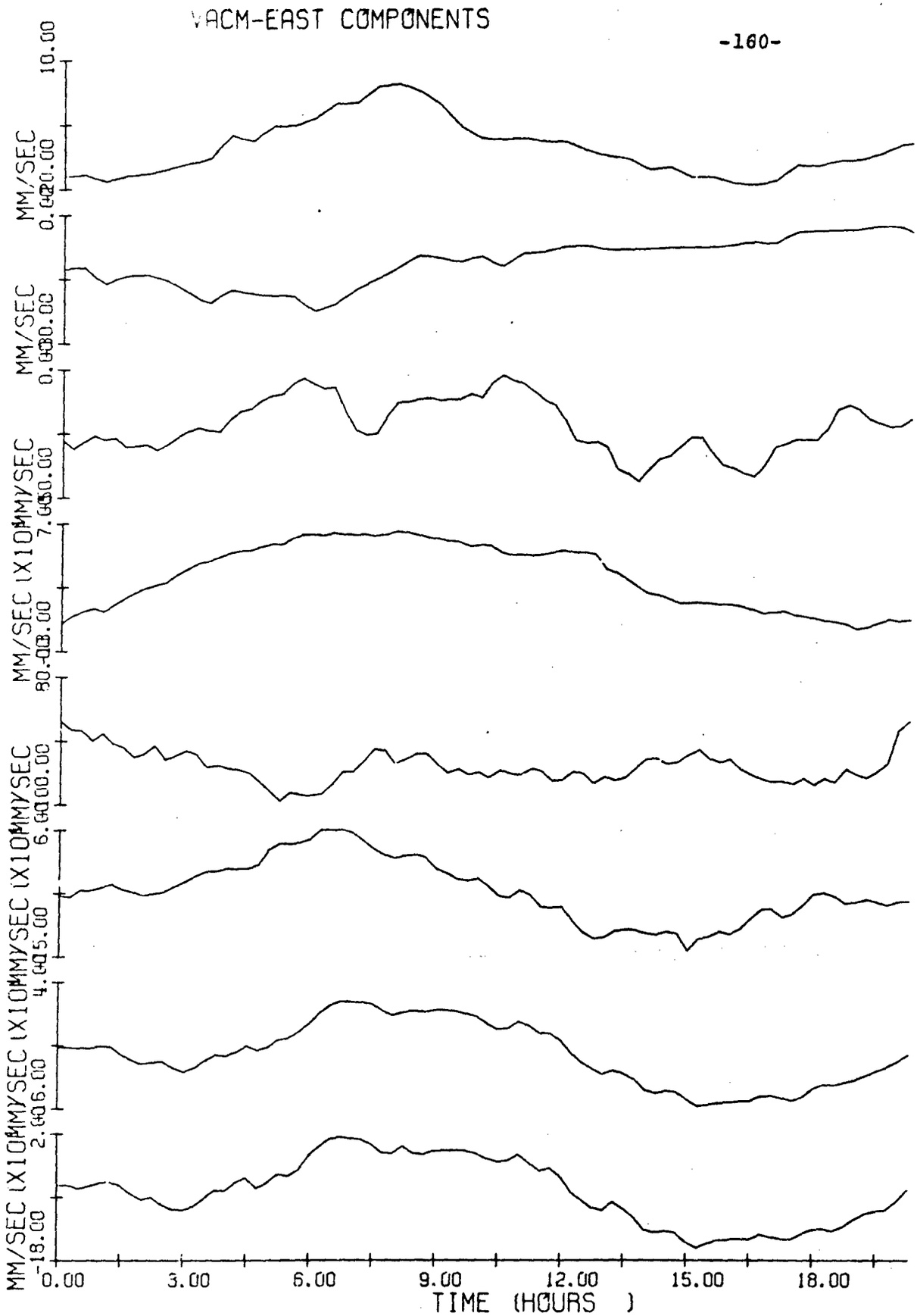


Figure 5.50 Relative Velocity Data for Subsurface Moored System

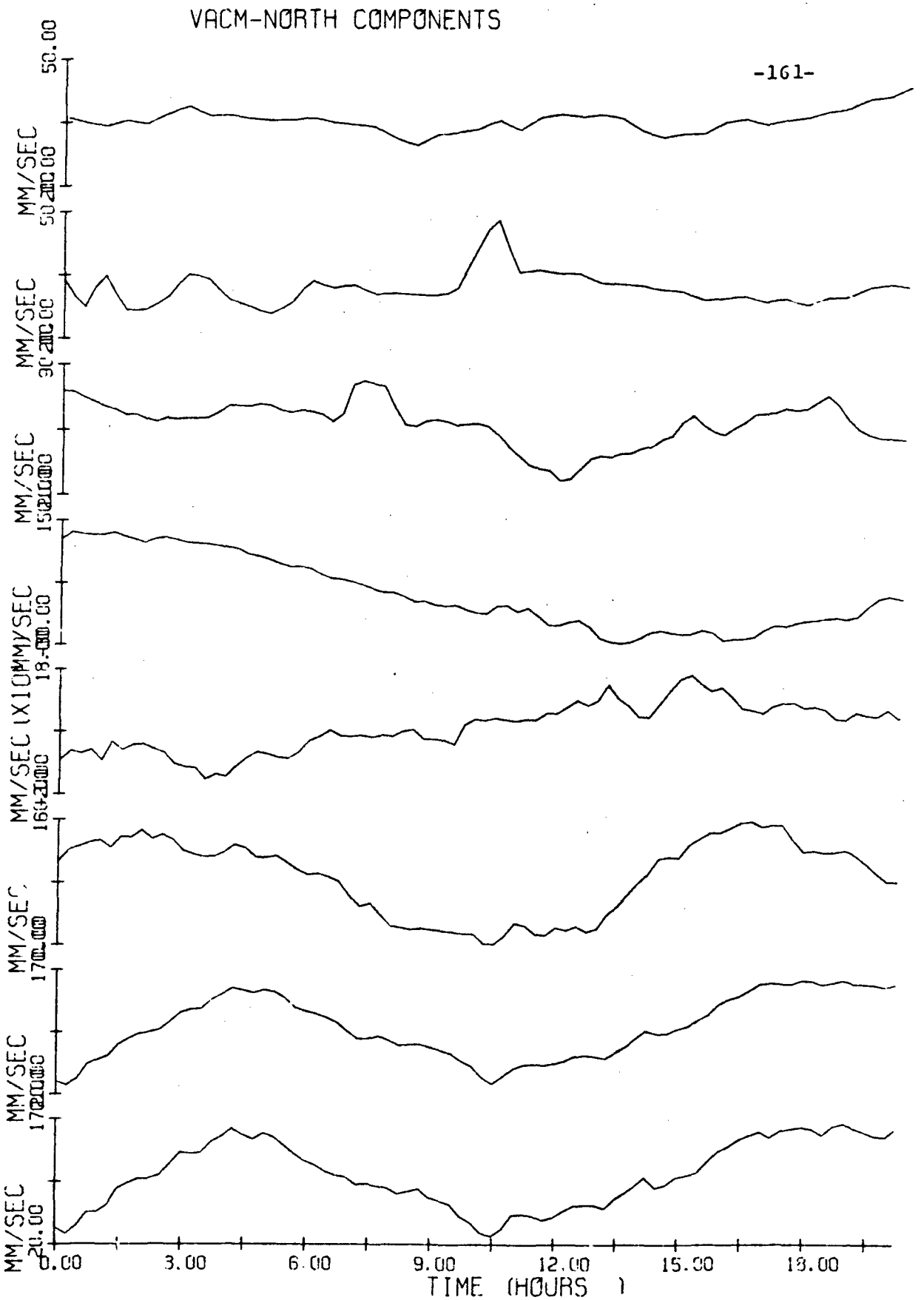


Figure 5.51 Relative Velocity Data for the Subsurface Moored System

current meter of this mooring was corrected for the effects of mooring motion, and power spectra of the uncorrected and corrected signals were compared. The relative velocity data as shown in Figures 5.50 and 5.51 was sampled every fifteen minutes at eight locations on this moori line. Computer program SSS3.FORT was used for this simulation and no surface wave was assumed present. Figures 5.52 and 5.53 show this same response with the computer program SSD3.FORT, which calculates and includes inertia forces. Overlay of the two responses shows no noticeable difference. Input data used with these two computer programs, describing the system are shown in Tables 5.7 and 5.8.

Response of the top of this mooring line to hypothetical absolute velocity data (Figures 5.54 and 5.55), is shown in Figures 5.56 and 5.57. This simulation was done with computer program SSD31.FORT and the input data shown in Table 5.9. Surface wave was again assumed to be absent in this simulation.

5.3 Surface Moored System

The three-dimensional computer program SD3.FORT is used for this simulation. The surface moored system simulated is shown in Figure 5.58. This system of spar

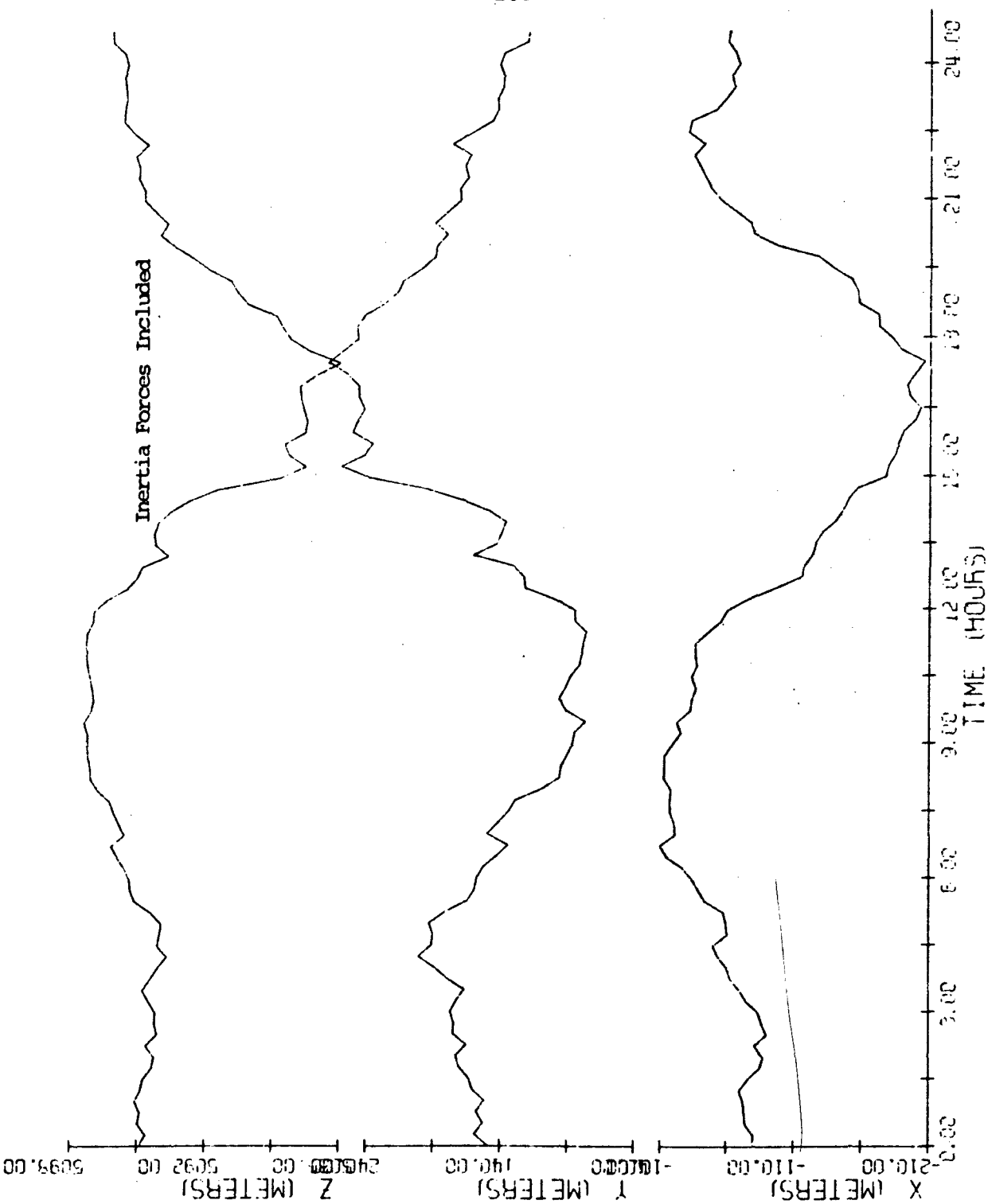


Figure 5.52 Motion of the Subsurface Moored System Top

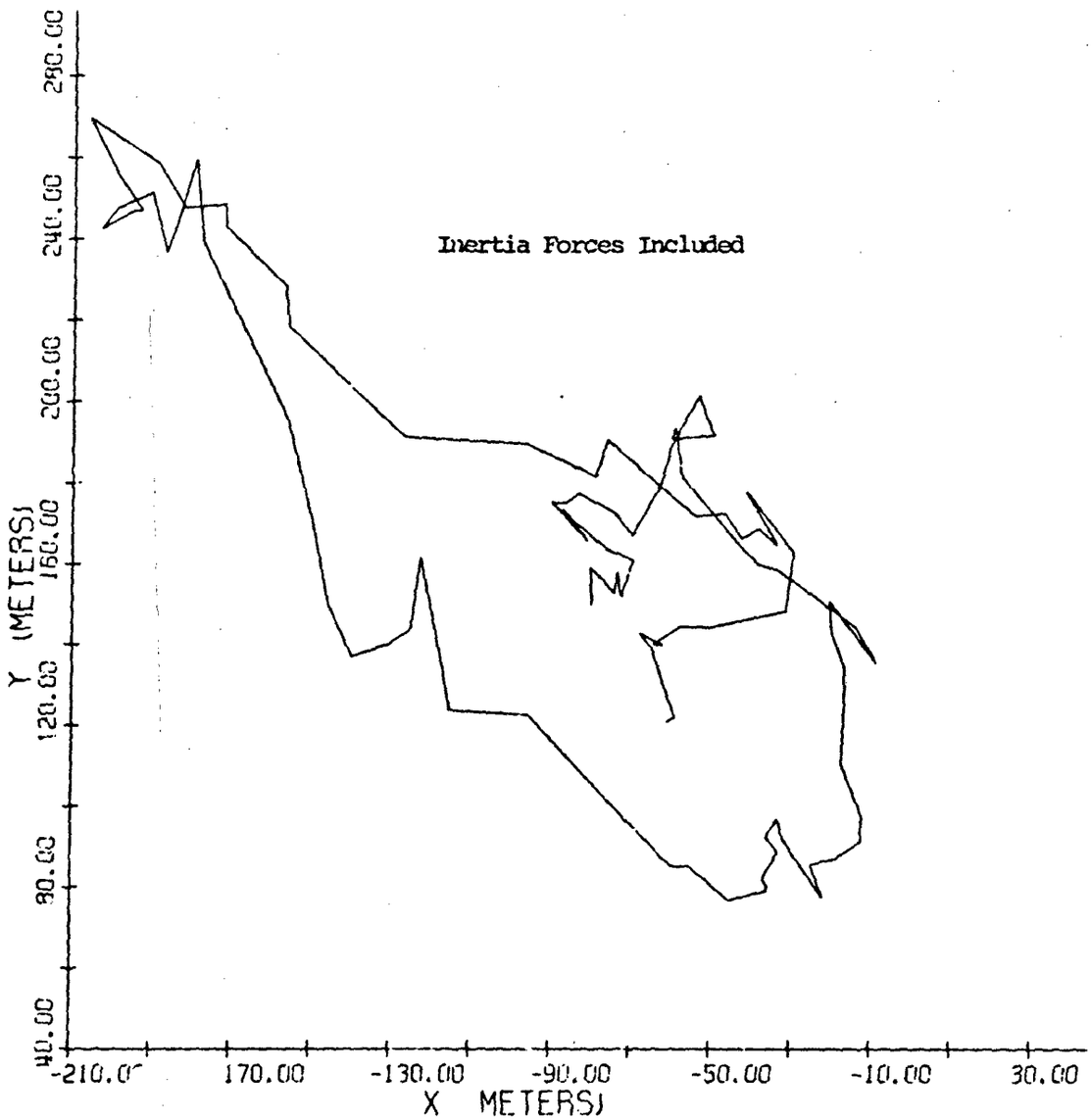


Figure 5.53 Trajectory of the Subsurface Moored System Top

```
 SSS3.DATA
00010    27    17    50  100     7  5462.     365.      3.
00020 0.       0.       0.       0.       45.      1.75     0.
00030 0.0      0.       0.       0.       976.     14.0     0.
00040 0.0      22.0     0.       0.       -107.00  4.62     0.13
00050 22.0     5.0      0.       0.       -160.    14.4     0.50
00060 123.0    5.50     0.       0.       -90.0    7.66     0.26
00070 123.0    0.       0.       0.       -24.     0.4      0.
00080 328.9    5.50     0.       0.       -90.0    7.66     0.26
00090 328.9    0.       0.       0.       -24.     0.4      0.
00100 533.4    0.       0.       0.       -24.     0.4      0.
00110 730.4    0.       0.       0.       -24.     0.4      0.
00120 1011.4   0.       0.       0.       585.6    8.4      0.
00130 1011.4   18.      0.       0.       -155.    10.65    0.34
00140 1529.9   0.       0.       0.       -24.     0.4      0.
00150 2009.9   0.       0.       0.       -24.     0.4      0.
00160 2493.4   0.       0.       0.       488.     7.0      0.
00170 2493.4   13.      0.       0.       -131.    9.5      0.31
00180 2508.4   0.       0.       0.       -24.     0.4      0.
00190 2993.9   0.       0.       0.       -20.     0.4      0.
00200 3498.9   0.       0.       0.       244.     3.5      0.
00210 3498.9   8.       0.       0.       -79.     5.76     0.15
00220 3508.9    0.      0.       0.        -24.     0.41     0.
00230 3885.9    0.      0.       0.        -24.     0.4      0.
00240 4791.9   2.5      0.       0.       -53.     4.6      0.12
00250 4796.4   0.       0.       0.       -24.     0.4      0.
00260 4860.9   0.       0.       0.       732.     10.5     0.
00270 4860.9   18.      0.       0.       -152.    7.15     0.2
00280 4898.9   4.5      0.       0.       -30.     0.93     0.03
00290 0.269      96.          0.239     0.154     10000000.
00300 0.269      200.40       0.239     0.154     10000000.
00310 0.269      199.00       0.239     0.154     10000000.
```

Table 5.7 Input Data for the Subsurface Moored System

```
00320 0.269     197.00    0.239     0.154     10000000.
00330 0.269     281.      0.239     0.154     10000000.
00340 0.269     500.50    0.239     0.154     10000000.
00350 0.375     480.00    0.148     0.0375    1123.
00360 0.375     483.50    0.148     0.0375    1110.
00370 0.5       2.0       0.262     0.0667    1578.
00380 0.375     485.50    0.148     0.0375    1558.
00390 0.375     505.00    0.148     0.0375    1544.
00400 0.50      2.0       0.262     0.0667    1765.
00410 0.375     377.0     0.148     0.0375    1749.
00420 0.375     906.00    0.148     0.0375    1736.
00430 0.5       2.0       0.262     0.0667    1690.
00440 0.375     64.5      0.148     0.0375    1672.
00450 0.75      20.       0.476     0.0457    2407.
00460 0.064     0.        0.        0.        1.0       1.4       0.028
00470 0.064     0.        0.        0.        1.0       1.4       0.028
00480 0.064     0.        0.        0.        1.0       1.4       0.028
00490 0.064     0.        0.        0.        1.0       1.4       0.028
00500 0.064     0.        0.        0.        1.0       1.4       0.028
00510 0.064     0.        0.        0.        1.0       1.4       0.028
00520 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00530 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00540 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00550 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00560 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00570 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00580 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00590 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00600 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00610 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00620 88.       1.94      74.8      1.87      1.0       1.4       0.028
00630  392.75    443.5     596.5     1048.5    1400.     3450.5    4664.
```

Table 5.7 (cont.) Input Data for the Subsurface Moored System

```
 SSD3.DATA
00010    27    17    50   100     7  5462.     365.      3.
00020 0.       0.        2.02     1.01     45.       1.75     0.
00030 0.0      0.       18.76     9.38     976.      14.0     0.
00040 0.0      22.0      3.70     3.70     -107.00   4.62     0.13
00050 22.0     5.0      10.25    10.25     -160.     14.4     0.50
00060 123.0    5.50      5.43     5.43     -90.0     7.66     0.26
00070 123.0    0.        2.02     1.01     -24.      0.4      0.
00080 328.9    5.50      5.43     5.43     -90.0     7.66     0.26
00090 328.9    0.        2.02     1.01     -24.      0.4      0.
00100 533.4    0.        2.02     1.01     -24.      0.4      0.
00110 730.4    0.        2.02     1.01     -24.      0.4      0.
00120 1011.4   0.       11.25     5.63     585.6     8.4      0.
00130 1011.4   18.       7.61     7.61     -155.     10.65    0.34
00140 1529.9   0.        2.02     1.01     -24.      0.4      0.
00150 2009.9   0.        2.02     1.01     -24.      0.4      0.
00160 2493.4   0.        9.38     4.69     488.      7.0      0.
00170 2493.4   13.       6.83     6.83     -131.     9.5      0.31
00180 2508.4   0.        2.02     1.01     -24.      0.4      0.
00190 2993.9   0.        2.02     1.01     -20.      0.4      0.
00200 3498.9   0.        4.69     2.35     244.      3.5      0.
00210 3498.9   8.        5.06     5.06     -79.      5.76     0.15
00220 3508.9    0.       2.02     1.01      -24.      0.41     0.
00230 3885.9    0.       2.02     1.01      -24.      0.4      0.
00240 4791.9   2.5       3.50     3.50     -53.      4.6      0.12
00250 4796.4   0.        2.02     1.01     -24.      0.4      0.
00260 4860.9   0.       14.07     7.03     732.      10.5     0.
00270 4860.9   18.       8.54     8.54     -152.     7.15     0.2
00280 4898.9   4.5       1.60     1.60     -30.      0.93     0.03
00290 0.269      96.          0.239      0.154      10000000.
00300 0.269      200.40       0.239      0.154      10000000.
00310 0.269      199.00       0.239      0.154      10000000.
```

Table 5.8 Input Data for the Subsurface Moored System

```
00320 0.269     197.00    0.239     0.154     10000000.
00330 0.269     281.      0.239     0.154     10000000.
00340 0.269     500.50    0.239     0.154     10000000.
00350 0.375     480.00    0.148     0.0375    1123.
00360 0.375     483.50    0.148     0.0375    1110.
00370 0.5       2.0       0.262     0.0667    1578.
00380 0.375     485.50    0.148     0.0375    1558.
00390 0.375     505.00    0.148     0.0375    1544.
00400 0.50      2.0       0.262     0.0667    1765.
00410 0.375     377.0     0.148     0.0375    1749.
00420 0.375     906.00    0.148     0.0375    1736.
00430 0.5       2.0       0.262     0.0667    1690.
00440 0.375     64.5      0.148     0.0375    1672.
00450 0.75      20.       0.476     0.0457    2407.
00460 0.064     0.        0.        0.        1.0       1.4       0.028
00470 0.064     0.        0.        0.        1.0       1.4       0.028
00480 0.064     0.        0.        0.        1.0       1.4       0.028
00490 0.064     0.        0.        0.        1.0       1.4       0.028
00500 0.064     0.        0.        0.        1.0       1.4       0.028
00510 0.064     0.        0.        0.        1.0       1.4       0.028
00520 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00530 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00540 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00550 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00560 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00570 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00580 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00590 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00600 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00610 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00620 88.       1.94      74.8      1.87      1.0       1.4       0.028
00630  392.75    443.5     596.5     1048.5    1400.     3450.5    4664.
```

Table 5.8 (cont.) Input Data for the Subsurface Moored System

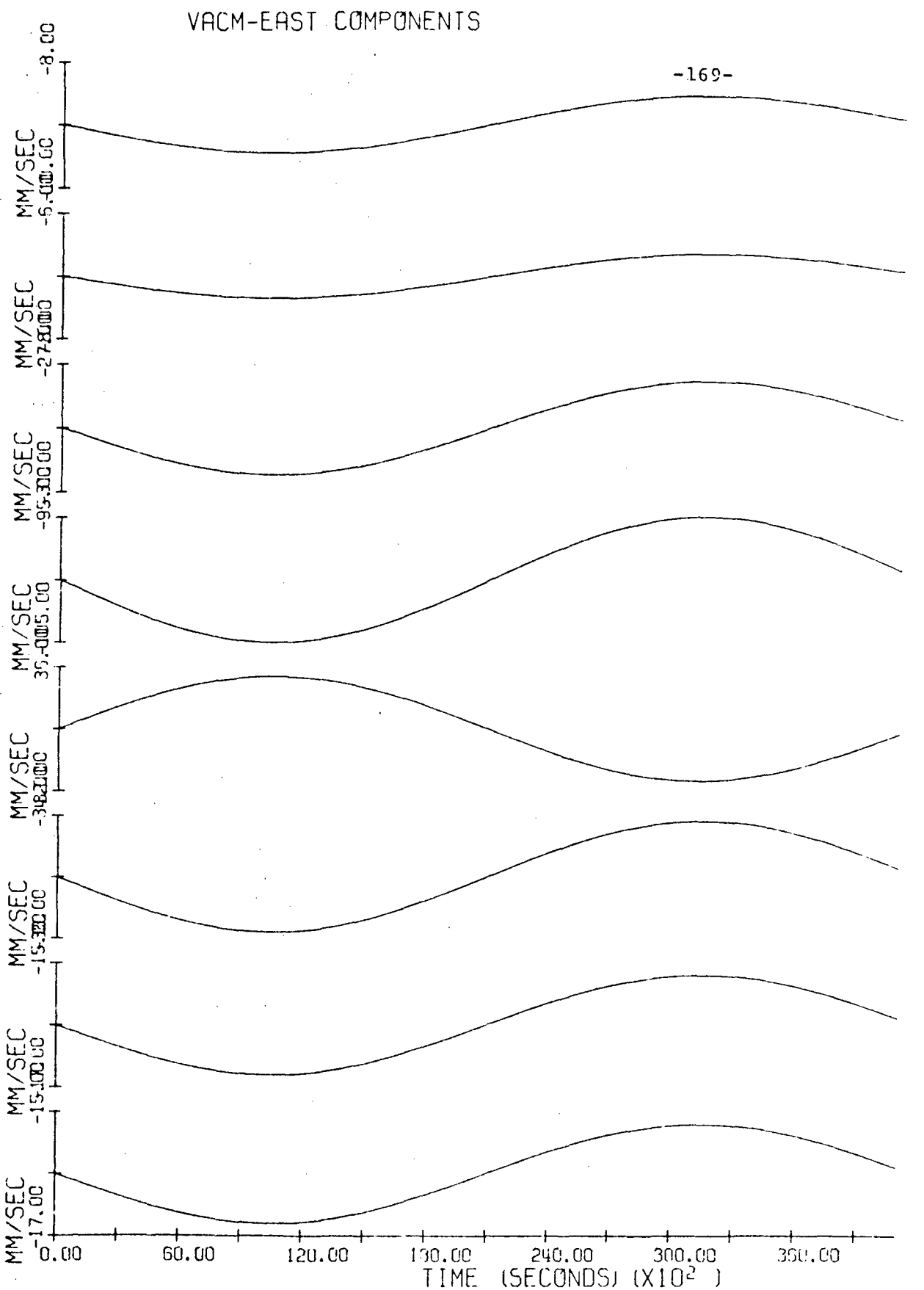


Figure 5.54 Absolute Velocity Data for the Subsurface Moored System

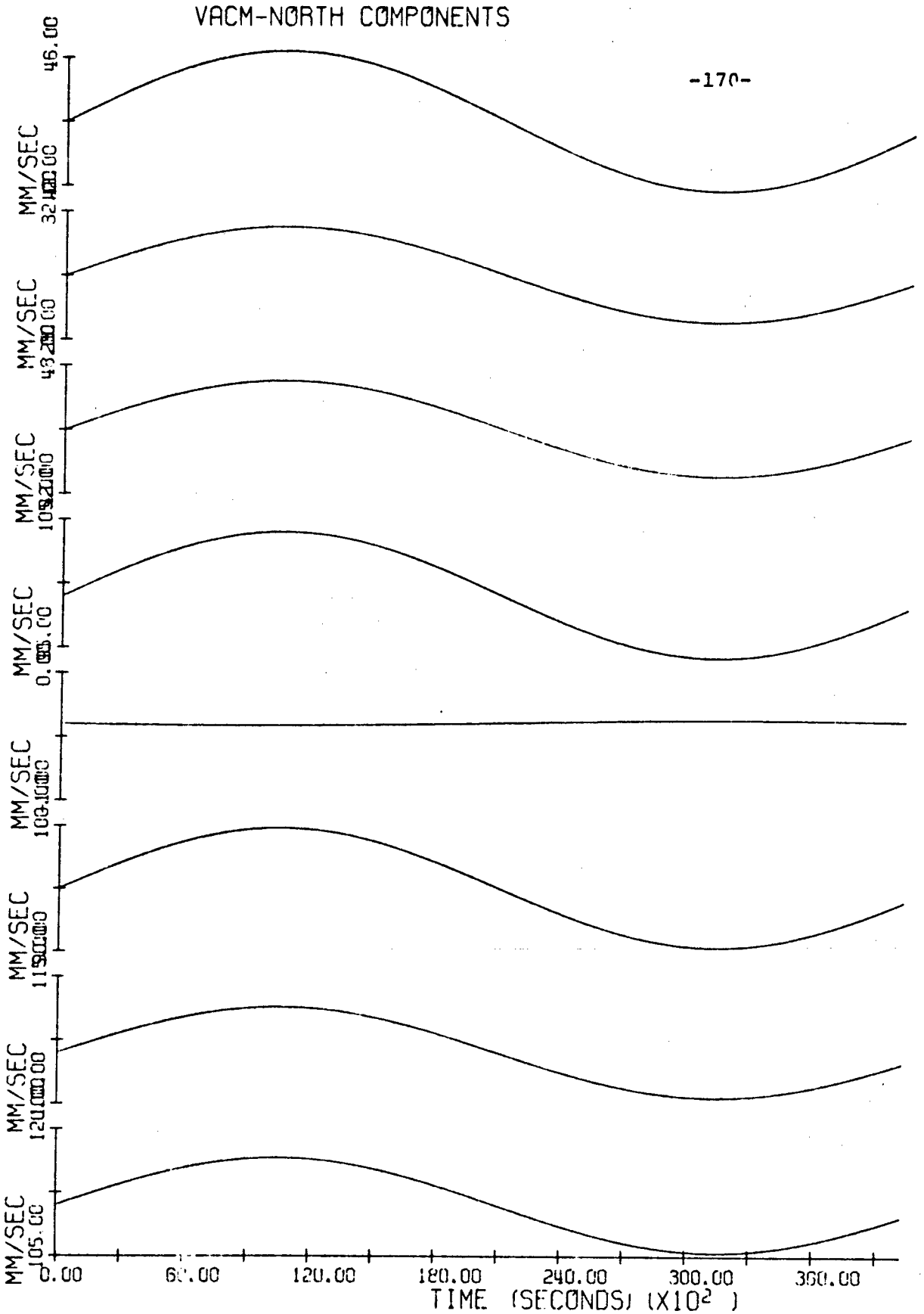


Figure 5.55 Absolute Velocity Data for the Subsurface Moored System

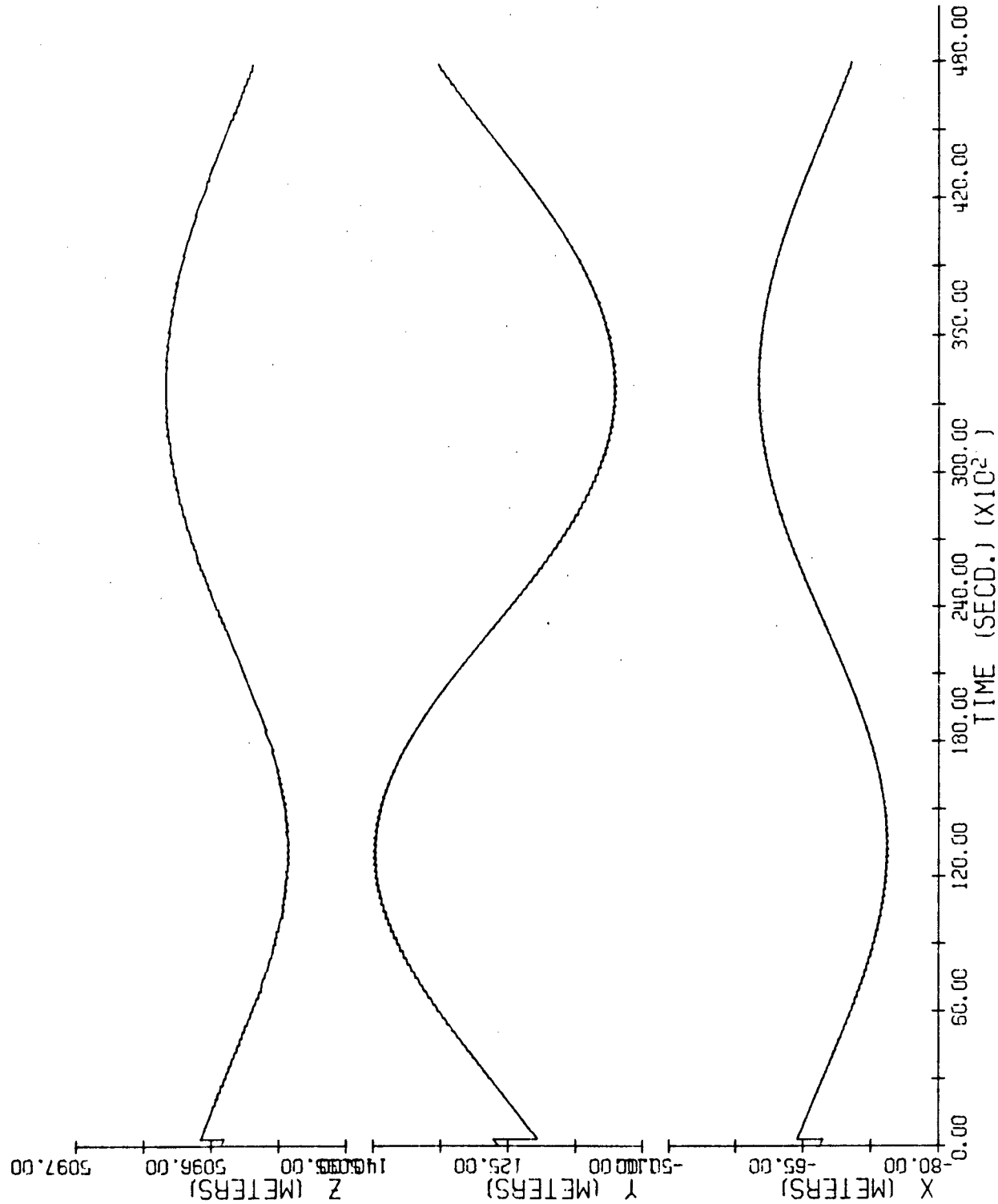


Figure 5.56 Motion of the Subsurface Moored System Top

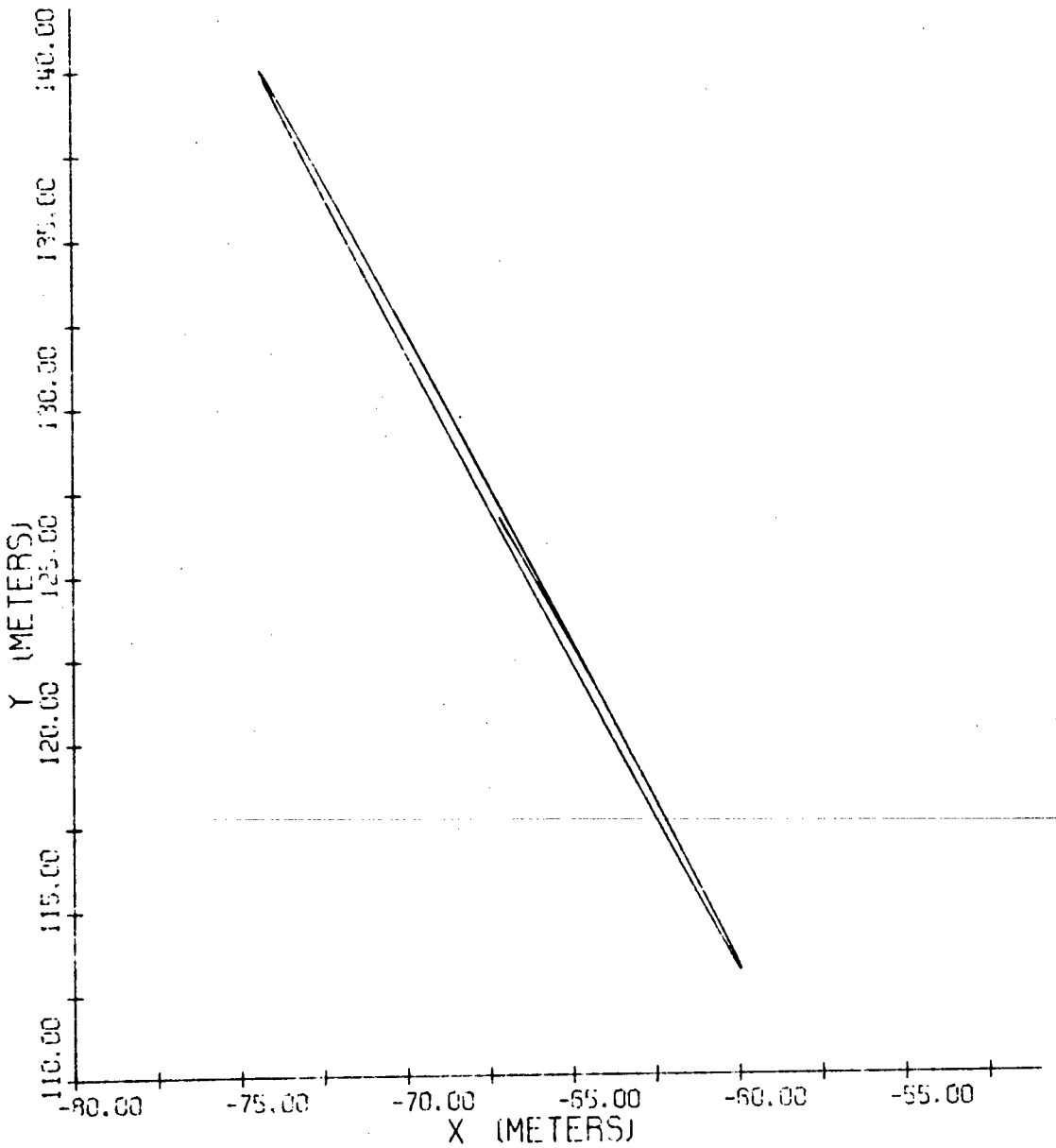


Figure 5.57 Trajectory of the Subsurface Moored System Top

```
 SSD31.DATA
00010     27    17   125     7   5462.     365.      3.          0.1
00020 0.        0.        2.02      1.01      45.       1.75      0.
00030 0.0       0.        18.76     9.38      976.      14.0      0.
00040 0.0       22.0      3.70      3.70      -107.00   4.62      0.13
00050 22.0      5.0       10.25     10.25     -160.     14.4      0.50
00060 123.0     5.50      5.43      5.43      -90.0     7.66      0.26
00070 123.0     0.        2.02      1.01      -24.      0.4       0.
00080 328.9     5.50      5.43      5.43      -90.0     7.66      0.26
00090 328.9     0.        2.02      1.01      -24.      0.4       0.
00100 533.4     0.        2.02      1.01      -24.      0.4       0.
00110 730.4     0.        2.02      1.01      -24.      0.4       0.
00120 1011.4    0.        11.25     5.63      585.6     8.4       0.
00130 1011.4    18.       7.61      7.61      -155.     10.65     0.34
00140 1529.9    0.        2.02      1.01      -24.      0.4       0.
00150 2009.9    0.        2.02      1.01      -24.      0.4       0.
00160 2493.4    0.        9.38      4.69      488.      7.0       0.
00170 2493.4    13.       6.83      6.83      -131.     9.5       0.31
00180 2508.4    0.        2.02      1.01      -24.      0.4       0.
00190 2993.9    0.        2.02      1.01      -20.      0.4       0.
00200 3498.9    0.        4.69      2.35      244.      3.5       0.
00210 3498.9    8.        5.06      5.06      -79.      5.76      0.15
00220 3508.9     0.       2.02      1.01       -24.      0.41      0.
00230 3885.9     0.       2.02      1.01       -24.      0.4       0.
00240 4791.9    2.5       3.50      3.50      -53.      4.6       0.12
00250 4796.4    0.        2.02      1.01      -24.      0.4       0.
00260 4860.9    0.        14.07     7.03      732.      10.5      0.
00270 4860.9    18.       8.54      8.54      -152.     7.15      0.2
00280 4898.9    4.5       1.60      1.60      -30.      0.93      0.03
00290 0.269       96.           0.239       0.154       10000000.
00300 0.269       200.40        0.239       0.154       10000000.
00310 0.269       199.00        0.239       0.154       10000000.
00320 0.269       197.00        0.239       0.154       10000000.
00330 0.269       281.          0.239       0.154       10000000.
00340 0.269       500.50        0.239       0.154       10000000.
```

Table 5.9 Input Data for the Subsurface Moored System

```
00350 0.375     480.00    0.148     0.0375    1123.
00360 0.375     483.50    0.148     0.0375    1110.
00370 0.5       2.0       0.262     0.0667    1578.
00380 0.375     485.50    0.148     0.0375    1558.
00390 0.375     505.00    0.148     0.0375    1544.
00400 0.50      2.0       0.262     0.0667    1765.
00410 0.375     377.0     0.148     0.0375    1749.
00420 0.375     906.00    0.148     0.0375    1736.
00430 0.5       2.0       0.262     0.0667    1690.
00440 0.375     64.5      0.148     0.0375    1672.
00450 0.75      20.       0.476     0.0457    2407.
00460 0.064     0.        0.        0.        1.0       1.4       0.028
00470 0.064     0.        0.        0.        1.0       1.4       0.028
00480 0.064     0.        0.        0.        1.0       1.4       0.028
00490 0.064     0.        0.        0.        1.0       1.4       0.028
00500 0.064     0.        0.        0.        1.0       1.4       0.028
00510 0.064     0.        0.        0.        1.0       1.4       0.028
00520 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00530 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00540 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00550 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00560 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00570 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00580 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00590 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00600 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00610 1250.     1.8       1350.     1.6       1.0       1.4       0.028
00620 88.       1.94      74.8      1.87      1.0       1.4       0.028
00630  392.75    443.5     596.5     1048.5    1400.     3450.5    4364.
00640  -16.     111.      0.        -16.      106.      0.        -36.
00650  95.      0.        34.       -0.4      0.        -100.     99.
00660  0.       -29.      45.        0.       -7.       30.        0.
00670  -9.      44.       0.
00680  0.000150 300.      300.      50000.
00690  1.0      0.0       0.0
```

Table 5.9 (cont.) Input Data for the Subsurface Moored System

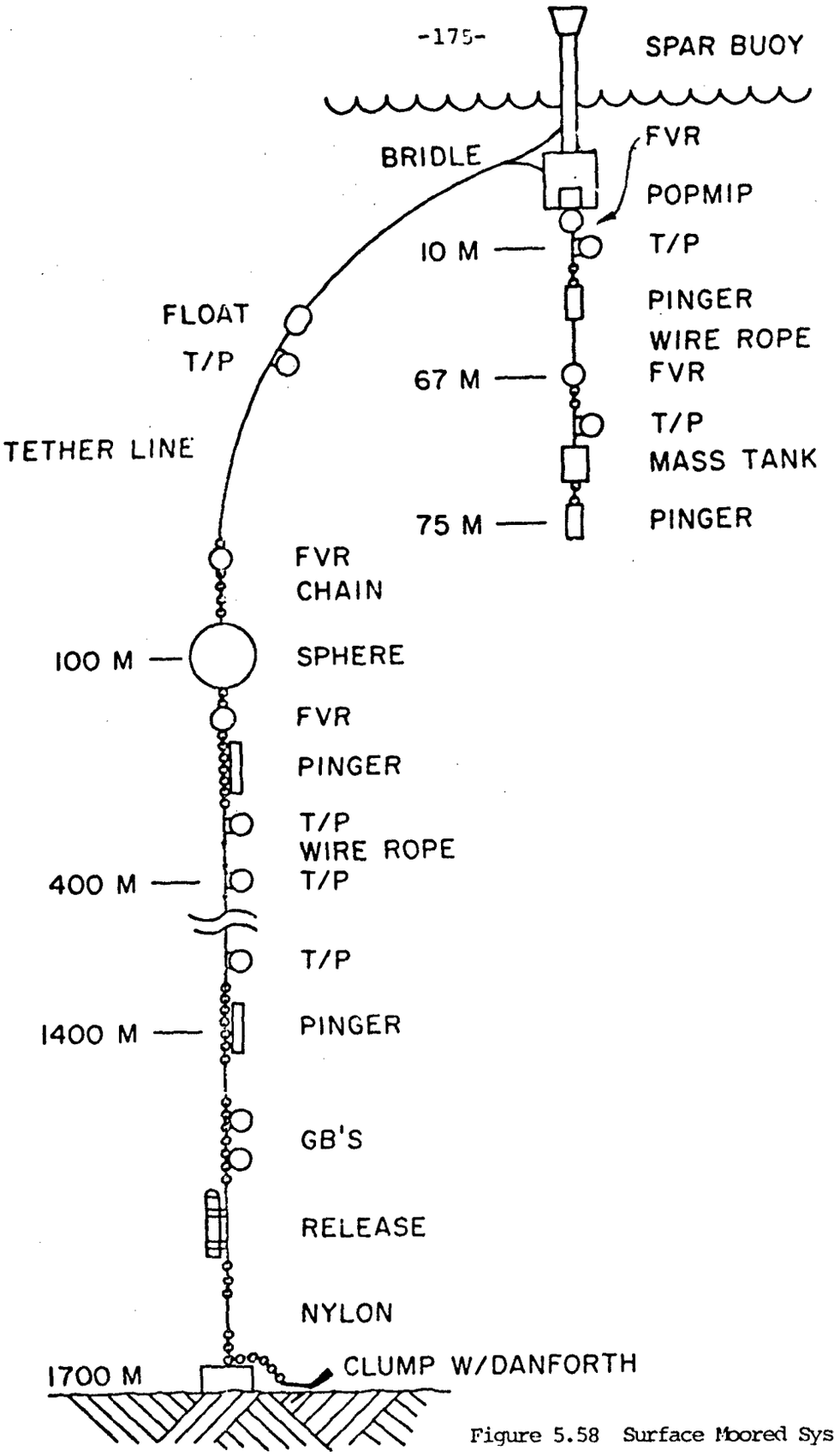
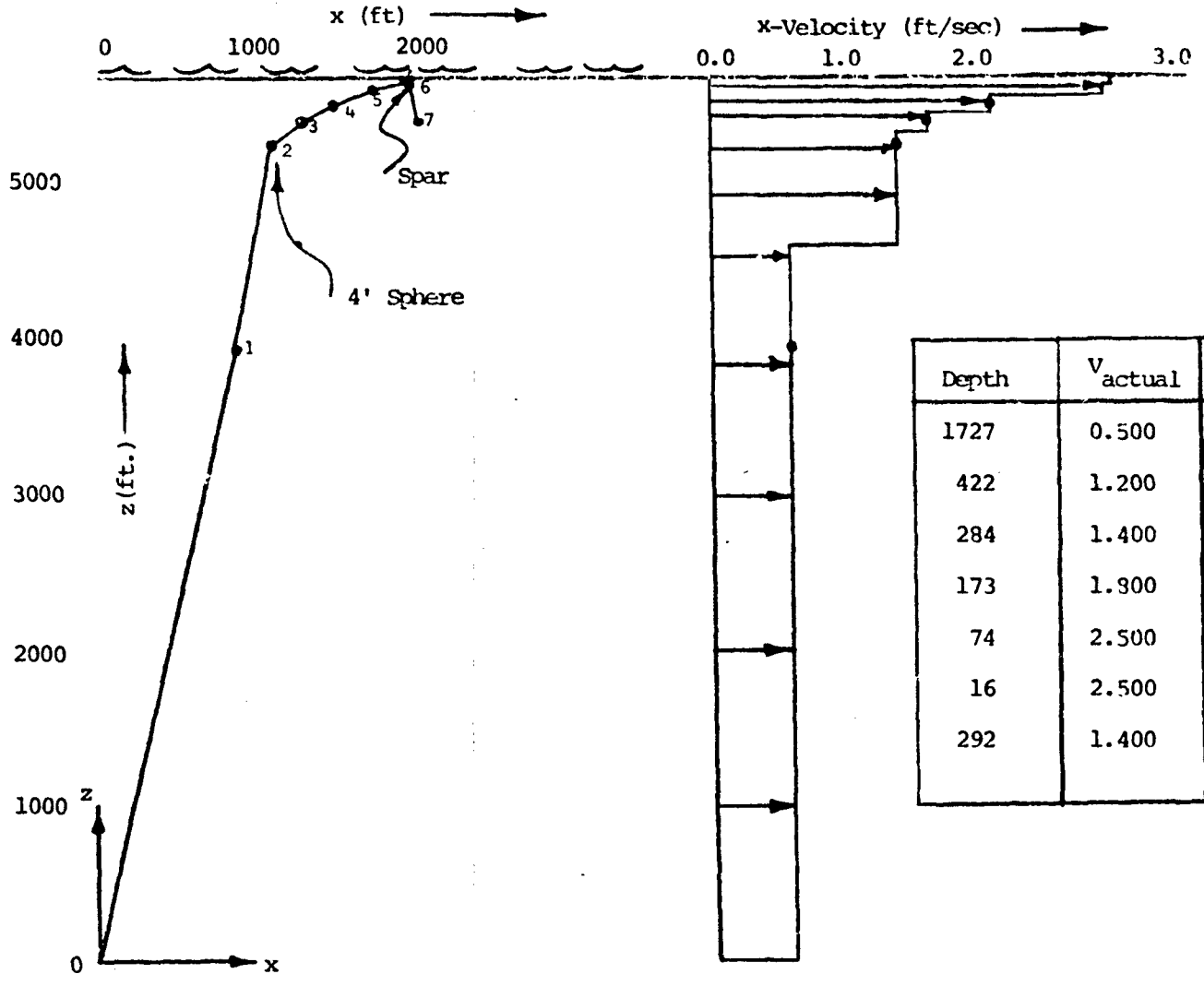


Figure 5.58 Surface Moored System

buoy/buoyant tether/subsurface mooring line with an instrument line hanging beneath the spar was deployed as part of the ONR/NDBO Mooring Dynamics Experiment during October 1976. The system is reduced to seven nodes for this simulation. These nodes are shown in Figure 5.59. Figure 5.59 shows the mean configuration of these nodes in the x-z plane. This configuration is the result of the static solution calculated by SD3.FORT using the equivalent velocity profile described in Section 4.1. The equivalent velocity profile and the actual velocity profile in the x-z plane is also shown in Figure 5.59. Figures 5.60 through 5.64 display time responses of these seven nodes to the input data presented in Table 5.10 which describes the moored system and the environment forcing this system. Forcing the system is a constant current profile and a surface wave of amplitude 1 foot, a frequency of 1 radian per second, and having a wave direction inclined at 0.1 radians with the x-axis of the earth fixed frame. Figures 5.60 through 5.62 are the plots of surge, sway and heave responses of the seven nodes. Figure 5.63 displays the roll, pitch and yaw of the tuned spar buoy (node 6), and Figure 5.64 displays tensions in the seven segments of mooring lines preceding the seven nodes.



Depth	V_{actual}	$V_{equivalent}$
1727	0.500	0.500
422	1.200	1.200
284	1.400	1.400
173	1.800	1.800
74	2.500	2.502
16	2.500	2.555
292	1.400	1.400

Figure 5.59 Static (Mean) Configuration of Surface Moored System

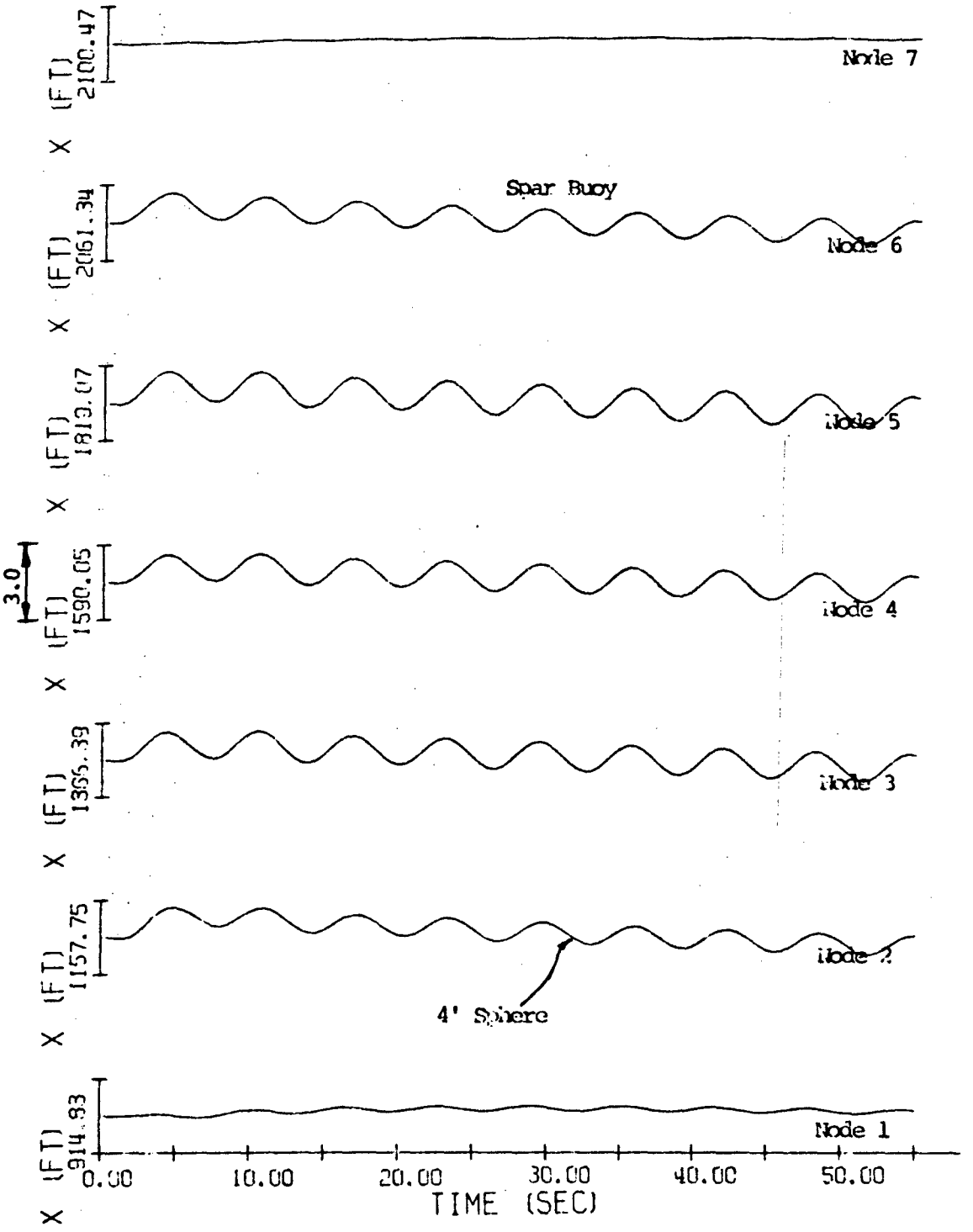


Figure 5.60 Surge Motions of a Surface Moored System

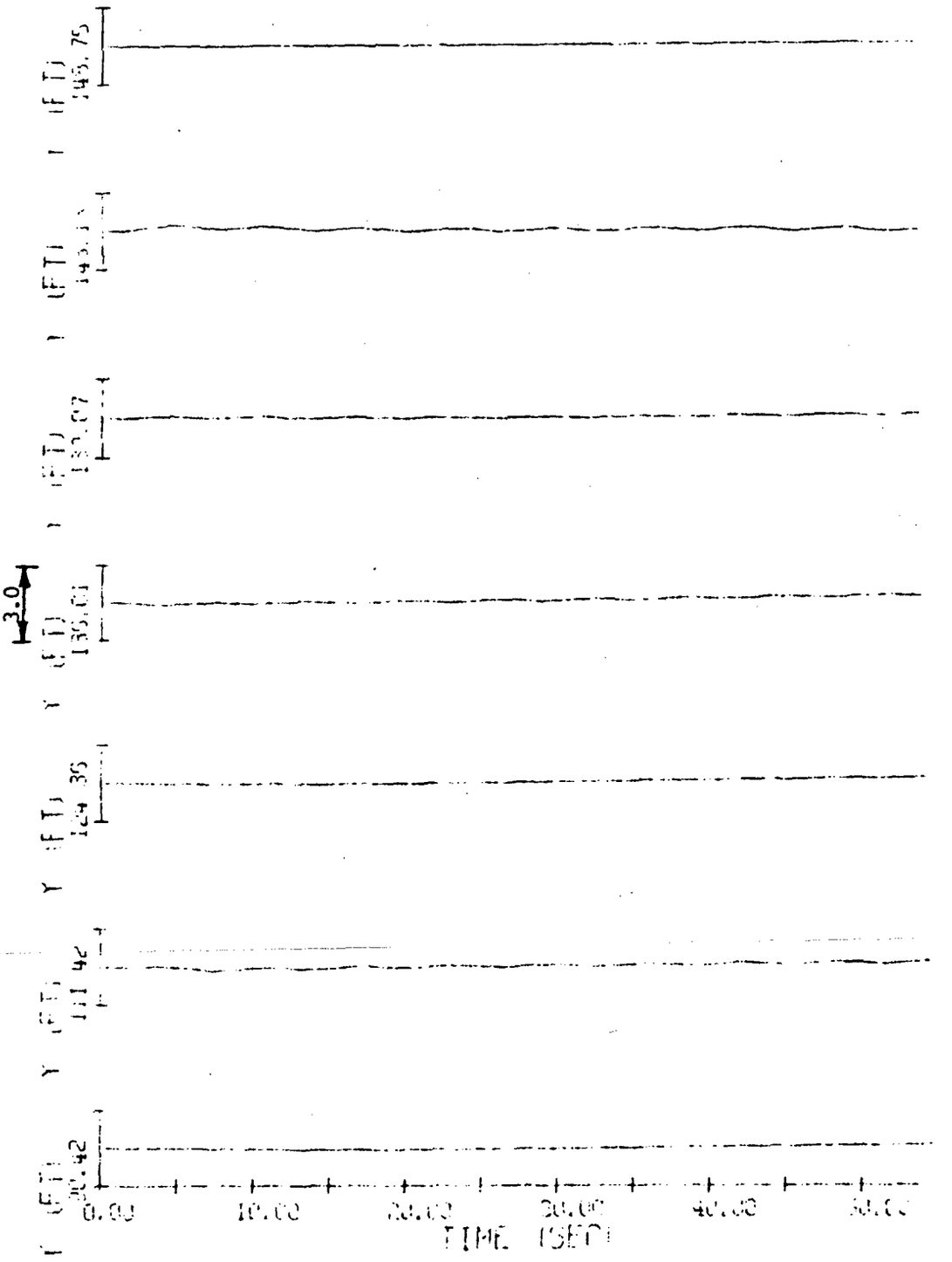


Figure 5.61 Sway Motions of a Surface Moored System

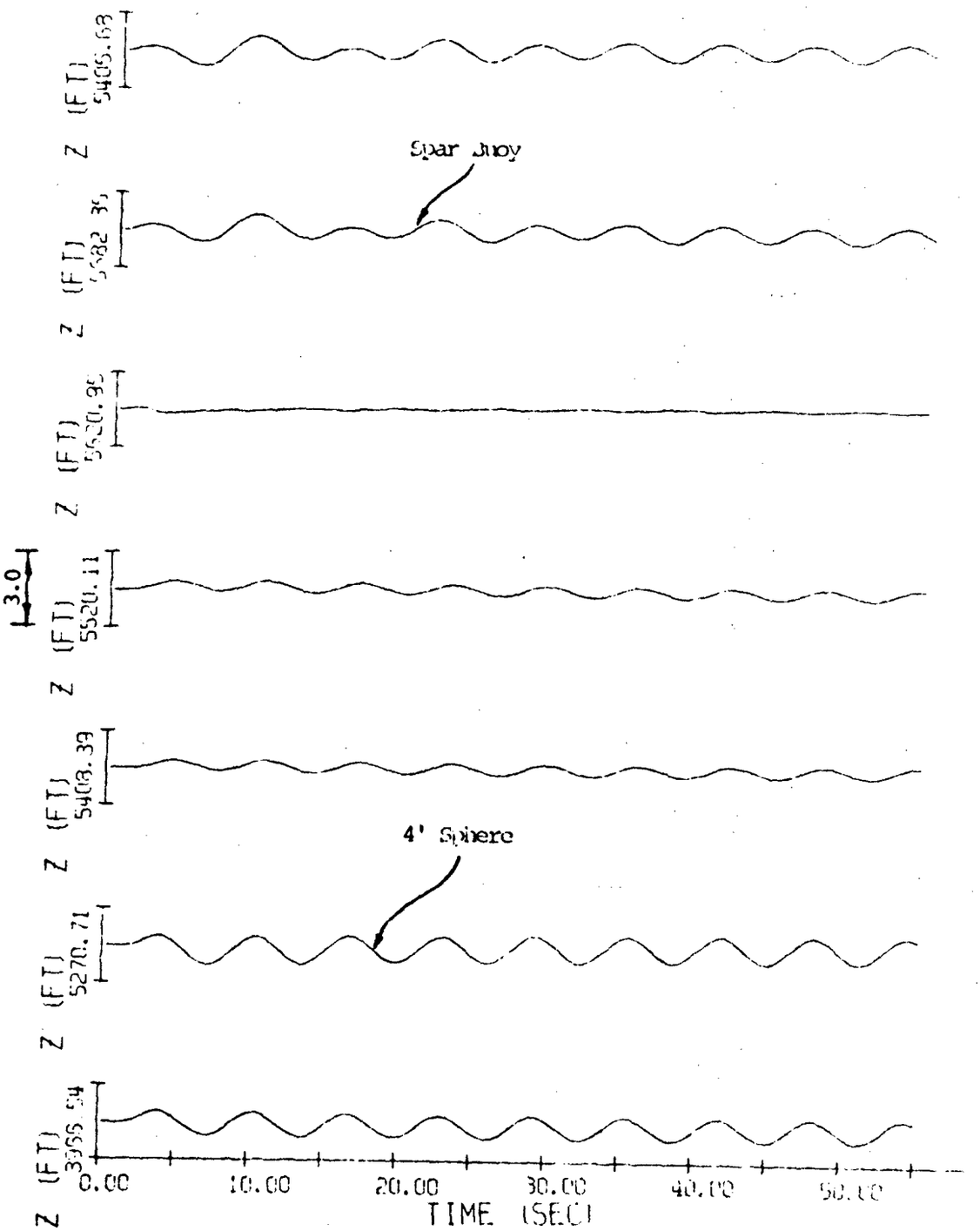


Figure 5.62 Heave Motions of a Surface Moored System

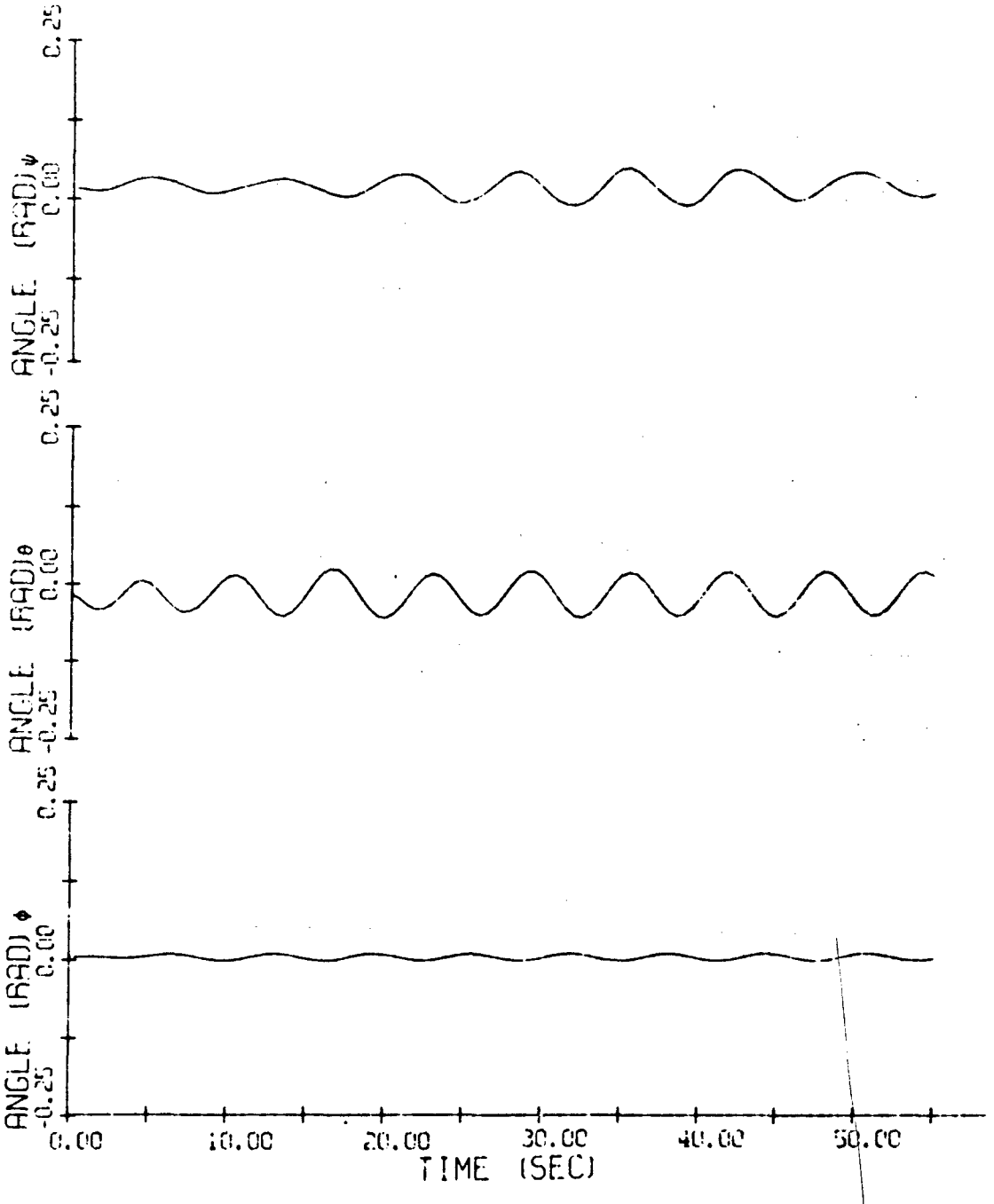


Figure 5.63 Roll, Pitch, and Yaw Motion of the Moored Spar Buoy

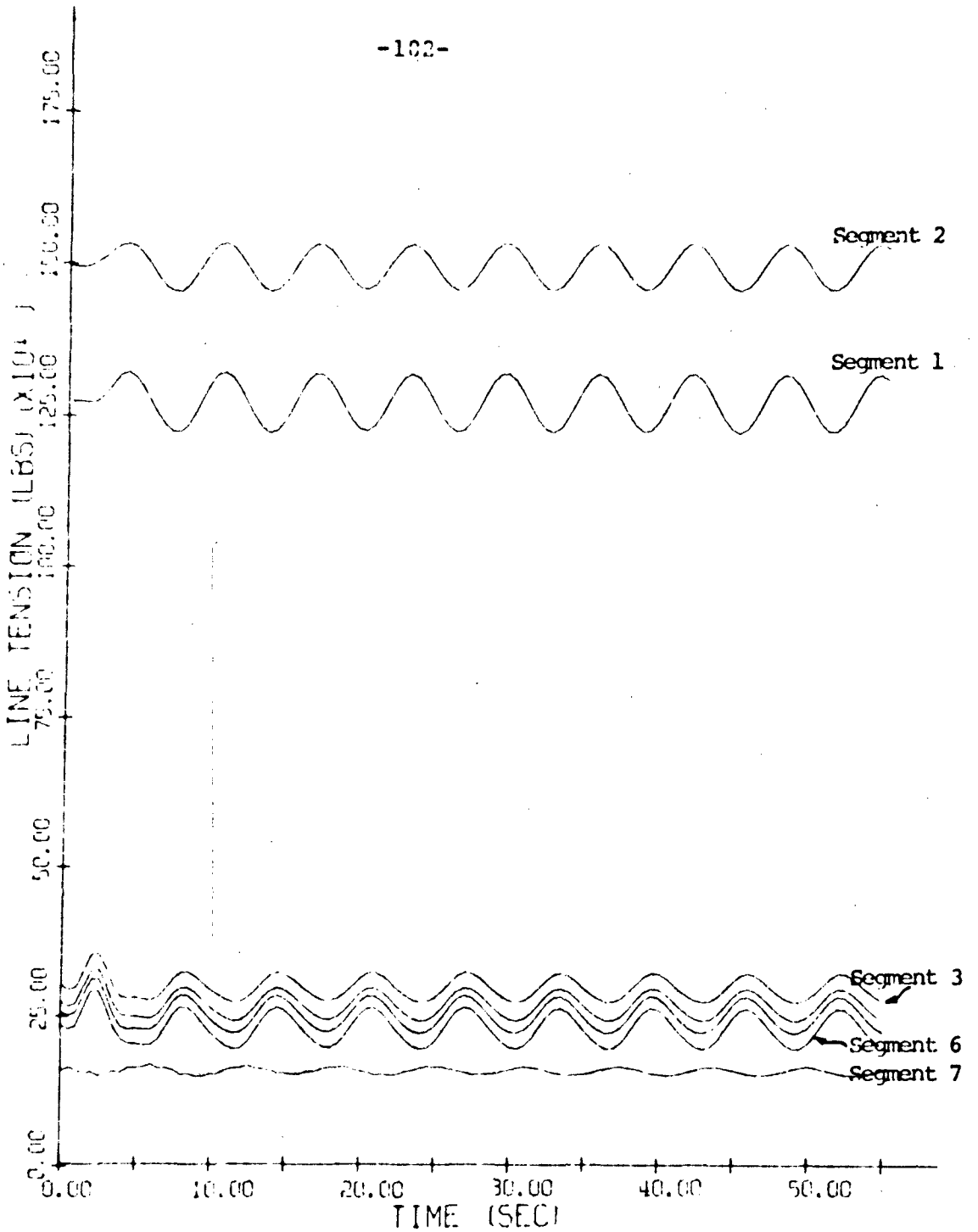


Figure 5.64 Tension Magnitudes in Different Segments of the Surface Moored System

```
 SMDE.DATA
00010     7     6     0    25     1     1 5700.      0.02       55.0       0.5
00020  1.7          0.5          3.35         5.11       30.        1.0        5.27
00021  1.0          1.0
00025  230.         16.71        2.93         12.86      6.99       1.19
00030  -1317.61     34.2         36.27        78.22      5.64       0.46
00035  -54.99       4.2          0.           6.1        6.1        0.
00040  -34.99       5.75         0.52         7.14       6.1        0.
00050  -54.99       4.2          0.           6.1        6.1        0.
00055  3545.62
00056  162.24       14.12        2.93         15.73      9.86       1.95
00060  4061.        102.         46.36        1.40       1.48       -1727.
00070  1322.        315.         28.51        0.85       7.78       -422.
00075  250.0        1000.        31.25        0.94       0.         -284.
00080  250.0        1000.        31.25        0.94       0.         -173.
00085  250.0        1000.        31.25        0.94       0.         -74.
00090  250.0        1000.        0.                                 -16.
00100  276.         1000.        10.25        0.31       1.48       -292.
00105  -0.5         0.0          3.00         0.0        0.0        -3.35
00110  0.5          0.2          0.           1.2        0.3        0.         1.4
00111  0.2          0.           1.8          0.4        0.         2.5        0.0
00112  0.           2.5          0.0          0.         1.4        0.2        0.
00120  1.0          1.           0.1
READY
```

Table 5.10 Input Data for the Surface Moored Buoy System

5.4 Drifting Drogued Buoy System

A submerged window shade drogue attached to a cylindrical spar buoy by means of an elastic nylon line is simulated on the computer by the three dimensional computer program SD3.FORT. The system is subjected to simple harmonic surface waves without any current profile. A 47 feet long 3/8" nylon line is attached to the center of the bottom of the buoy. The spar buoy is 30 feet long and 0.66 foot in diameter. Its static draft with no lines attached is 19.43 feet and weighs 426 pounds in air. The distance between the center of gravity and the bottom end of the buoy is 8.69 feet. The window shade drogue is made of nylon cloth (9 1/4 oz/yd^2) and has the dimensions of 7.5' x 32'. Its approximate thickness is 0.02". The drogue has two metallic bars, one at each end. The bottom bar also acts as the dead weight, keeping the drogue in tension. For computer simulation the drogue was divided into four strips and the parameters of each strip lumped at a node. Tensions in three links between these four nodes and the tension in the nylon tether line were simulated on the computer. Figure 5.65 shows the system. Figures 5.66 through 5.70 show the time histories of the response of the system in a surface wave of amplitude 1 foot and circular frequency of 1 radian per second, and having a wave direction inclined at 0.25 radians with the x-axis. No current profile is present. Figures 5.66

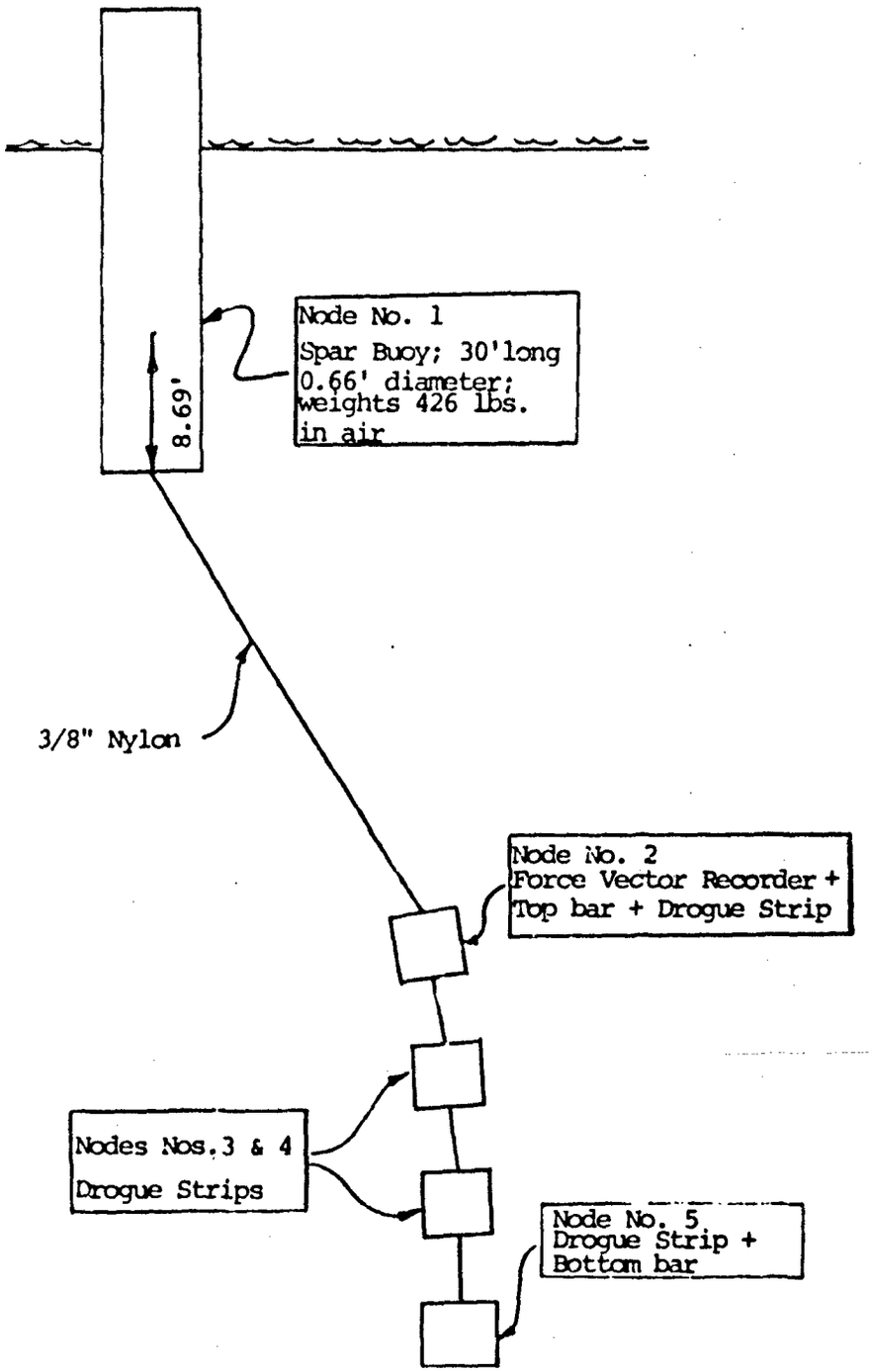


Figure 5.65 Drifting Drogued Buoy System

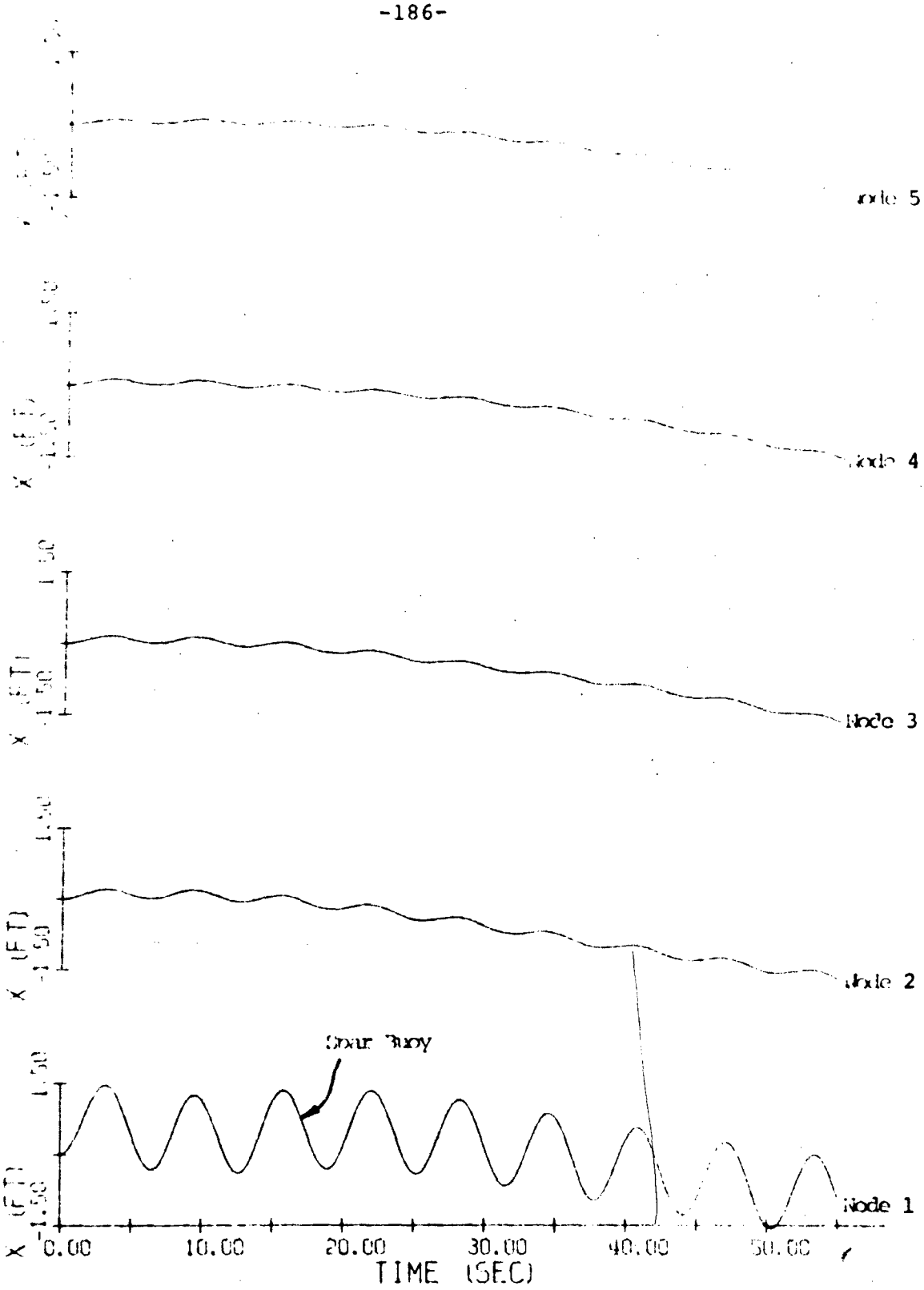


Figure 5.66 Surge Motions of the Drifting Moored System

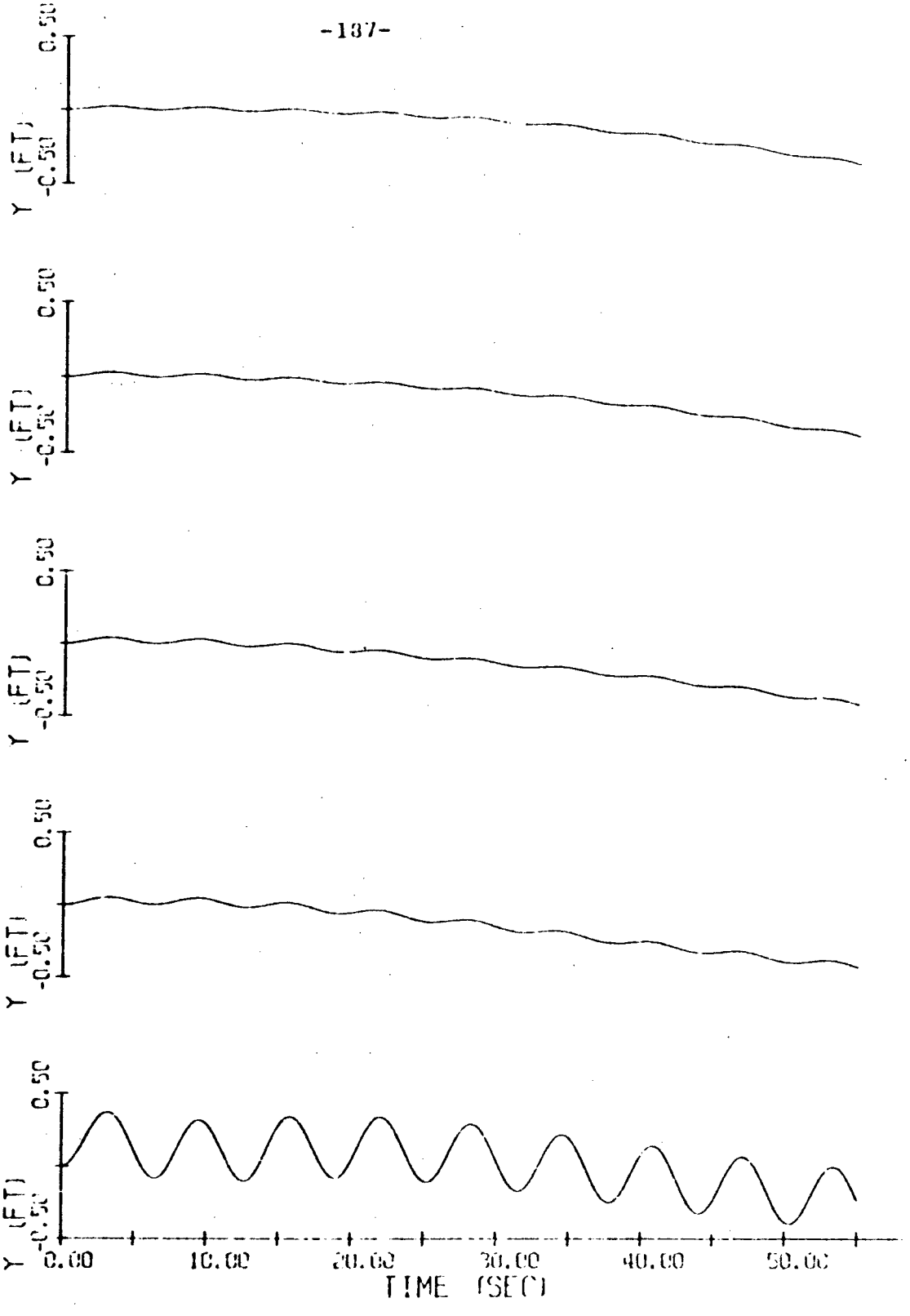


Figure 5.67 Sway Motions of the Drifting Drogued System

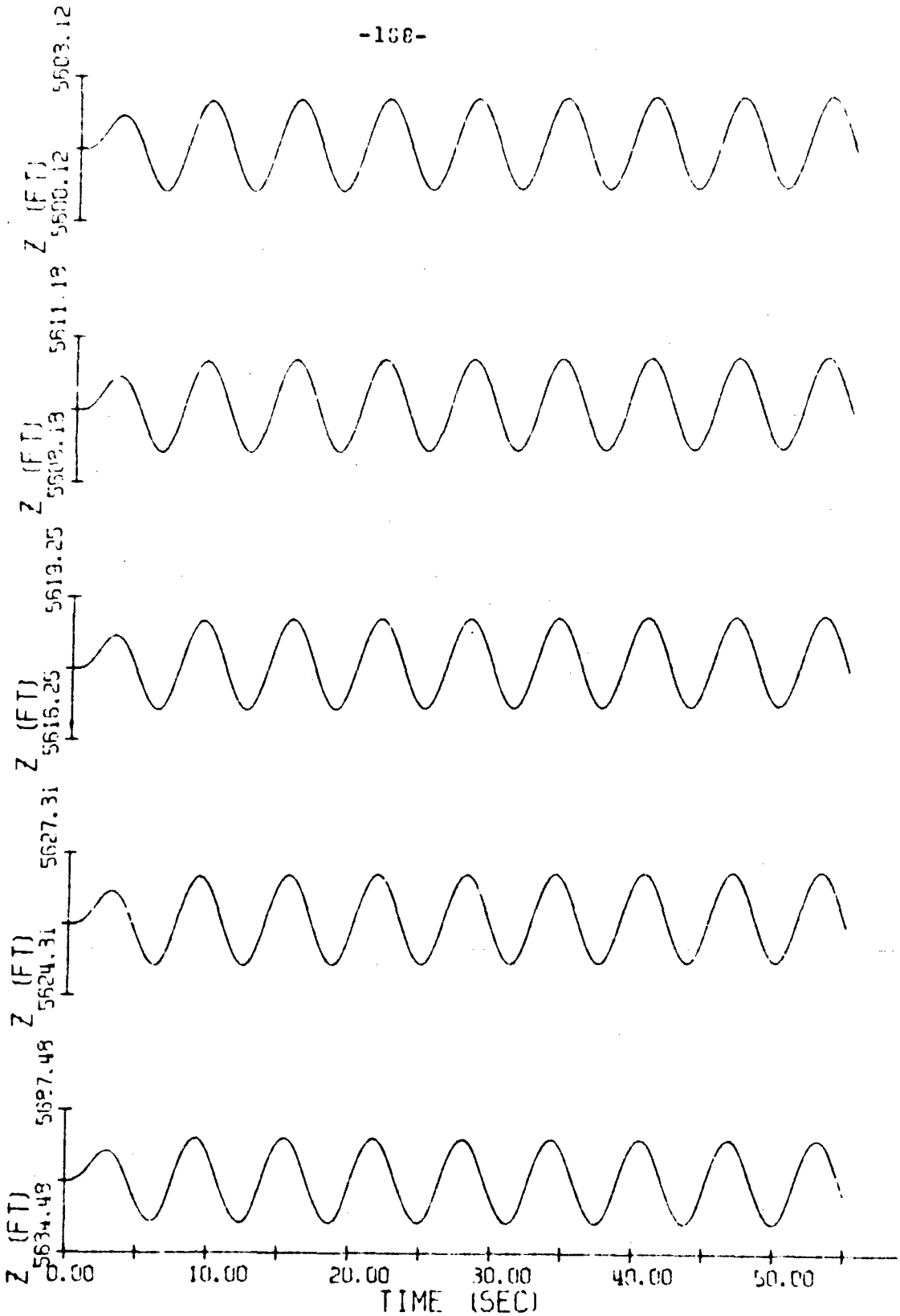


Figure 5.68 Heave Motions of the Drifting Droqued System

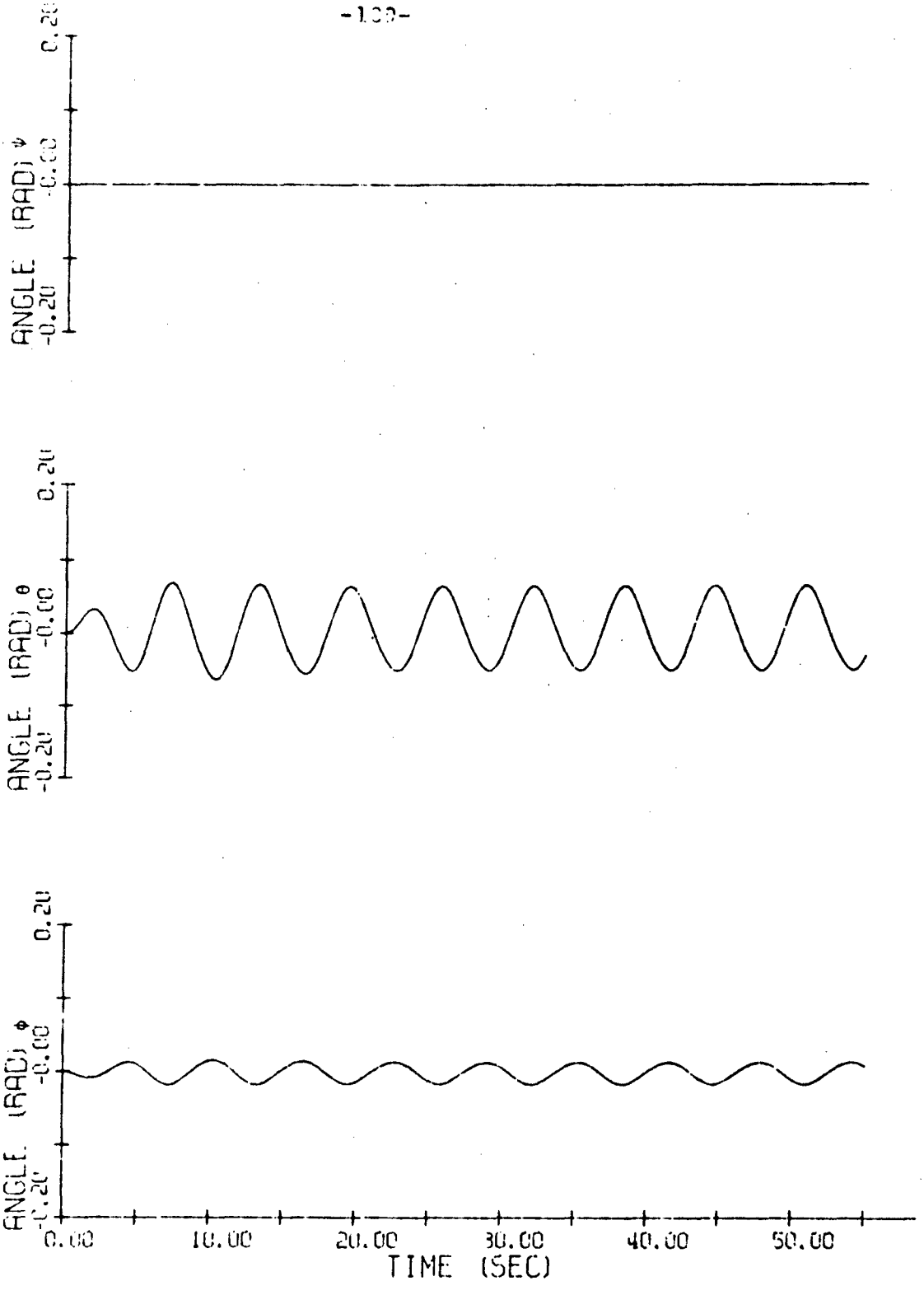


Figure 5.09 Roll, Pitch, and Yaw Motion of the Drifting Drogued Spar Buoy

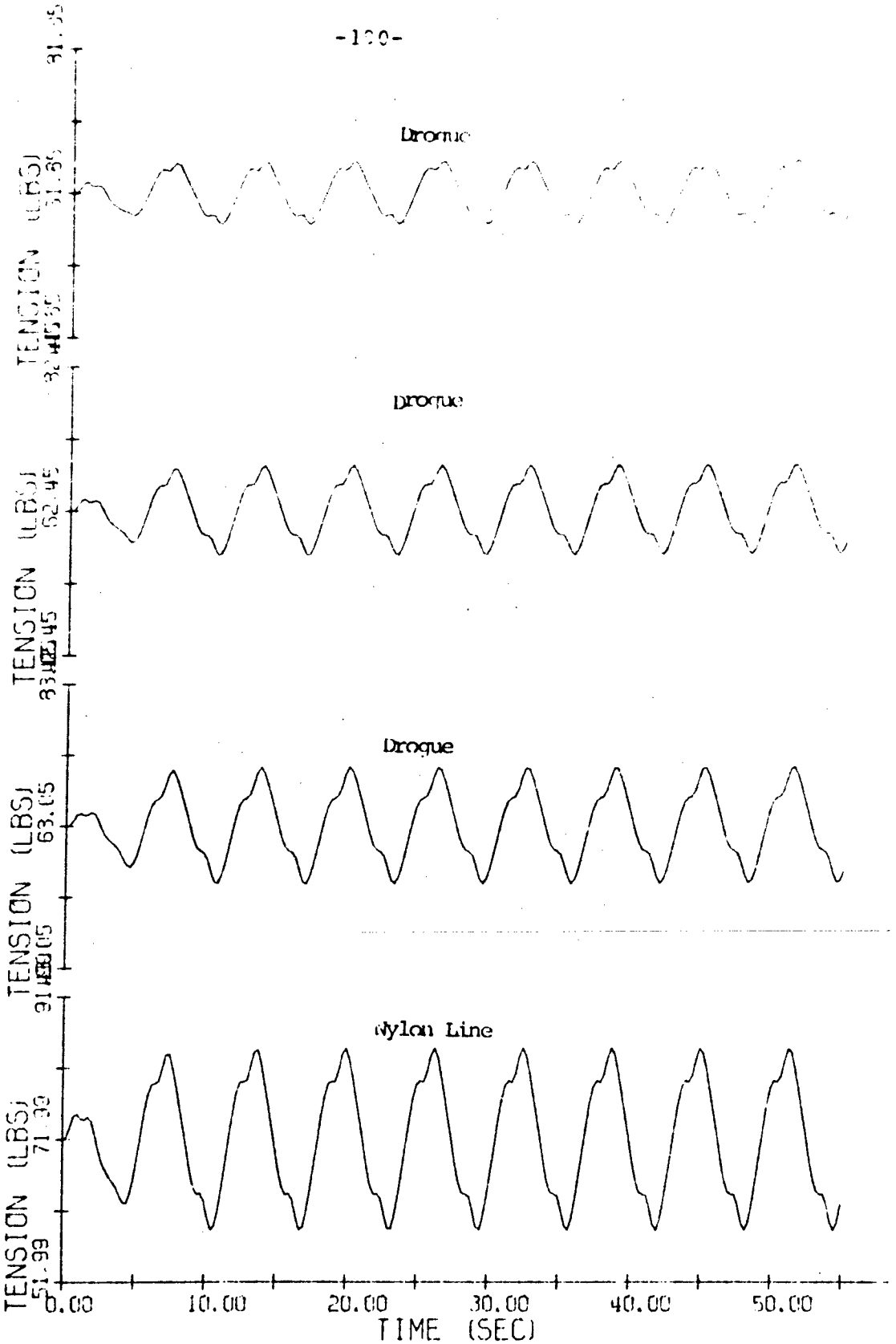


Figure 5.70 Tension Magnitude in Different Segments of the Drifting Drogued Buoy System

through 5.68 show the surge, sway and heave coordinates of the spar buoy C.G. and the four nodes on the drogue. Figure 5.69 shows the roll, pitch, and yaw angles of the buoy and Figure 5.70 shows the tension magnitudes in the nylon line below the buoy and the three segments of the drogue. The input data used for this simulation is presented in Table 5.11.

```
 SDDB.DATA
00010      5     1     2    25     1     2 5700.      0.010      55.0       0.25
00020   0.33        0.33          8.69        6.46       30.       1.0       5.27
00021    1.0         1.0
00030   426.
00031   8.94        3.0           1.1         705.1      704.       0.
00032   0.6         0.1           0.          704.       704.       0.
00033   0.6         0.1           0.          704.       704.       0.
00034   61.85       2.6           0.5         704.5      704.       0.
00060   5500.       0.0                                             -14.
00062   51.         150.          114.5       2.3        2.         -74.
00064   8.          1000.0        114.5       2.3        0.         -82.
00066   8.          1000.0        114.5       2.3        0.         -90.
00100   8.          1000.0        114.5       2.3        1.6        -98.
00105   0.0         0.0           0.0         0.0        0.0        -8.69
00112   0.0         0.0           0.0         0.00       0.0        0.0       0.00
00113   0.0         0.0           0.00        0.0        0.0        0.00      0.0
00114   0.0
00120   1.          1.0           0.25
READY
```

Table 5.11 Input Data for the Drifting Drogued Buoy System

6.0 SUMMARY

Dynamics of moored and drifting buoy systems in a three-dimensional space has been presented in this report. Formulation of mathematical models, which are programmed on the inhouse computer AMDAHL 470 V6, along with time-domain computer simulations of these systems are presented. Four case studies (freely floating spar buoy; a single point subsurface moored system; a spar buoy plus instrument line/buoyant tether/subsurface mooring; a freely drifting spar buoy attached to a window shade drogue) are simulated with forcing being supplied by a velocity profile and a fully developed surface wave field. One (a single point subsurface moored system) of these four case studies was evaluated with full scale ocean test data (CHHABRA, DAHLEN and FROIDEVAUX, 1974; CHHABRA, 1976). Simulations of other three case studies can now be readily evaluated with full scale ocean test data.

For the two-dimensional analysis of a freely floating spar buoy, where the assumption of small motions is not made, three sets of computer simulations are compared with the undamped linear analytical solution. This comparison is presented in Figures 5.8 through 5.11 for a cylindrical spar and Figures 5.31 through 5.34 for a tuned spar buoy. This comparison shows the effects of viscous drag and varying wave amplitude on the response

of a freely floating spar buoy. Plots for the undamped linear analytical solution and the computer simulation where wave amplitude is small and viscous drag is neglected are almost identical. For the case of a cylindrical spar, when viscous drag is added, the surge magnification vs Kh plots show a scatter near the heave and pitch natural frequencies (Figure 5.8). This emphasizes the coupling effects. Surge drift vs. Kh plots in Figure 5.9 show a markedly different responses near these same frequencies and viscous damping is evident in heave and pitch magnification plots (Figures 5.10 and 5.11). The non-linearity in the response with respect to wave amplitude is also evident. These plots also show a heave-pitch coupling near their resonance frequencies. Similar observations can be made for the case of the tuned spar.

For the three-dimensional analysis of a freely floating spar buoy, comparison is made with the two-dimensional analysis for the same input data. The only appreciable difference is observed in the surge drift, which can be attributed directly to the small motion assumption of the three-dimensional analysis.

Simulation of the spar buoy plus instrument line tethered to a subsurface mooring line shows a noteworthy transmittal of motion from the surface buoy to the subsurface line, by the tether. The tether line seems to

move mainly in the tangential direction due to the high viscous drag force in the transverse direction. As a result the combined surge, sway, and heave motions of the spar are transmitted to the subsurface line. Hence, the surge/sway motions of the spar (even if heave was negligible) could be transmitted to the subsurface line as heave. Therefore some modifications of the system may be necessary to minimize longitudinal motions of the subsurface line.

Simulation of the freely drifting spar buoy attached to a window shade drogue shows drift of the buoy and the drogue even though no current (except surface wave) is present. This drift is due to the same reasons as postulated for the freely floating spar buoy.

Full scale ocean test data for the spar buoy plus instrument line/buoyant tether/subsurface mooring is now available (October 1976 ONR/NDBO Mooring Dynamics Experiment). Using this and other data which might be available in the future; the next logical step is the evaluation of the mathematical models of the remaining three case studies.

APPENDIX A

Computer Program Listings — Page

```
 SD3.FORT
00010 C
00020 C     3 D DYNAMIC ANALYSIS OF A SPAR BUOY, A SURFACE MOORED SYSTEM,
00030 C     AND A DRIFTING DROGUED SPAR BUOY.
00040 C     ------------------------------------------------------------
00050 C     THE UNITS USED IN THIS ANALYSIS ARE FEET/POUNDS/SECONDS.
00060 C     ------------------------------------------------------------
00070 C     CASE STUDY 1--MOOR=0--SPAR BUOY FREELY FLOATING IN A SURFACE WAVE.
00080 C     CASE STUDY 2--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
00090 C     MOORING LINE; NM=NB.
00100 C     CASE STUDY 3--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
00110 C     MOORING LINE AND A LUMPED MASS INSTRUMENT LINE HANGING FROM THE
00120 C     BUOY; NM NOT EQUAL TO NB.THIS CASE STUDY SIMULATES THE MOORING
00130 C     DYNAMICS EXPERIMENT ALTERNATE MOORING SYSTEM CONFIGURATION.
00140 C     CASE STUDY 4--MOOR=2--SPAR BUOY ATTACHED TO A WINDOW SHADE DROGUE
00150 C     NB=1.
00160 C     ------------------------------------------------------------
00170 C
00180 C*****START OF THE MAIN PROGRAM**********************************************
00190       IMPLICIT REAL*8(A-H,O-Z)
00200       COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
00210      1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
00220      17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
00230      127),WWD1,WWD2,HMAX,HO,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRHO1,RRH
00240      102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,SO1,SO2,BB1,BB2,CSB,SNB,TMASS,Y
00250      1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,OEXP,AW,AWX,A
00260      1WY,AMP,WE,T,DT,P1,P2,QO,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
00270      1D
00280  100  FORMAT(12F10.2)
00290  101  FORMAT(7F10.0)
00300  102  FORMAT(6I5,4F10.0)
00310  103  FORMAT(6F10.0)
```

```
00320 C*****GENERAL DATA*****************************************************
00330       READ(5,102)NM,NB,ND,NST,NC,MOOR,DEPTH,DT,TMAX,T2
00340 C*****BUOY DATA********************************************************
00350       READ(5,101)RD1,RD2,ZCG,RGYR,HMAX,CDL,HST,CDP,ALPHA
00360 C*****MOORING LINE DATA************************************************
00370       READ(5,103)(W(I),CM(I),CMS(I),CMD(I),CMN(I),CMT(I),I=1,NM)
00380       READ(5,103)(SL(I),EK(I),CDIN(I),CDIT(I),CDIA(I),DEP(I),I=1,NM)
00390 C*****ATTACHMENT DATA**************************************************
00400       READ(5,101)XC1,YC1,ZC1,XC2,YC2,ZC2
00410 C*****CURRENT PROFILE DATA*********************************************
00420       READ(5,101)((V(IJ,I),IJ=1,3),I=1,NM)
00430       NM1=NM-NB
00440       NN=3*NB
00450       NB1=NB-1
00460       NB2=NB+1
00470       NM2=NM+1
00480       NDF=3*NM+3
00490 C*****INITIALIZE PHYSICAL CONSTANTS*************************************
00500       PI=4.*DATAN(1.D0)
00510       SO1=PI*RD1*RD1
00520       SO2=PI*RD2*RD2
00530       RHO=1.9905
00540       GR=32.2
00550       TMASS=W(NB)/GR
00560       YNERT1=TMASS*RGYR*RGYR
00570       RRHO1=RHO*CDL*RD1*HST
00580       RRHO2=RHO*CDL*RD2
00590       BB1=RHO*SO1
00600       BB2=RHO*SO2
00610       YNERT2=(BB1*RD1*RD1*HST+BB2*RD2*RD2*(HMAX-HST))/2.
00620       WWD1=BB1*GR*HST
00630       WWD2=BB2*GR
```

```
00640       VVO=1.33*RHO*ALPHA*(RD1**3+(RD1-RD2)**3)
00650 C*****START LOOP FOR DYNAMIC RESPONSE TO DIFFERENT SURFACE WAVES******
00660       DO 12 ICC=1,NC
00670 C*****SURFACE WAVE DATA*************************************************
00680       READ(5,101)WE,AMP,BETA
00690       CSB=DCOS(BETA)
00700       SNB=DSIN(BETA)
00710 C*****INITIALIZE DYNAMIC VARIABLES**************************************
00720       DO 1 I=1,NDF
00730       F(I)=0.
00740       YD(I)=0.
00750       Y(I)=0.
00760       TC(I)=0.
00770       DD(I)=0.
00780       DO 1 J=1,NM
00790     1 GK(J,I)=0.
00800       AW=WE*WE/GR
00810       AWX=AW*CSB
00820       AWY=AW*SNB
00830       T3=0.
00840 C*****FIND THE CURRENT PROFILE FOR STATIC SOLUTION**********************
00850       DO 4 I=1,NM
00860       DO 2 IS=1,3
00870     2 VV(IS,I)=0.
00880       ZX1=WE*AMP*DEXP(AW*DEP(I))
00890       DO 3 J=1,12
00900       VN(1)=V(1,I)+ZX1*CSB*DSIN(PI*J/6.)
00910       VN(2)=V(2,I)+ZX1*SNB*DSIN(PI*J/6.)
00920       VN(3)=V(3,I)+ZX1*DCOS(PI*J/6.)
00930       VEM=DSQRT(VN(1)**2+VN(2)**2+VN(3)**2)
00940       DO 3 IJK=1,3
00950     3 VV(IJK,I)=VV(IJK,I)+VEM*VN(IJK)
```

```
00960       VEM1=(VV(1,I)**2+VV(2,I)**2+VV(3,I)**2)**0.25
00970       DO 4 IKJ=1,3
00980       VV(IKJ,I)=VV(IKJ,I)/(VEM1*12.**0.5)
00990     4 IF(DABS(VV(IKJ,I)).LE.0.000001)VV(IKJ,I)=0.
01000       WRITE(6,100)((VV(IJ,I),IJ=1,3),I=1,NM)
01010       TC(NDF+1)=0.
01020       TC(NDF+2)=0.
01030       TC(NDF+3)=0.
01040       IF(NM.EQ.NB)GO TO 8
01050       CS(NDF+1)=0.
01060       CS(NDF+2)=0.
01070       CS(NDF+3)=-1.
01080       IF(MOOR.EQ.)GO TO 8
01090 C*****STATIC SOLUTION OF THE LUMPED MASSES (INSTRUMENT LINE)***********
01100       DO 7 I=1,NM1
01110       J=NM2-I
01120       K=J+1
01130       II=3*K+1
01140       VN(1)=VV(1,J)*(1.-CS(II)*CS(II))-VV(2,J)*CS(II)*CS(II+1)
01150      1-VV(3,J)*CS(II)*CS(II+2)
01160       VN(2)=VV(2,J)*(1.-CS(II+1)*CS(II+1))-VV(3,J)*CS(II+1)*CS(II+2)
01170      1-VV(1,J)*CS(II+1)*CS(II)
01180       VN(3)=VV(3,J)*(1.-CS(II+2)*CS(II+2))-VV(1,J)*CS(II+2)*CS(II)
01190      1-VV(2,J)*CS(II+2)*CS(II+1)
01200       VT=VV(1,J)*CS(II)+VV(2,J)*CS(II+1)+VV(3,J)*CS(II+2)
01210       VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
01220       VNM1=DSQRT(VV(1,J)*VV(1,J)+VV(2,J)*VV(2,J)+VV(3,J)*VV(3,J))
01230       DDTT=CDIT(J)*VT*DABS(VT)
01240       DO 5 IC=1,3
01250       KK=II+IC-4
01260       TC(KK)=TC(KK+3)+CDIA(J)*VNM1*VV(IC,J)+DDTT*CS(KK+3)+CDIN(J)*VNM*VN
01270      1(IC)
```

```
01280     5  CONTINUE
01290        TC(II-1)=TC(II-1)-W(J)
01300        TNN(J)=DSQRT(TC(II-3)**2+TC(II-2)**2+TC(II-1)**2)
01310        DO 6 IC=1,3
01320        KK=II-4+IC
01330     6  CS(KK)=TC(KK)/TNN(J)
01340     7  TL(J)=SL(J)+TNN(J)/EK(J)
01350     8  CALL STATIC
01360        IF(MOOR.GT.0)GO TO 9
01370        CALL BUOYS
01380        CALL MINVER(NDF,GM)
01390     9  T=0.
01400    10  SS=AWX*X(NN-2)+AWY*X(NN-1)-WE*T
01410        SNK=DSIN(SS)
01420        CSK=DCOS(SS)
01430        OEXP=DEXP(-HO*AW)
01440        HI=HO-AMP*SNK
01450        IF(CDL.NE.0.0)CALL DBDRG
01460        IF(MOOR.GT.0)CALL BUOYS
01470        CALL MATRIX
01480        IF(T.LT.T3)GO TO 11
01490        WRITE(6,100)T
01500        WRITE(6,100)(X(I),I=1,NDF)
01510        IF(MOOR.EQ.1)WRITE(6,100)(TN(K),K=1,NM)
01520        IF(MOOR.EQ.2)WRITE(6,100)(TN(K),K=2,NM)
01530        IF(T.GT.TMAX)GO TO 12
01540        T3=T3+T2
01550    11  T=T+DT
01560        IF(MOOR.GT.0)CALL NEXT
01570        GO TO 10
01580    12  CONTINUE
01590        CALL EXIT
```

```
01600       END
01610       IMPLICIT REAL*8(A-H,O-Z)
01620       SUBROUTINE STATIC
01630 C     THIS SUBROUTINE CALCULATES THE STATIC SOLUTION TO THE FOUR CASE
01640 C     STUDIES OUTLINED IN THE MAIN PROGRAM.
01650 C     ------------------------------------------------------------
01660 C
01670       COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
01680      1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
01690      17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
01700      127),WWD1,WWD2,HMAX,HO,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRHO1,RRH
01710      102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,SO1,SO2,BB1,BB2,CSB,SNB,TMASS,Y
01720      1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,OEXP,AW,AWX,A
01730      1WY,AMP,WE,T,DT,P1,P2,QO,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
01740      1D
01750  100  FORMAT(12F10.2)
01760  102  FORMAT (///10X,65HSTATIC CALCULATION TERMINATED -- BUOY HAS EXCEED
01770      1ED MAXIMUM DRAFT     )
01780  103  FORMAT (///10X,100HSTATIC CALCULATION TERMINATED -- NUMBER OF ITER
01790      1ATIONS TO FIND SYSTEM SOLUTION HAS EXCEEDED MAXIMUM (  I3,1H) )
01800       IF(MOOR.EQ.0)GO TO 26
01810       ITMAX=100
01820       IT=0
01830       IF(MOOR.EQ.1)GO TO 7
01840 C
01850 C*****CASE STUDY 4 ************************************************
01860 C
01870 C*****INITIALIZE VARIABLES*****************************************
01880       IS2=0
01890       DC3=0.
01900       VE=0.
01910       FXE=0.02*HMAX*RD2*V(1,1)*V(1,1)+0.01
```

```
01920       DVE=DABS(V(1,NM))/2.
01930 C*****START STATIC VELOCITY ITERATION LOOP*******************************
01940    1  IT=IT+1
01950       IF(IT.GT.ITMAX)GO TO 27
01960       DO 3 I=1,NM1
01970       J=NM2-I
01980       K=J+1
01990       II=3*K+1
02000       U1=VV(1,J)-VE
02010       U2=VV(2,J)
02020       U3=VV(3,J)
02030       VN(1)=U1*(1.-CS(II)*CS(II))-U2*CS(II)*CS(II+1)-U3*CS(II)*CS(II+2)
02040       VN(2)=U2*(1.-CS(II+1)*CS(II+1))-U3*CS(II+1)*CS(II+2)-U1*CS(II+1)*C
02050      1S(II)
02060       VN(3)=U3*(1.-CS(II+2)*CS(II+2))-U1*CS(II+2)*CS(II)-U2*CS(II+2)*CS(
02070      1II+1)
02080       VT=U1*CS(II)+U2*CS(II+1)+U3*CS(II+2)
02090       VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
02100       VNM1=DSQRT(U1*U1+U2*U2+U3*U3)
02110       DDTT=CDIT(J)*VT*DABS(VT)
02120       TC(II-3)=TC(II)+CDIA(J)*VNM1*U1+DDTT*CS(II)+CDIN(J)*VNM*VN(1)
02130       TC(II-2)=TC(II+1)+CDIA(J)*VNM1*U2+DDTT*CS(II+1)+CDIN(J)*VNM*VN(2)
02140       TC(II-1)=TC(II+2)-W(J)+CDIA(J)*VNM1*U3+DDTT*CS(II+2)+CDIN(J)*VNM*V
02150      1N(3)
02160       TNN(J)=DSQRT(TC(II-3)**2+TC(II-2)**2+TC(II-1)**2)
02170       DO 2 IC=1,3
02180       KK=II-4+IC
02190    2  CS(KK)=TC(KK)/TNN(J)
02200    3  TL(J)=SL(J)+TNN(J)/EK(J)
02210       HO=HST+(W(1)-TC(9)-DC3-WWD1)/WWD2
02220       IF(HO.GT.0.99*HMAX)GO TO 28
02230       HOD=HO-HST
```

```
02240        VNM1=DSQRT((VV(1,1)-VE)**2+VV(2,1)**2)
02250        RRHO=RRHO1+RRHO2*HOD
02260        DC1=RRHO*VNM1*(VV(1,1)-VE)
02270        DC2=RRHO*VNM1*VV(2,1)
02280        DC3=(0.02*RRHO+0.5*RHO*CDP*(2.*SO1-SO2))*DABS(VV(3,1))*VV(3,1)
02290        FX=DC1+TC(7)
02300        WRITE(6,100)VE,HO,FX
02310        IF(DABS(FX).LE.FXE)GO TO 4
02320        IS3=DSIGN(1.0D0,FX)
02330        IF(IS2+IS3.EQ.0)DVE=DVE/2.
02340        VE=VE+IS3*DVE
02350        IS2=IS3
02360        GO TO 1
02370     4  YD(1)=VE
02380        DO 5 I=2,NM
02390        II=3*I+1
02400     5  YD(II)=VE
02410        X(1)=0.
02420        X(2)=0.
02430        X(3)=ZCG-HO+DEPTH
02440        B=WWD1+WWD2*HOD
02450        TNN(1)=0.
02460        DO 6 IC=1,3
02470     6  CS(IC)=0.
02480        GO TO 18
02490  C
02500  C*****CASE STUDIES 2 AND 3 ********************************************
02510  C
02520     7  INDEX=1
02530        NBB=NB-1
02540        HO=HST+(W(NB)-TC(NN+6)-WWD1)/WWD2
02550        DH=0.
```

```
02560       WADD=W(NBB)/10.
02570       VNM1=DSQRT(VV(1,NB)*VV(1,NB)+VV(2,NB)*VV(2,NB))
02580 C******INITILIZE ERROR TOLERANCE**********************************
02590       ER= 0.0010*DEPTH
02600 C******START CABLE (BUOY DRAFT) ITERATION LOOP*********************
02610     8 IT=IT+1
02620       DH=DH/2.
02630       IF (IT.GT.ITMAX) GO TO 27
02640       IF(HO.GT.0.99*HMAX)GO TO 28
02650       HOD=HO-HST
02660       B=WWD1+WWD2*HOD
02670       RRHO=RRHO1+RRHO2*HOD
02680       DC1=RRHO*VNM1*VV(1,NB)
02690       DC2=RRHO*VNM1*VV(2,NB)
02700       DC3=(0.02*RRHO+0.5*RHO*CDP*(2.*SO1-SO2))*DABS(VV(3,NB))*VV(3,NB)
02710       TC(NN-2)=DC1+TC(NN+4)
02720       TC(NN-1)=DC2+TC(NN+5)
02730       TC(NN)=DC3+B-W(NB)+TC(NN+6)
02740       TNN(NB)=DSQRT(TC(NN-2)*TC(NN-2)+TC(NN-1)*TC(NN-1)+TC(NN)*TC(NN))
02750       TL(NB)=SL(NB)+TNN(NB)/EK(NB)
02760       DO 9 IC=1,3
02770       KK=NN+IC-3
02780     9 CS(KK)=TC(KK)/TNN(NB)
02790       IF(NB.EQ.1)GO TO 14
02800    10 DO 13 I=1,NB1
02810       J=NB-I
02820       K=J+1
02830       II=J*3
02840       VN(1)=VV(1,J)*(1.-CS(II+1)*CS(II+1))-VV(2,J)*CS(II+1)*CS(II+2)
02850      1-VV(3,J)*CS(II+1)*CS(II+3)
02860       VN(2)=VV(2,J)*(1.-CS(II+2)*CS(II+2))-VV(3,J)*CS(II+2)*CS(II+3)
02870      1-VV(1,J)*CS(II+2)*CS(II+1)
```

```
02880       VN(3)=VV(3,J)*(1.-CS(II+3)*CS(II+3))-VV(1,J)*CS(II+3)*CS(II+1)
02890      1-VV(2,J)*CS(II+3)*CS(II+2)
02900       VT=VV(1,J)*CS(II+1)+VV(2,J)*CS(II+2)+VV(3,J)*CS(II+3)
02910       VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
02920       VNM2=DSQRT(VV(1,J)*VV(1,J)+VV(2,J)*VV(2,J)+VV(3,J)*VV(3,J))
02930       DDTT=CDIT(J)*VT*DABS(VT)
02940       DO 11 IC=1,3
02950       KK=II+IC-3
02960       TC(KK)=TC(KK+3)+CDIA(J)*VNM2*VV(IC,J)+DDTT*CS(KK+3)+CDIN(J)*VNM*VN
02970      1(IC)
02980    11 CONTINUE
02990       TC(II)=TC(II)-W(J)
03000       TNN(J)=DSQRT(TC(II)**2+TC(II-2)**2+TC(II-1)**2)
03010       DO 12 IC=1,3
03020       KK=II-3+IC
03030    12 CS(KK)=TC(KK)/TNN(J)
03040    13 TL(J)=SL(J)+TNN(J)/EK(J)
03050    14 ZJ1=0.
03060       DO 15 I=1,NB
03070       II=I*3
03080    15 ZJ1=ZJ1+TL(I)*CS(II)
03090       DEPTH1=ZJ1+(HO-ZCG-ZC1)
03100       EE=DEPTH-DEPTH1
03110       WRITE(6,100)HO,EE,(W(NCC),NCC=1,NB)
03120       EEE=DABS(EE)
03130       IF(EEE.LE.ER)GO TO 17
03140       IF(INDEX.EQ.1.AND.EE.LT.0.)GO TO 25
03150       IF(INDEX.EQ.2)GO TO 16
03160       INDEX=2
03170       DH=(HMAX-HO)/2.
03180 C*****ITERATION ON BUOY DRAFT*****
03190       HO=HO+DH
```

```
03200        GO TO 8
03210    16  HO=HO+EEE*DH/EE
03220        GO TO 8
03230    17  DEPTH=DEPTH1
03240        WRITE(6,100)DEPTH
03250 C****STATIC SOLUTION HAS CONVERGED--COMPUTE ALL ANGLES AND COORDINATES
03260    18  AL1=(SO1*HST*(HST/2.-ZCG)+SO2*HOD*((HO+HST)/2.-ZCG))/(SO1*HST+SO2*
03270       1HOD)
03280        AL2=(HST/2.-ZCG)+RD2*HOD*HO/(2.*(RD1*HST+RD2*HOD))
03290        GM(1,1)=B*AL1+DC3*AL2-TC(NN-1)*YC1-TC(NN)*ZC1+TC(NN+5)*YC2+TC(NN+6
03300       1)*ZC2
03310        GM(1,2)=TC(NN-1)*XC1-TC(NN+5)*XC2
03320        GM(1,3)=TC(NN)*XC1-TC(NN+6)*XC2
03330        GM(2,1)=TC(NN-2)*YC1-TC(NN+4)*YC2
03340        GM(2,2)=B*AL1+DC3*AL2-TC(NN-2)*XC1-TC(NN)*ZC1+TC(NN+4)*XC2
03350       1+TC(NN+6)*ZC2
03360        GM(2,3)=TC(NN)*YC1-TC(NN+6)*YC2
03370        GM(3,1)=-DC1*AL2+TC(NN-2)*ZC1-TC(NN+4)*ZC2
03380        GM(3,2)=-DC2*AL2+TC(NN-1)*ZC1-TC(NN+5)*ZC2
03390        GM(3,3)=-TC(NN-2)*XC1-TC(NN-1)*YC1+TC(NN+4)*XC2+TC(NN+5)*YC2
03400        IF(GM(3,3).EQ.0.)GM(3,3)=0.00001
03410        VN(1)=-DC2*AL2+TC(NN-1)*ZC1-TC(NN)*YC1-TC(NN+5)*ZC2+TC(NN+6)*YC2
03420        VN(2)=DC1*AL2-TC(NN-2)*ZC1+TC(NN)*XC1+TC(NN+4)*ZC2-TC(NN+6)*XC2
03430        VN(3)=TC(NN-2)*YC1-TC(NN-1)*XC1-TC(NN+4)*YC2+TC(NN+5)*XC2
03440        CALL MINVER(3,GM)
03450        DO 19 IC=1,3
03460        X(NN+IC)=0.
03470        DO 19 IC1=1,3
03480    19  X(NN+IC)=X(NN+IC)+GM(IC,IC1)*VN(IC1)
03490        IF(MOOR.EQ.2)GO TO 22
03500        X(1)=TL(1)*CS(1)
03510        X(2)=TL(1)*CS(2)
```

```
03520         X(3)=TL(1)*CS(3)
03530         IF(NB.EQ.1)GO TO 21
03540         DO 20 I=2,NB
03550         II=3*I
03560         X(II-2)=X(II-5)+TL(I)*CS(II-2)
03570         X(II-1)=X(II-4)+TL(I)*CS(II-1)
03580    20   X(II)=X(II-3)+TL(I)*CS(II)
03590    21   XJ1=X(NN-2)
03600         X(NN-2)=XJ1-XC1
03610         YJ1=X(NN-1)
03620         X(NN-1)=YJ1-YC1
03630         X(NN)=ZJ1-ZC1
03640         IF(NM.EQ.NB)GO TO 24
03650    22   XJ2=X(NN-2)+XC2
03660         YJ2=X(NN-1)+YC2
03670         ZJ2=X(NN)+ZC2
03680         X(NN+4)=XJ2+TL(NB2)*CS(NN+4)
03690         X(NN+5)=YJ2+TL(NB2)*CS(NN+5)
03700         X(NN+6)=ZJ2+TL(NB2)*CS(NN+6)
03710         IF(NB2.EQ.NM)GO TO 24
03720         NB3=NB2+1
03730         DO 23 I=NB3,NM
03740         II=3*I+1
03750         X(II)=X(II-3)+TL(I)*CS(II)
03760         X(II+1)=X(II-2)+TL(I)*CS(II+1)
03770    23   X(II+2)=X(II-1)+TL(I)*CS(II+2)
03780    24   WRITE(6,100)HO,X(NN+1),X(NN+2),X(NN+3)
03790         WRITE(6,100)(X(I),I=1,NDF)
03800         RETURN
03810 C*****IF PART OF THE MOORING LINE IS FLOATING ON THE SURFACE FIND IT****
03820    25   W(NBB)=W(NBB)-WADD
03830         IF(W(NBB).LT.0.)GO TO 10
```

```
03840        W(NBB)=0.
03850        NBB=NBB-1
03860        GO TO 25
03870 C
03880 C*****CASE STUDY 1 ************************************************
03890 C
03900    26  HO=HST+(W(1)-WWD1)/WWD2
03910        WAW=AW*HO
03920        OEXP=DEXP(-WAW)
03930        HOD=HO-HST
03940        BK1=(BB1-BB2)*HST
03950        BK2=BB2*HO
03960        P1=BK2*(0.5*HO-ZCG)+BK1*(0.5*HST-ZCG)
03970        P2=BK2*(HO*HO/3.-HO*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
03980        QQ2=DEXP(-HOD*AW)
03990        Q0=(BB1*(QQ2-OEXP)+BB2*(1.-QQ2))/AW
04000        Q1=(BB1*(QQ2*(HST-1./AW-ZCG)+OEXP*(1./AW+ZCG))+BB2*(HO-1./AW-ZCG-Q
04010       1Q2*(HST-1./AW-ZCG)))/AW
04020        X(1)=-AMP*CSB*Q0/TMASS
04030        X(2)=-AMP*SNB*Q0/TMASS
04040        X(3)=ZCG-HO+DEPTH
04050        X(4)=-WE*WE*AMP*SNB*(2.*Q1-P1*Q0/TMASS)/(P1*GR-WE*WE*(YNERT1+P2-P1
04060       1*P1/(2.*TMASS)))
04070        X(5)=WE*WE*AMP*CSB*(2.*Q1-P1*Q0/TMASS)/(P1*GR-WE*WE*(YNERT1+P2-P1*
04080       1P1/(2.*TMASS)))
04090        X(6)=0.
04100        YD(1)=VV(1,1)
04110        YD(2)=VV(2,1)
04120        YD(3)=WE*AMP*(RHO*SO2*GR-WE*WE*Q0)/(RHO*SO2*GR-WE*WE*(TMASS+VV0))
04130        XAW=X(1)/AMP
04140        YAW=X(2)/AMP
04150        ZAW=YD(3)/(WE*AMP)
```

```
04160        PAW=X(5)/(AW*AMP)
04170        RAW=X(4)/(AW*AMP)
04180        WRITE(6,100)WAW,XAW,YAW,ZAW,RAW,PAW
04190        RETURN
04200    27  WRITE(6,103)ITMAX
04210        CALL EXIT
04220    28  WRITE(6,102)
04230        CALL EXIT
04240        END
04250        IMPLICIT REAL*8(A-H,O-Z)
04260        SUBROUTINE DBDRG
04270 C      THIS SUBROUTINE CALCULATES THE VISCOUS DRAG FORCE ON THE
04280 C      SPAR BUOY DURING ITS DYNAMIC MOTIONS. INTEGRATION OF FORCES
04290 C      IS DONE ALONG THE AXIS OF THE SPAR.
04300 C      ------------------------------------------------------------
04310 C
04320        COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
04330       1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
04340       17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
04350       127),WWD1,WWD2,HMAX,HO,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRHO1,RRH
04360       102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,SO1,SO2,BB1,BB2,CSB,SNB,TMASS,Y
04370       1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,CDF,PI,RHO,GR,OEXP,AW,AWX,A
04380       1WY,AMP,WE,T,DT,P1,P2,QO,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
04390       1D
04400 C*****COEFFICIENTS OF THE VELOCITY EQUATION ARE CALCULATED BELOW***
04410        V1=V(1,NB)-YD(NN-2)+X(NN+3)*(V(2,NB)-YD(NN-1))-X(NN+2)*(V(3,NB)-YD
04420       1(NN))
04430        V2=WE*AMP*CSB*SNK
04440        V3=-X(NN+3)*(V(1,NB)-YD(NN-2))+V(2,NB)-YD(NN-1)+X(NN+1)
04450       1*(V(3,NB)-YD(NN))
04460        V4=WE*AMP*SNB*SNK
04470        V5=X(NN+2)*(V(1,NB)-YD(NN-2))-X(NN+1)*(V(2,NB)-YD(NN-1))
```

```
04480       1+V(3,NB)-YD(NN)
04490        V6=-WE*AMP*CSK
04500        VTB=V5-V6*OEXP
04510        VTS=V5-V6*OEXP*DEXP(AW*HST)
04520 C*****PRESSURE DRAG DUE TO BOTTOM END AND STEP OF THE SPAR IS CALCULATED
04530        DP=0.5*RHO*CDP*(SO1*DABS(VTB)*VTB+(SO1-SO2)*DABS(VTS)*VTS)
04540        DNT1=0.
04550        DNT2=0.
04560        DDTT=0.
04570        DM1=0.
04580        DM2=0.
04590 C*****THE VISCOUS DRAG FORCE AND MOMENT ON THE SPAR IS CALCULATED****
04600        RD=RD1
04610        NDS1=NST*HST/HI
04620        NDS2=NST-NDS1
04630        NDS=NDS1
04640        DZ=HST/NDS
04650        ZZ=DZ/2.-ZCG
04660        RHOO=RHO*CDL
04670        DO 2 INZ=1,2
04680        DO 1 IZ=1,NDS
04690        XXE=DEXP(AW*(ZCG+ZZ))
04700        VN1=V1-V2*OEXP*XXE-ZZ*YD(NN+2)
04710        VN2=V3-V4*OEXP*XXE+ZZ*YD(NN+1)
04720        VNM=DSQRT(VN1*VN1+VN2*VN2)
04730        DN1=VNM*VN1*RD*DZ
04740        DN2=VNM*VN2*RD*DZ
04750        DNT1=DNT1+DN1
04760        DNT2=DNT2+DN2
04770        VTT=V5-V6*OEXP*XXE
04780        DDTT=DDTT+VTT*DABS(VTT)*RD*DZ
04790        DM1=DM1-ZZ*DN2
```

```
04800        DM2=DM2+ZZ*DN1
04810     1  ZZ=ZZ+DZ
04820        RD=RD2
04830        NDS=NDS2
04840        DZ=(HI-HST)/NDS
04850        ZZ=HST+DZ/2.-ZCG
04860     2  CONTINUE
04870        DD(NN-2)=RHOO*(DNT1-X(NN+3)*DNT2+X(NN+2)*(DDTT*0.02+DP/RHOO))
04880        DD(NN-1)=RHOO*(DNT1*X(NN+3)+DNT2-X(NN+1)*(DDTT*0.02+DP/RHOO))
04890        DD(NN)=RHOO*(-DNT1*X(NN+2)+DNT2*X(NN+1)+DDTT*0.02+DP/RHOO)
04900        DD(NN+1)=RHOO*(DM1-DM2*X(NN+3))
04910        DD(NN+2)=RHOO*(DM1*X(NN+3)+DM2)
04920        DD(NN+3)=RHOO*(-DM1*X(NN+2)+DM2*X(NN+1))
04930        RETURN
04940        END
04950        IMPLICIT REAL*8(A-H,O-Z)
04960        SUBROUTINE BUOYS
04970  C     THIS SUBROUTINE CALCULATES THE MASS MATRIX FOR THE SPAR BUOY
04980  C     ------------------------------------------------------------------
04990  C
05000        COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
05010       1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
05020       17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
05030       127),WWD1,WWD2,HMAX,HO,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRHO1,RRH
05040       102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,SO1,SO2,BB1,BB2,CSB,SNB,TMASS,Y
05050       1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,DEXP,AW,AWX,A
05060       1WY,AMP,WE,T,DT,P1,P2,QO,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
05070       1D
05080        DO 1 I=1,NDF
05090        DO 1 J=1,NDF
05100     1  GM(I,J)=0.
05110        BK1=(BB1-BB2)*HST
```

```
05120        BK2=BB2*HO
05130        P0=BB1*HST+BB2*(HO-HST)
05140        P1=BK2*(0.5*HO-ZCG)+BK1*(0.5*HST-ZCG)
05150        P2=BK2*(HO*HO/3.-HO*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
05160        QQ2=DEXP(-HOD*AW)
05170        Q0=(BB1*(QQ2-DEXP)+BB2*(1.-QQ2))/AW
05180        Q1=(BB1*(QQ2*(HST-1./AW-ZCG)+DEXP*(1./AW+ZCG))+BB2*(HO-1./AW-ZCG-Q
05190       1Q2*(HST-1./AW-ZCG)))/AW
05200 C****COEFFICIENTS IN EQUATIONS OF BUOY MOTION*************************
05210        GM(NN-2,NN-2)=TMASS+P0
05220        GM(NN-2,NN)=X(NN+2)*(VVO-P0)
05230        GM(NN-2,NN+2)=P1
05240        GM(NN-1,NN-1)=GM(NN-2,NN-2)
05250        GM(NN-1,NN)=X(NN+1)*(P0-VVO)
05260        GM(NN-1,NN+1)=-GM(NN-2,NN+2)
05270        GM(NN,NN-2)=GM(NN-2,NN)
05280        GM(NN,NN-1)=GM(NN-1,NN)
05290        GM(NN,NN)=TMASS+VVO
05300        GM(NN+1,NN)=-P1*X(NN+1)
05310        GM(NN+1,NN-1)=GM(NN-1,NN+1)
05320        GM(NN+1,NN+1)=YNERT1+P2
05330        GM(NN+2,NN-2)=GM(NN-2,NN+2)
05340        GM(NN+2,NN)=-P1*X(NN+2)
05350        GM(NN+2,NN+2)=GM(NN+1,NN+1)
05360        GM(NN+3,NN-2)=-GM(NN+1,NN)
05370        GM(NN+3,NN-1)=-GM(NN+2,NN)
05380        GM(NN+3,NN+3)=YNERT2
05390        RETURN
05400        END
05410        IMPLICIT REAL*8(A-H,O-Z)
05420        SUBROUTINE NEXT
05430 C      THIS SUBROUTINE UPDATES THE GEOMETRY OF THE SYSTEM DURING
```

```
05440 C      ITS DYNAMIC MOTIONS.
05450 C      ------------------------------------------------------------------
05460 C
05470        COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
05480       1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
05490       17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
05500       127),WWD1,WWD2,HMAX,HO,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRHO1,RRH
05510       102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,SO1,SO2,BB1,BB2,CSB,SNB,TMASS,Y
05520       1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,OEXP,AW,AWX,A
05530       1WY,AMP,WE,T,DT,P1,P2,QO,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
05540       1D
05550        IF(MOOR.EQ.2)GO TO 3
05560        XJ1=X(NN-2)+XC1
05570        YJ1=X(NN-1)+YC1
05580        ZJ1=X(NN)+ZC1
05590        IF(NB.EQ.1)GO TO 1
05600        TL(1)=DSQRT(X(1)**2+X(2)**2+X(3)**2)
05610        CS(1)=X(1)/TL(1)
05620        CS(2)=X(2)/TL(1)
05630        CS(3)=X(3)/TL(1)
05640        TL(NB)=DSQRT((XJ1-X(NN-5))**2+(YJ1-X(NN-4))**2+(ZJ1-X(NN-3))**2)
05650        CS(NN-2)=(XJ1-X(NN-5))/TL(NB)
05660        CS(NN-1)=(YJ1-X(NN-4))/TL(NB)
05670        CS(NN)=(ZJ1-X(NN-3))/TL(NB)
05680        GO TO 2
05690    1   TL(1)=DSQRT(XJ1**2+YJ1**2+ZJ1**2)
05700        CS(1)=XJ1/TL(1)
05710        CS(2)=YJ1/TL(1)
05720        CS(3)=ZJ1/TL(1)
05730    2   IF(NM.EQ.NB)GO TO 4
05740    3   XJ2=X(NN-2)+XC2
05750        YJ2=X(NN-1)+YC2
```

```
05760          ZJ2=X(NN)+ZC2
05770          TL(NB2)=DSQRT((X(NN+4)-XJ2)**2+(X(NN+5)-YJ2)**2+(X(NN+6)-ZJ2)**2)
05780          CS(NN+4)=(X(NN+4)-XJ2)/TL(NB2)
05790          CS(NN+5)=(X(NN+5)-YJ2)/TL(NB2)
05800          CS(NN+6)=(X(NN+6)-ZJ2)/TL(NB2)
05810      4   DO 5 I=2,NM
05820          IF(I.EQ.NB.OR.I.EQ.NB2)GO TO 5
05830          II=3*I+1
05840          IF(I.LT.NB)II=II-3
05850          TL(I)=DSQRT((X(II)-X(II-3))**2+(X(II+1)-X(II-2))**2+(X(II+2)-X(II-
05860         11))**2)
05870          CS(II)=(X(II)-X(II-3))/TL(I)
05880          CS(II+1)=(X(II+1)-X(II-2))/TL(I)
05890          CS(II+2)=(X(II+2)-X(II-1))/TL(I)
05900      5   CONTINUE
05910          RETURN
05920          END
05930          IMPLICIT REAL*8(A-H,O-Z)
05940          SUBROUTINE MINVER(N,C)
05950 C        THIS SUBROUTINE CALCULATES THE INVERSE OF A MATRIX 'C'
05960 C        OF ORDER 'N'
05970 C        ------------------------------------------------------------------
05980 C
05990          DIMENSION C(27,27),H(27),G(27)
06000          NN=N-1
06010          C(1,1)=1.0/C(1,1)
06020          DO 6 M=1,NN
06030          K=M+1
06040          DO 1 I=1,M
06050          G(I)=0.
06060          DO 1 J=1,M
06070      1   G(I)=G(I)+C(I,J)*C(J,K)
```

```
06080        D=0.
06090        DO 2 I=1,M
06100     2  D=D+C(K,I)*G(I)
06110        E=C(K,K)-D
06120        C(K,K)=1.0/E
06130        DO 3 I=1,M
06140     3  C(I,K)=-G(I)*C(K,K)
06150        DO 4 J=1,M
06160        H(J)=0.
06170        DO 4 I=1,M
06180     4  H(J)=H(J)+C(K,I)*C(I,J)
06190        DO 5 J=1,M
06200     5  C(K,J)=-H(J)*C(K,K)
06210        DO 6 I=1,M
06220        DO 6 J=1,M
06230     6  C(I,J)=C(I,J)-G(I)*C(K,J)
06240        RETURN
06250        END
06260        IMPLICIT REAL*8(A-H,O-Z)
06270        SUBROUTINE MATRIX
06230 C      THIS SUBROUTINE CALCULATES THE REMAINING ELEMENTS OF THE GLOBAL
06290 C      MATRICES AND INTEGRATES THE 'NDF' SIMULTANEOUS DIFFERENTIAL
06300 C      EQUATIONS OF MOTION BY THE METHOD OF RECTANGULAR RULE.
06310 C      ------------------------------------------------------------------
06320 C
06330        COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
06340       1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
06350       17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
06360       127),WWD1,WWD2,HMAX,HO,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRHO1,RRH
06370       102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,SO1,SO2,BB1,BB2,CSB,SNB,TMASS,Y
06380       1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,OEXP,AW,AWX,A
06390       1WY,AMP,WE,T,DT,P1,P2,QO,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
```

```
      1D
C*****WAVE EXCITING PLUS HYDROSTATIC FORCES ON THE SPAR BUOY*******
      F(NN-2)=2.*WE*WE*AMP*CSB*Q0*CSK
      F(NN-1)=2.*WE*WE*AMP*SNB*Q0*CSK
      F(NN)=WE*WE*AMP*Q0*SNK+GR*(BB1*HST+BB2*(HI-HST)-TMASS)
      F(NN+1)=-WE*WE*2.*AMP*SNB*Q1*CSK-GR*P1*X(NN+1)
      F(NN+2)=WE*WE*2.*AMP*CSB*Q1*CSK-GR*P1*X(NN+2)
      IF(MOOR.EQ.0)GO TO 11
C*****STIFFNESS MATRIX FOR THE MOORING LINES*********************
      AA1=XC1-YC1*X(NN+3)+ZC1*X(NN+2)
      AA2=XC2-YC2*X(NN+3)+ZC2*X(NN+2)
      AB1=XC1*X(NN+3)+YC1-ZC1*X(NN+1)
      AB2=XC2*X(NN+3)+YC2-ZC2*X(NN+1)
      AC1=-XC1*X(NN+2)+YC1*X(NN+1)+ZC1
      AC2=-XC2*X(NN+2)+YC2*X(NN+1)+ZC2
      GK(1,1)=EK(1)*CS(1)
      GK(1,2)=EK(1)*CS(2)
      GK(1,3)=EK(1)*CS(3)
      GK(NB,NN+1)=EK(NB)*(-AC1*CS(NN-1)+AB1*CS(NN))
      GK(NB,NN+2)=EK(NB)*(AC1*CS(NN-2)-AA1*CS(NN))
      GK(NB,NN+3)=EK(NB)*(-AB1*CS(NN-2)+AA1*CS(NN-1))
      IF(NM.EQ.1)GO TO 5
      IF(NM.EQ.NB)GO TO 1
      GK(NB2,NN-2)=-EK(NB2)*CS(NN+4)
      GK(NB2,NN-1)=-EK(NB2)*CS(NN+5)
      GK(NB2,NN)=-EK(NB2)*CS(NN+6)
      GK(NB2,NN+1)=-EK(NB2)*(-AC2*CS(NN+5)+AB2*CS(NN+6))
      GK(NB2,NN+2)=-EK(NB2)*(AC2*CS(NN+4)-AA2*CS(NN+6))
      GK(NB2,NN+3)=-EK(NB2)*(-AB2*CS(NN+4)+AA2*CS(NN+5))
      GK(NB2,NN+4)=-GK(NB2,NN-2)
      GK(NB2,NN+5)=-GK(NB2,NN-1)
      GK(NB2,NN+6)=-GK(NB2,NN)
```

```
06720     1  DO 2 I=2,NM
06730        IF(I.EQ.NB2)GO TO 2
06740        II=3*I
06750        IF(I.GT.NB)II=II+3
06760        GK(I,II-5)=-EK(I)*CS(II-2)
06770        GK(I,II-4)=-EK(I)*CS(II-1)
06780        GK(I,II-3)=-EK(I)*CS(II)
06790        GK(I,II)=-GK(I,II-3)
06800        GK(I,II-1)=-GK(I,II-4)
06810        GK(I,II-2)=-GK(I,II-5)
06820     2  CONTINUE
06830 C*****MASS MATRIX FOR THE MOORING LINES*********************************
06840        DO 4 I=1,NM
06850        IF(I.EQ.NB)GO TO 4
06860        II=3*I+1
06870        IF(I.LT.NB)II=II-3
06880        GM(II,II)=CM(I)+CMS(I)+CMN(I)*(1.-CS(II+3)**2)+CMT(I)*CS(II+3)**2
06890        GM(II+1,II+1)=CM(I)+CMS(I)+CMN(I)*(1.-CS(II+4)**2)+CMT(I)*CS(II+4)
06900       1**2
06910        GM(II+2,II+2)=CM(I)+CMS(I)+CMN(I)*(1.-CS(II+5)**2)+CMT(I)*CS(II+5)
06920       1**2
06930        GM(II,II+1)=(CMT(I)-CMN(I))*CS(II+4)*CS(II+3)
06940        GM(II+1,II)=GM(II,II+1)
06950        GM(II,II+2)=(CMT(I)-CMN(I))*CS(II+5)*CS(II+3)
06960        GM(II+1,II+2)=(CMT(I)-CMN(I))*CS(II+5)*CS(II+4)
06970        GM(II+2,II)=GM(II,II+2)
06980        GM(II+2,II+1)=GM(II+1,II+2)
06990        WWE=GR*AMP*DEXP(AW*(X(II+2)-DEPTH))
07000        SWE=AWX*X(II)+AWY*X(II+1)-WE*T
07010        SNK1=DSIN(SWE)
07020        CSK1=DCOS(SWE)
07030        PHX=WWE*AWX*CSK1
```

```
07040       PHY=WWE*AWY*CSK1
07050       PHZ=WWE*AW*SNK1
07060 C*****VISCOUS DRAG FORCE ARRAY FOR THE MOORING LINES*******************
07070       U1=V(1,I)-YD(II)-AWX*SNK1*WWE/WE
07080       U2=V(2,I)-YD(II+1)-AWY*SNK1*WWE/WE
07090       U3=V(3,I)-YD(II+2)+AW*CSK1*WWE/WE
07100       UT1=U1*CS(II+3)+U2*CS(II+4)+U3*CS(II+5)
07110       VN(1)=U1*(1.-CS(II+3)*CS(II+3))-U2*CS(II+3)*CS(II+4)-U3*CS(II+3)*
07120      1CS(II+5)
07130       VN(2)=U2*(1.-CS(II+4)*CS(II+4))-U3*CS(II+4)*CS(II+5)-U1*CS(II+4)*
07140      1CS(II+3)
07150       VN(3)=U3*(1.-CS(II+5)*CS(II+5))-U1*CS(II+5)*CS(II+3)-U2*CS(II+5)*
07160      1CS(II+4)
07170       DDTT=CDIT(I)*UT1*DABS(UT1)
07180       VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
07190       VNM1=DSQRT(U1*U1+U2*U2+U3*U3)*CDIA(I)
07200       DD(II)=CDIN(I)*VNM*VN(1)+DDTT*CS(II+3)+VNM1*U1
07210       DD(II+1)=CDIN(I)*VNM*VN(2)+DDTT*CS(II+4)+VNM1*U2
07220       DD(II+2)=CDIN(I)*VNM*VN(3)+DDTT*CS(II+5)+VNM1*U3
07230       IF(MOOR.EQ.2.AND.I.GE.ND)GO TO 3
07240 C*****FROUDE KRYLOV FORCE ARRAY FOR THE MOORING LINES*****************
07250       F(II)=CMD(I)*PHX
07260       F(II+1)=CMD(I)*PHY
07270       F(II+2)=-W(I)+CMD(I)*PHZ
07280       GO TO 4
07290     3 F(II)=CMD(I)*(PHX*(1.-CS(II+3)*CS(II+3))-PHY*CS(II+3)*CS(II+4)-PHZ
07300      1*CS(II+3)*CS(II+5))
07310       F(II+1)=CMD(I)*(PHY*(1.-CS(II+4)*CS(II+4))-PHZ*CS(II+4)*CS(II+5)-P
07320      1HX*CS(II+4)*CS(II+3))
07330       F(II+2)=CMD(I)*(PHZ*(1.-CS(II+5)*CS(II+5))-PHX*CS(II+5)*CS(II+3)-P
07340      1HY*CS(II+5)*CS(II+4))-W(I)
07350     4 CONTINUE
```

```
C*****TENSION FORCE ARRAY FOR THE MOORING LINES**********************
    5  DO 7 K=1,NM
       TG=0.
       DO 6 IK=1,NDF
    6  TG=TG+GK(K,IK)*Y(IK)
       TNN(K)=TG+TNN(K)
       TN(K)=TNN(K)
       IF(TN(K).LE.0.)TN(K)=0.
       KK=3*K+1
       IF(K.LE.NB)KK=KK-3
       TC(KK)=TN(K)*CS(KK)
       TC(KK+2)=TN(K)*CS(KK+2)
    7  TC(KK+1)=TN(K)*CS(KK+1)
       TC(NN+1)=-AC1*TC(NN-1)+AB1*TC(NN)+AC2*TC(NN+5)-AB2*TC(NN+6)
       TC(NN+2)=AC1*TC(NN-2)-AA1*TC(NN)-AC2*TC(NN+4)+AA2*TC(NN+6)
       TC(NN+3)=-AB1*TC(NN-2)+AA1*TC(NN-1)+AB2*TC(NN+4)-AA2*TC(NN+5)
       IF(NM.EQ.1)GO TO 11
       IF(MOOR.EQ.2)GO TO 9
       NZ=NN-3
       DO 8 M=1,NZ
    8  TC(M)=TC(M)-TC(M+3)
    9  TC(NN-2)=TC(NN-2)-TC(NN+4)
       TC(NN-1)=TC(NN-1)-TC(NN+5)
       TC(NN)=TC(NN)-TC(NN+6)
       IF(NM.EQ.NB)GO TO 11
       NZ=NN+4
       DO 10 M=NZ,NDF
   10  TC(M)=TC(M)-TC(M+3)
C*****SOLUTION OF THE DIFFERENTIAL EQUATIONS**************************
       CALL MINVER(NDF,GM)
   11  DO 13 M=1,NDF
       YDD(M)=0.
```

```
07680          DO 12 K=1,NDF
07690     12   YDD(M)=YDD(M)+GM(M,K)*(DD(K)-TC(K)+F(K))
07700          YD(M)=YD(M)+YDD(M)*DT
07710          Y(M)=YD(M)*DT
07720     13   X(M)=X(M)+Y(M)
07730          HO=HO-Y(NN)
07740          HOD=HO-HST
07750          RETURN
07760          END
READY
```

```
 SD2.FORT
00010 C
00020 C      2 D DYNAMIC ANALYSIS OF A SPAR BUOY,A SURFACE MOORED SYSTEM,
00030 C      AND A DRIFTING DROGUED SPAR BUOY.
00040 C      ------------------------------------------------------------
00050 C      THE UNITS USED IN THIS ANALYSIS ARE FEET/POUNDS/SECONDS.
00060 C      ------------------------------------------------------------
00070 C      CASE STUDY 1--MOOR=0--SPAR BUOY FREELY FLOATING IN A SURFACE WAVE.
00080 C      CASE STUDY 2--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
00090 C      MOORING LINE; NM=NB.
00100 C      CASE STUDY 3--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
00110 C      MOORING LINE AND A LUMPED MASS INSTRUMENT LINE HANGING FROM
00120 C      THE BUOY; NM NOT EQUAL TO NB.
00130 C      CASE STUDY 4--MOOR=2--SPAR BUOY ATTACHED TO A WINDOW SHADE
00140 C      DROGUE; NB=1.
00150 C      ------------------------------------------------------------
00160 C
00170 C*****START OF THE MAIN PROGRAM*********************************************
00180        IMPLICIT REAL*8(A-H,O-Z)
00190        COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
00200       1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
00210       1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
00220       1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SO1,SO2,CS1,SN1,RD1,RD2,XJ1,Y
00230       1J1,XJ2,YJ2,BB1,BB2,HMAX,HO,HST,HOD,RRHO1,RRHO2,TMASS,YNERT,ZCG,HI,
00240       1THO,CDP,CDL,CDN,COT,ALPHA,PI,RHO,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,O
00250       1K,VVV(8),OEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
00260       1,NN,ND
00270   100  FORMAT(12F10.2)
00280   101  FORMAT(7F10.0)
00290   102  FORMAT(6I5,4F10.0)
00300   103  FORMAT(6F10.0)
00310 C*****GENERAL DATA*********************************************************
```

```
00320       READ(5,102)NM,NB,ND,NST,NC,MOOR,DEPTH,DT,TMAX,T2
00330 C*****BUOY DATA*************************************************************
00340       READ(5,101)RD1,RD2,ZCG,RGYR,HMAX,CDL,HST,CON,COT,CDF,ALPHA
00350 C*****MOORING LINE DATA*****************************************************
00360       READ(5,103)(W(I),CM(I),CMS(I),CMD(I),CMN(I),CMT(I),I=1,NM)
00370       READ(5,103)(SL(I),EK(I),CDIN(I),CDIT(I),CDIA(I),DEP(I),I=1,NM)
00380 C*****ATTACHMENT DATA*******************************************************
00390       READ(5,101)XC1,ZC1,XC2,ZC2
00400 C*****CURRENT PROFILE DATA**************************************************
00410       READ(5,101)(V(I),I=1,NM)
00420       NM1=NM-NB
00430       NN=NB+NB
00440       NB1=NB-1
00450       NB2=NB+1
00460       NM2=NM+1
00470       NDF=2*NM+1
00480 C*****INITIALIZE PHYSICAL CONSTANTS*****************************************
00490       PI=4.*DATAN(1.D0)
00500       S01=PI*RD1*RD1
00510       S02=PI*RD2*RD2
00520       RHO=1.9905
00530       GR=32.2
00540       TMASS=W(NB)/GR
00550       YNERT=TMASS*RGYR*RGYR
00560       RRHO1=RHO*CDL*RD1*HST
00570       RRHO2=RHO*CDL*RD2
00580       BB1=RHO*S01
00590       BB2=RHO*S02
00600       WWD1=BB1*GR*HST
00610       WWD2=BB2*GR
00620       VV0=1.33*RHO*ALPHA*RD1**3
00630       VV1=1.33*RHO*ALPHA*(RD1-RD2)**3
```

```
C*****START LOOP FOR DYNAMIC RESPONSE TO DIFFERENT SURFACE WAVES******
      DO 8 ICC=1,NC
      READ(5,101)WE,AMP
C*****INITIALIZE DYNAMIC VARIABLES************************************
      DC1=0.
      DC2=0.
      DC3=0.
      DO 1 I=1,NDF
      YD(I)=0.
      TC(I)=0.
      Y(I)=0.
      DO 1 J=1,NM
    1 GK(J,I)=0.
      AW=WE*WE/GR
      T3=0.
C*****FIND THE CURRENT PROFILE FOR STATIC SOLUTION*********************
      DO 3 I=1,NM
      VV(I)=0.
      ZX4=0.
      ZX1=WE*AMP*DEXP(AW*DEP(I))
      DO 2 J=1,12
      ZX3=ZX1*DCOS(PI*J/6.)
      ZX2=V(I)+ZX1*DSIN(PI*J/6.)
      VEM=DSQRT(ZX2**2+ZX3**2)
      ZX4=ZX4+ZX3*VEM
    2 VV(I)=VV(I)+ZX2*VEM
      VEM1=(ZX4**2+VV(I)**2)**0.25
      VV(I)=VV(I)/(VEM1*12.**0.5)
    3 IF(DABS(VV(I)).LE.0.000001)VV(I)=0.
      WRITE(6,100)(VV(I),I=1,NM)
      TC(NDF+1)=0.
      TC(NDF+2)=0.
```

```
00960       IF(NM.EQ.NB)GO TO 5
00970       CS(NM2)=0.
00980       SN(NM2)=-1.
00990       IF(MOOR.EQ.2)GO TO 5
01000 C*****STATIC SOLUTION OF THE LUMPED MASSES (INSTRUMENT LINE)***********
01010       DO 4 I=1,NM1
01020       J=NM2-I
01030       K=J+1
01040       II=J+J
01050       VN=VV(J)*SN(K)
01060       VT=VV(J)*CS(K)
01070       DI=CDIA(J)*VV(J)*DABS(VV(J))
01080       DDTT=CDIT(J)*VT*DABS(VT)
01090       DN=CDIN(J)*VN*DABS(VN)
01100       TC(II)=TC(II+2)+DI+DDTT*CS(K)+DN*SN(K)
01110       TC(II+1)=TC(II+3)+DDTT*SN(K)-DN*CS(K)-W(J)
01120       TNN(J)=DSQRT(TC(II)**2+TC(II+1)**2)
01130       CS(J)=TC(II)/TNN(J)
01140       SN(J)=TC(II+1)/TNN(J)
01150     4 TL(J)=SL(J)+TNN(J)/EK(J)
01160     5 CALL STATIC
01170       T=0.
01180     6 SS=AW*X(NN-1)-WE*T
01190       SNK=DSIN(SS)
01200       CSK=DCOS(SS)
01210       OK=AW*CS1
01220       OEXP=DEXP(-HO*CK)
01230       HI=HO-AMP*SNK/CS1
01240       IF(CDL.NE.0.0)CALL DBDRG
01250       CALL BUOYS
01260       CALL MATRIX
01270       IF(T.LT.T3)GO TO 7
```

```
01280         WRITE(6,100)T
01290         WRITE(6,100)(X(I),I=1,NDF)
01300         IF(MOOR.EQ.1)WRITE(6,100)(TN(K),K=1,NM)
01310         IF(MOOR.EQ.2)WRITE(6,100)(TN(K),K=2,NM)
01320         IF(T.GT.TMAX)GO TO 8
01330         T3=T3+T2
01340      7  T=T+DT
01350         IF(MOOR.GT.0)CALL NEXT
01360         GO TO 6
01370      8  CONTINUE
01380         CALL EXIT
01390         END
01400         IMPLICIT REAL*8(A-H,O-Z)
01410         SUBROUTINE STATIC
01420 C       THIS SUBROUTINE CALCULATES THE STATIC SOLUTION OF THE FOUR CASE
01430 C       STUDIES OUTLINED IN THE MAIN PROGRAM
01440 C       ------------------------------------------------------------------
01450 C
01460         COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
01470        1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
01480        1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
01490        1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SO1,SO2,CS1,SN1,RD1,RD2,XJ1,Y
01500        1J1,XJ2,YJ2,BB1,BB2,HMAX,HO,HST,HOD,RRHO1,RRHO2,TMASS,YNERT,ZCG,HI,
01510        1THO,CDF,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,O
01520        1K,VVV(8),OEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
01530        1,NN,ND
01540   100   FORMAT(9F12.3)
01550   101   FORMAT(///9X111HSTATIC CALCULATION TERMINATED -- NUMBER OF ITERATI
01560        1ONS TO DETERMINE BUOY EQUILIBRIUM CONDITION EXCEEDS MAXIMUM (  I3,
01570        1   1H))
01580   102   FORMAT (///10X,65HSTATIC CALCULATION TERMINATED -- BUOY HAS EXCEED
01590        1ED MAXIMUM DRAFT )
```

```
01600  103  FORMAT (///10X,100HSTATIC CALCULATION TERMINATED -- NUMBER OF ITER
01610      1ATIONS TO FIND STATIC SOLUTION HAS EXCEEDED MAXIMUM (  I3,1H))
01620       IF(MOOR.EQ.0)GO TO 33
01630       DATA ITMAX,IT1MAX/100,100/
01640       IT1=0
01650       INDEX=1
01660       EMAX=.01*W(NB)*(RD1+RD2)
01670       IF(MOOR.EQ.1)GO TO 8
01680 C
01690 C*****CASE STUDY 4 ***************************************************
01700 C
01710 C*****INITIALIZE VARIABLES*******************************************
01720       IS2=0
01730       DC2=0.
01740       VE=0.
01750       FXE=0.02*HMAX*RD2*V(1)*V(1)+0.01
01760       DVE=DABS(V(NM))/2.
01770       X(3)=0.
01780       CS1=1.
01790       SN1=0.
01800       DTHI=0.05
01810 C*****START STATIC VELOCITY ITERATION LOOP*****************************
01820    1  INDEX=INDEX+1
01830    2  IT1=IT1+1
01840       IF(IT1.GT.IT1MAX)GO TO 30
01850       DTH=DTHI
01860       IT=0
01870       IS=0
01880       DO 3 IC=1,NM
01890    3  VVV(IC)=VV(IC)-VE
01900       DO 4 IC=2,NM
01910       K=NM-IC+3
```

```
01920          J=K-1
01930          II=J+J
01940          VN=VVV(J)*SN(K)
01950          VT=VVV(J)*CS(K)
01960          DN=CDIN(J)*VN*DABS(VN)
01970          DDTT=CDIT(J)*VT*DABS(VT)
01980          DI=CDIA(J)*VVV(J)*DABS(VVV(J))
01990          TC(II)=TC(II+2)+DI+DDTT*CS(K)+DN*SN(K)
02000          TC(II+1)=TC(II+3)+DDTT*SN(K)-DN*CS(K)-W(J)
02010          TNN(J)=DSQRT(TC(II)*TC(II)+TC(II+1)*TC(II+1))
02020          CS(J)=TC(II)/TNN(J)
02030          SN(J)=TC(II+1)/TNN(J)
02040      4   TL(J)=SL(J)+TNN(J)/EK(J)
02050          IF(INDEX.EQ.3)GO TO 11
02060          HO=HST+(W(1)-TC(5)-DC2-WWD1)/WWD2
02070          RRHO=RRHO1+RRHO2*(HO-HST)
02080          VN=VVV(1)*CS1
02090          VT=VVV(1)*SN1
02100          DDTT=(0.02*RRHO+0.5*RHO*CDP*(2.*SO1-SO2))*DABS(VT)*VT
02110          DN=RRHO*VN*DABS(VN)
02120          DC1=DN*CS1+DDTT*SN1
02130      5   FX=DC1+TC(4)
02140          WRITE(6,100)VE,HO,FX
02150          IF(DABS(FX).LE.FXE)GO TO 6
02160          IS3=DSIGN(1.0D0,FX)
02170          IF(IS2+IS3.EQ.0)DVE=DVE/2.
02180          VE=VE+IS3*DVE
02190          IS2=IS3
02200          GO TO 2
02210      6   IF(INDEX.EQ.2)GO TO 1
02220          WRITE(6,100)VE,HO,E
02230          YD(1)=VE
```

```
02240        DO 7 I=2,NM
02250        II=I+I
02260     7  YD(II)=VE
02270        X(1)=0.
02280        X(2)=(ZCG-HO)*CS1+DEPTH
02290        TNN(1)=0.
02300        CS(1)=0.
02310        SN(1)=0.
02320        GO TO 25
02330 C
02340 C*****CASE STUDIES 2 AND 3 ***********************************************
02350 C
02360     8  NBB=NB-1
02370        VVV(NB)=VV(NB)
02380        HO=HST+(W(NB)-TC(NN+3)-WWD1)/WWD2
02390        DH=0.
02400        WADD=W(NBB)/10.
02410 C*****INITILIZE ERROR TOLERANCES**************************************
02420        ER= 0.0010*DEPTH
02430     9  HO=HO+DH
02440        X(NN+1)=0.0
02450        DTHI=0.05
02460        IT1=0
02470 C*****START CABLE (BUOY DRAFT) ITERATION LOOP*****************************
02480    10  IT1=IT1+1
02490        IF(IT1.GT.IT1MAX)GO TO 30
02500        DTH=DTHI
02510        IT=0
02520        IS=0
02530    11  E=1.D25
02540 C*****START BUOY ITERATION*************************************************
02550    12  IF(MOOR.EQ.2)HO=HST+(W(1)-TC(5)-DC2-WWD1)/WWD2
```

```
      IF(HO.GT.0.99*HMAX)GO TO 31
      HOD=HO-HST
      B=WWD1+WWD2*HOD
      E1=E
      IT=IT+1
      IF(IT.GT.ITMAX)GO TO 32
      RRHO=RRHO1+RRHO2*HOD
      CS1=DCOS(X(NN+1))
      SN1=DSIN(X(NN+1))
      VN=VVV(NB)*CS1
      VT=VVV(NB)*SN1
      DDTT=(0.C2*RRHO+0.5*RHO*CDP*(2.*SO1-SO2))*DABS(VT)*VT
      DN=RRHO*VN*DABS(VN)
      DC1=DN*CS1+DDTT*SN1
      DC2=DDTT*CS1-DN*SN1
      DC3=DN*(HST/2.-ZCG)+RRHO2*HO*HOD*VN*DABS(VN)/2.
      IF(MOOR.EQ.2)GO TO 13
      TC(NN-1)=DC1+TC(NN+2)
      TC(NN)=DC2+B-W(NB)+TC(NN+3)
   13 ARMX1=ZC1*SN1+XC1*CS1
      ARMZ1=ZC1*CS1-XC1*SN1
      ARMX2=ZC2*SN1+XC2*CS1
      ARMZ2=ZC2*CS1-XC2*SN1
C*****CALCULATE UNBALANCED MOMENT ON BUOY*****************************
      BGZ=SN1*(WWD1*(HST/2.-ZCG)+WWD2*HOD*((HO+HST)/2.-ZCG))
      E=DC3-BGZ-TC(NN-1)*ARMZ1+TC(NN)*ARMX1+TC(NN+2)*ARMZ2-TC(NN+3)*ARMX
     12
      IF(MOOR.EQ.1)WRITE(6,100)HO,X(NN+1),DH,E
      IF(MOOR.EQ.2)WRITE(6,100)VE,HO,X(3),E
C*****TEST FOR CONVERGENCE*******************************************
      IF (DABS(E).LT.EMAX) GO TO 16
      IS1=DSIGN(1.DO,E)
```

```
C*****TEST FOR CHANGE IN SIGN OF UNBALANCED MOMENT (ERROR)***************
      IF((IS1+IS).EQ.0) GO TO 14
      IF(DABS(E).GT.DABS(E1)) GO TO 15
C*****NO CHANGE IN SIGN - CONTINUE IN SAME DIRECTION**********************
      X(NN+1)=X(NN+1)+IS1*DTH
      IS=IS1
      GO TO 12
C*****CHANGE IN SIGN -- GO BACK HALF A STEP*******************************
  14  DTH=DTH/2.
      X(NN+1)=X(NN+1)+IS1*DTH
      IS=IS1
      GO TO 12
C*****ERROR IS GROWING - ITERATE IN OPPOSITE DIRECTION********************
  15  DTH=-DTH
      X(NN+1)=X(NN+1)+IS1*DTH*2.
      IS=IS1
      GO TO 12
  16  IF(MOOR.EQ.2)GO TO 5
C*****BUOY-CABLE SOLUTION HAS CONVERGED - START INTEGRATION DOWN CABLE
      DTHI=DTHI/2.
      DH=DH/2.
      TNN(NB)=DSQRT(TC(NN-1)*TC(NN-1)+TC(NN)*TC(NN))
      TL(NB)=SL(NB)+TNN(NB)/EK(NB)
      CS(NB)=TC(NN-1)/TNN(NB)
      SN(NB)=TC(NN)/TNN(NB)
      IF(NB.EQ.1)GO TO 19
  17  DO 18 I=1,NB1
      J=NB-I
      K=J+1
      II=J+J
      VN=VV(J)*SN(K)
      VT=VV(J)*CS(K)
```

```
      DI=CDIA(J)*VV(J)*DABS(VV(J))
      DDTT=CDIT(J)*VT*DABS(VT)
      DN=CDIN(J)*VN*DABS(VN)
      TC(II-1)=TC(II+1)+DI+DDTT*CS(K)+DN*SN(K)
      TC(II)=TC(II+2)+DDTT*SN(K)-DN*CS(K)-W(J)
      TNN(J)=DSQRT(TC(II-1)**2+TC(II)**2)
      CS(J)=TC(II-1)/TNN(J)
      SN(J)=TC(II)/TNN(J)
   18 TL(J)=SL(J)+TNN(J)/EK(J)
   19 YJ1=0.
      DO 20 I=1,NB
   20 YJ1=YJ1+TL(I)*SN(I)
      DEPTH1=YJ1+(HO-ZCG-ZC1)*CS1+XC1*SN1
      EE=DEPTH-DEPTH1
      WRITE(6,100)HO,X(NN+1),EE,(W(NCC),NCC=1,NB)
      EEE=DABS(EE)
C*****TEST FOR CONVERGENCE*********************************************
      IF(EEE.LE.ER)GO TO 22
      IF(INDEX.EQ.1.AND.EE.LT.0.)GO TO 29
      IF(INDEX.EQ.2)GO TO 21
      INDEX=2
      DH=(HMAX-HO)/2.
      GO TO 9
   21 HO=HO+EEE*DH/EE
      GO TO 10
C*****STATIC SOLUTION HAS CONVERGED- COMPUTE ALL COORDINATES************
   22 DEPTH=DEPTH1
      WRITE(6,100)DEPTH
      X(1)=TL(1)*CS(1)
      X(2)=TL(1)*SN(1)
      IF(NB.EQ.1)GO TO 24
      DO 23 I=2,NB
```

```
03520          II=I+I
03530          X(II-1)=X(II-3)+TL(I)*CS(I)
03540     23   X(II)=X(II-2)+TL(I)*SN(I)
03550     24   XJ1=X(NN-1)
03560          X(NN-1)=XJ1-XC1*CS1-ZC1*SN1
03570          X(NN)=YJ1-ZC1*CS1+XC1*SN1
03580          IF(NM.EQ.NB)GO TO 27
03590     25   XJ2=X(NN-1)+XC2*CS1+ZC2*SN1
03600          YJ2=X(NN)+ZC2*CS1-XC2*SN1
03610          X(NN+2)=XJ2+TL(NB2)*CS(NB2)
03620          X(NN+3)=YJ2+TL(NB2)*SN(NB2)
03630          IF(NB2.EQ.NM)GO TO 27
03640          NB3=NB2+1
03650          DO 26 I=NB3,NM
03660          II=I+I
03670          X(II)=X(II-2)+TL(I)*CS(I)
03680     26   X(II+1)=X(II-1)+TL(I)*SN(I)
03690     27   WRITE(6,100)HO,X(NN+1),TC(NN-1),TC(NN),E
03700          WRITE(6,100)(X(I),I=1,NDF)
03710     28   RETURN
03720 C*****PART OF THE MOORING LINE IS FLOATING ON THE SURFACE(FIND IT)*****
03730     29   W(NBB)=W(NBB)-WADD
03740          IF(W(NBB).LT.0.)GO TO 17
03750          W(NBB)=0.
03760          NBB=NBB-1
03770          GO TO 29
03780     30   WRITE(6,103)IT1MAX
03790          CALL EXIT
03800     31   WRITE(6,102)
03810          CALL EXIT
03820     32   WRITE(6,101)ITMAX
03830          CALL EXIT
```

```
C
C*****CASE STUDY 1 ************************************************************
C
  33  HO=HST+(W(1)-WWD1)/WWD2
      WAW=AW*HO
      OEXP=DEXP(-WAW)
      HOD=HO-HST
      BK1=(BB1-BB2)*HST
      BK2=BB2*HO
      P1=BK2*(0.5*HO-ZCG)+BK1*(0.5*HST-ZCG)
      P2=BK2*(HO*HO/3.-HO*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
      QQ2=DEXP(-HOD*AW)
      Q0=(BB1*(QQ2-OEXP)+BB2*(1.-QQ2))/AW
      Q1=(BB1*(QQ2*(HST-1./AW-ZCG)+OEXP*(1./.W+ZCG))+BB2*(HO-1./AW-ZCG-Q
     1Q2*(HST-1./AW-ZCG)))/AW
      YD(1)=VV(1)
      YD(2)=WE*AMP*(RHO*SO2*GR-WE*WE*Q0)/(RHO*SO2*GR-WE*WE*(TMASS+VVO+VV
     11))
      X(1)=-AMP*Q0/TMASS
      X(2)=ZCG-HO+DEPTH
      X(3)=WE*WE*AMP*(2.*Q1-P1*Q0/TMASS)/(P1*GR-WE*WE*(YNERT+P2-P1*P1/(2
     1.*TMASS)))
      XAW=X(1)/AMP
      ZAW=YD(2)/(WE*AMP)
      TAW=X(3)/(AW*AMP)
      SN1=DSIN(X(3))
      CS1=DCOS(X(3))
      WRITE(6,100)WAW,XAW,ZAW,TAW
      RETURN
      END
      IMPLICIT REAL*8(A-H,O-Z)
      SUBROUTINE DBDRG
```

```
04160 C      THIS SUBROUTINE CALCULATES THE VISCOUS DRAG FORCE ON THE
04170 C      SPAR BUOY DURING ITS DYNAMIC MOTIONS. INTEGRATION OF FORCES
04180 C      IS DONE ALONG THE AXIS OF THE SPAR.
04190 C      ------------------------------------------------------------------
04200 C
04210        COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
04220       1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
04230       1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
04240       1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SO1,SO2,CS1,SN1,RD1,RD2,XJ1,Y
04250       1J1,XJ2,YJ2,BB1,BB2,HMAX,HO,HST,HOD,RRHO1,RRHO2,TMASS,YNERT,ZCG,HI,
04260       1THO,CDP,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,O
04270       1K,VVV(8),OEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
04280       1,NN,ND
04290 C*****COEFFICIENTS OF THE VELOCITY EQUATION ARE CALCULATED BELOW********
04300        V1=(V(NB)-YD(NN-1))*CS1+YD(NN)*SN1
04310        V2=WE*AMP*(SNK*CS1+CSK*SN1)
04320        V3=(V(NB)-YD(NN-1))*SN1-YD(NN)*CS1
04330        V4=WE*AMP*(SNK*SN1-CSK*CS1)
04340        VTB=V3-V4*OEXP
04350        VTS=V3-V4*OEXP*DEXP(OK*HST)
04360 C*****PRESSURE DRAG DUE TO BOTTOM END AND STEP OF THE SPAR IS CALCULATED
04370        DP=0.5*RHO*CDP*(SO1*DABS(VTB)*VTB+(SO1-SO2)*DABS(VTS)*VTS)
04380        DN=0.
04390        DDTT=0.
04400        DM=0.
04410 C*****THE VISCOUS DRAG FORCE AND MOMENT ON THE SPAR BUOY ARE CALCULATED
04420        RD=RD1
04430        NDS1=NST*HST/HI
04440        NDS2=NST-NDS1
04450        NDS=NDS1
04460        DZ=HST/NDS
04470        ZZ=DZ/2.-ZCG
```

```
04480         RHOO=RHO*CDL
04490         DO 2 INZ=1,2
04500         DO 1 IZ=1,NDS
04510         XXE=DEXP(OK*(ZZ+ZCG))
04520         VNN=V1-V2*OEXP*XXE-ZZ*YD(NN+1)
04530         DNN=VNN*DABS(VNN)*RD*DZ
04540         DN=DN+DNN
04550         VTT=V3-V4*OEXP*XXE
04560         DDTT=DDTT+VTT*DABS(VTT)*RD*DZ
04570         DM=DM+DNN*ZZ
04580       1 ZZ=ZZ+DZ
04590         RD=RD2
04600         NDS=NDS2
04610         DZ=(HI-HST)/NDS
04620         ZZ=HST+DZ/2.-ZCG
04630       2 CONTINUE
04640         DN=RHOO*DN
04650         DDTT=0.02*RHOO*DDTT
04660         DC3=RHOO*DM
04670 C*****DC1&DC2 ARE THE VISCOUS DRAG FORCES IN SURGE AND HEAVE DIRECTIONS*
04680         DC1=DN*CS1+(DDTT+DP)*SN1
04690         DC2=(DDTT+DP)*CS1-DN*SN1
04700         RETURN
04710         END
04720         IMPLICIT REAL*8(A-H,O-Z)
04730         SUBROUTINE BUOYS
04740 C       THIS SUBROUTINE CALCULATES THE MASS MATRIX FOR THE SPAR BUOY,
04750 C       PART(EXCEPT VISCOUS) OF THE (FA) MATRIX AS DD, AND (FD+FG)
04760 C       ARRAYS AS XW,ZW,AND EMW.
04770 C       ------------------------------------------------------------
04780 C
04790         COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
```

```
04800       1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
04810       1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
04820       1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SO1,SO2,CS1,SN1,RD1,RD2,XJ1,Y
04830       1J1,XJ2,YJ2,BB1,BB2,HMAX,HO,HST,HOD,RRHO1,RRHO2,TMASS,YNERT,ZCG,HI,
04840       1THO,CDP,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,O
04850       1K,VVV(8),OEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
04860       1,NN,ND
04870         DO 1 I=1,NDF
04880         DO 1 J=1,NDF
04890      1  GM(I,J)=0.
04900         BK1=(BB1-BB2)*HST
04910         BK2=BB2*HO
04920         P0=BB1*HST+BB2*(HO-HST)
04930         P1=BK2*(0.5*HO-ZCG)+BK1*(0.5*HST-ZCG)
04940         P2=BK2*(HO*HO/3.-HO*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
04950         QQ2=DEXP(-HOD*OK)
04960         Q0=(BB1*(QQ2-OEXP)+BB2*(1.-QQ2))/OK
04970         Q1=(BB1*(QQ2*(HST-1./OK-ZCG)+OEXP*(1./OK+ZCG))+BB2*(HO-1./OK-
04980       1ZCG-QQ2*(HST-1./OK-ZCG)))/OK
04990 C*****COEFFICIENTS IN EQUATIONS OF BUOY MOTION(MASS MATRIX)***********
05000         GM(NN-1,NN-1)=TMASS+P0*(CON*CS1*CS1+COT*SN1*SN1)+(VVO+VV1)*SN1*SN
05010       11
05020         GM(NN-1,NN)=P0*SN1*CS1*(COT-CON)+(VVO+VV1)*SN1*CS1
05030         GM(NN-1,NN+1) = P1*CS1*CON
05040         GM(NN,NN-1)=GM(NN-1,NN)
05050         GM(NN,NN)=TMASS+P0*(CON*SN1*SN1+COT*CS1*CS1)+(VVO+VV1)*CS1*CS1
05060         GM(NN,NN+1)=-P1*SN1*CON
05070         GM(NN+1,NN-1)=GM(NN-1,NN+1)
05080         GM(NN+1,NN)=GM(NN,NN+1)
05090         GM(NN+1,NN+1) = YNERT+P2*CON
05100 C*****PART OF THE (FA) ARRAY*****************************************
05110         DD(NN-1)=(COT*P1*SN1-VVO*SN1*ZCG+VV1*SN1*(HST-ZCG))*YD(NN+1)*YD(NN
```

```
05120       1+1)
05130        DD(NN)=(COT*P1*CS1-VVO*CS1*ZCG+VV1*CS1*(HST-ZCG))*YD(NN+1)*YD(NN+1
05140       1)
05150        DD(NN+1)=0.
05160 C*****WAVE EXCITATION AND HYDROSTATICS FORCES ON THE SPAR***************
05170        XW=WE*WE*AMP*QO*(CSK*(1.+CON*CS1*CS1+COT*SN1*SN1)+SNK*SN1*CS1*(COT
05180       1-CON))
05190        ZW=WE*WE*AMP*QO*(SNK*(1.+CON*SN1*SN1+COT*CS1*CS1)+CSK*SN1*CS1*(COT
05200       1-CON))+GR*(BB1*HST+BB2*(HI-HST)-TMASS)
05210        EMW=WE*WE*AMP*Q1*(1.+CON)*(CSK*CS1-SNK*SN1)-GR*P1*SN1
05220        RETURN
05230        END
05240        IMPLICIT REAL*8(A-H,O-Z)
05250        SUBROUTINE NEXT
05260 C      THIS SUBROUTINE UPDATES THE GEOMETRY OF THE MOORING SYSTEM
05270 C      DURING ITS DYNAMIC MOTIONS.
05280 C      ---------------------------------------------------------------
05290 C
05300        COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
05310       1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
05320       1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
05330       1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SO1,SO2,CS1,SN1,RD1,RD2,XJ1,Y
05340       1J1,XJ2,YJ2,BB1,BB2,HMAX,HO,HST,HOD,RRHO1,RRHO2,TMASS,YNERT,ZCG,HI,
05350       1THO,CDP,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,O
05360       1K,VVV(8),OEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
05370       1,NN,ND
05380        IF(MOOR.EQ.2)GO TO 3
05390        XJ1=X(NN-1)+ZC1*SN1+XC1*CS1
05400        YJ1=X(NN)+ZC1*CS1-XC1*SN1
05410        IF(NB.EQ.1)GO TO 1
05420        TL(1)=DSQRT(X(1)**2+X(2)**2)
05430        CS(1)=X(1)/TL(1)
```

```
05440        SN(1)=X(2)/TL(1)
05450        TL(NB)=DSQRT((XJ1-X(NN-3))**2+(YJ1-X(NN-2))**2)
05460        CS(NB)=(XJ1-X(NN-3))/TL(NB)
05470        SN(NB)=(YJ1-X(NN-2))/TL(NB)
05480        GO TO 2
05490     1  TL(1)=DSQRT(XJ1**2+YJ1**2)
05500        CS(1)=XJ1/TL(1)
05510        SN(1)=YJ1/TL(1)
05520     2  IF(NM.EQ.NB)GO TO 4
05530     3  XJ2=X(NN-1)+ZC2*SN1+XC2*CS1
05540        YJ2=X(NN)+ZC2*CS1-XC2*SN1
05550        TL(NB2)=DSQRT((X(NN+2)-XJ2)**2+(X(NN+3)-YJ2)**2)
05560        CS(NB2)=(X(NN+2)-XJ2)/TL(NB2)
05570        SN(NB2)=(X(NN+3)-YJ2)/TL(NB2)
05580     4  DO 5 I=2,NM
05590        IF(I.EQ.NB.OR.I.EQ.NB2)GO TO 5
05600        II=I+I
05610        IF(I.LT.NB)II=II-1
05620        TL(I)=DSQRT((X(II)-X(II-2))**2+(X(II+1)-X(II-1))**2)
05630        CS(I)=(X(II)-X(II-2))/TL(I)
05640        SN(I)=(X(II+1)-X(II-1))/TL(I)
05650     5  CONTINUE
05660        DO 6 I=1,NM
05670     6  IF(SN(I).LT.-1.)SN(I)=-1.0
05680        RETURN
05690        END
05700        IMPLICIT REAL*8(A-H,O-Z)
05710        SUBROUTINE MINVER(N,C)
05720  C     THIS SUBROUTINE CALCULATES THE INVERSE OF MATRIX 'C' OF ORDER 'N'
05730  C     ----------------------------------------------------------------
05740  C
05750        DIMENSION C(17,17),H(17),G(17)
```

```
05760        NN=N-1
05770        C(1,1)=1.0/C(1,1)
05780        DO 6 M=1,NN
05790        K=M+1
05800        DO 1 I=1,M
05810        G(I)=0.
05820        DO 1 J=1,M
05830     1  G(I)=G(I)+C(I,J)*C(J,K)
05840        D=0.
05850        DO 2 I=1,M
05860     2  D=D+C(K,I)*G(I)
05870        E=C(K,K)-D
05880        C(K,K)=1.0/E
05890        DO 3 I=1,M
05900     3  C(I,K)=-G(I)*C(K,K)
05910        DO 4 J=1,M
05920        H(J)=0.
05930        DO 4 I=1,M
05940     4  H(J)=H(J)+C(K,I)*C(I,J)
05950        DO 5 J=1,M
05960     5  C(K,J)=-H(J)*C(K,K)
05970        DO 6 I=1,M
05980        DO 6 J=1,M
05990     6  C(I,J)=C(I,J)-G(I)*C(K,J)
06000        RETURN
06010        END
06020        IMPLICIT REAL*8(A-H,O-Z)
06030        SUBROUTINE MATRIX
06040 C      THIS SUBROUTINE CALCULATES ALL THE REMAINING ELEMENTS OF THE
06050 C      GLOBAL MATRICES AND INTEGRATES THE 'NDF' SIMULTANEOUS DIFFERENTIAL
06060 C      EQUATIONS OF MOTION BY THE METHOD OF RECTANGULAR RULE.
06070 C      ------------------------------------------------------------------
```

```
06080 C
06090       COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
06100      1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
06110      1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
06120      1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SC1,SC2,CS1,SN1,RD1,RD2,YJ1,
06130      1J1,XJ2,YJ2,BB1,BB2,HMAX,HO,HST,HOD,RRHO1,RRHO2,TMASS,YNERT,ZCG,HI
06140      1THO,CDP,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,C
06150      1K,VVV(8),DEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
06160      1,NN,ND
06170 C*****WAVE EXCITING + HYDROSTATIC + VISCOUS DRAG FORCES ON THE SPAR*****
06180       F(NN-1)=DC1+XW
06190       F(NN)=ZW+DC2
06200       F(NN+1)=DC3+EMW
06210       IF(MOOR.EQ.0)GO TO 11
06220 C*****STIFFNESS MATRIX FOR THE MOORING LINES*****************************
06230       AA1=XC1*CS1+ZC1*SN1
06240       AA2=XC2*CS1+ZC2*SN1
06250       AB1=-XC1*SN1+ZC1*CS1
06260       AB2=-XC2*SN1+ZC2*CS1
06270       GK(1,1)=EK(1)*CS(1)
06280       GK(1,2)=EK(1)*SN(1)
06290       GK(NB,NN+1)=EK(NB)*(AB1*CS(NB)-AA1*SN(NB))
06300       IF(NM.EQ.1)GO TO 5
06310       IF(NM.EQ.NB)GO TO 1
06320       GK(NB2,NN-1)=-EK(NB2)*CS(NB2)
06330       GK(NB2,NN)=-EK(NB2)*SN(NB2)
06340       GK(NB2,NN+1)=-EK(NB2)*(AB2*CS(NB2)-AA2*SN(NB2))
06350       GK(NB2,NN+2)=-GK(NB2,NN-1)
06360       GK(NB2,NN+3)=-GK(NB2,NN)
06370     1 DO 2 I=2,NM
06380       IF(I.EQ.NB2)GO TO 2
06390       II=I+I
```

```
      IF(I.GT.NB)II=II+1
      GK(I,II-3)=-EK(I)*CS(I)
      GK(I,II-2)=-EK(I)*SN(I)
      GK(I,II)=-GK(I,II-2)
      GK(I,II-1)=-GK(I,II-3)
    2 CONTINUE
C*****MASS MATRIX FOR THE MOORING LINES*********************************
      DO 4 I=1,NM
      IF(I.EQ.NB)GO TO 4
      II=I+I
      IF(I.LT.NB)II=II-1
      GM(II,II)=CM(I)+CMS(I)+CMN(I)*SN(I)**2+CMT(I)*CS(I)**2
      GM(II+1,II+1)=CM(I)+CMS(I)+CMN(I)*CS(I)**2+CMT(I)*SN(I)**2
      GM(II,II+1)=(CMT(I)-CMN(I))*SN(I)*CS(I)
      GM(II+1,II)=GM(II,II+1)
      WWE=WE*AMP*DEXP(AW*(X(II+1)-DEPTH))
      SWE=AW*X(II)-WE*T
      SNK1=DSIN(SWE)
      CSK1=DCOS(SWE)
C*****VISCOUS DRAG FORCES ON THE MOORING LINES**********************
      U1=V(I)-YD(II)-WWE*SNK1
      U2=-YD(II+1)+WWE*CSK1
      UT1=U1*CS(I+1)+U2*SN(I+1)
      UN1=U1*SN(I+1)-U2*CS(I+1)
      DN=CDIN(I)*UN1*DABS(UN1)
      DDTT=CDIT(I)*UT1*DABS(UT1)
      U3=DSQRT(U1**2+U2**2)*CDIA(I)
      DD(II)=DN*SN(I+1)+DDTT*CS(I+1)+U3*U1
      DD(II+1)=DDTT*SN(I+1)-DN*CS(I+1)+U3*U2
      IF(MOOR.EQ.2.AND.I.GE.ND)GO TO 3
      F(II)=CMD(I)*WE*WWE*CSK1
      F(II+1)=CMD(I)*WE*WWE*SNK1-W(I)
```

```
06720        GO TO 4
06730     3  F(II)=WE*WWE*CMD(I)*SN(I)*(CSK1*SN(I)-SNK1*CS(I))
06740        F(II+1)=WE*WWE*CMD(I)*CS(I)*(SNK1*CS(I)-CSK1*SN(I))-W(I)
06750     4  CONTINUE
06760     5  DO 7 K=1,NM
06770        TG=0.
06780        DO 6 IK=1,NDF
06790     6  TG=TG+GK(K,IK)*Y(IK)
06800        TNN(K)=TG+TNN(K)
06810        TN(K)=TNN(K)
06820        IF(TN(K).LE.0.)TN(K)=0.
06830        KK=K+K
06840        IF(K.LE.NB)KK=KK-1
06850        TC(KK)=TN(K)*CS(K)
06860     7  TC(KK+1)=TN(K)*SN(K)
06870        TC(NN+1)=-(TC(NN-1)*(-ZC1*CS1+XC1*SN1)+TC(NN)*(-XC1*CS1-ZC1*
06880       1SN1)+TC(NN+2)*(-ZC2*CS1+XC2*SN1)-TC(NN+3)*(-XC2*CS1-ZC2*SN1))
06890        IF(NM.EQ.1)GO TO 11
06900        IF(MOOR.EQ.2)GO TO 9
06910        NZ=NN-2
06920        DO 8 M=1,NZ
06930     8  TC(M)=TC(M)-TC(M+2)
06940     9  TC(NN-1)=TC(NN-1)-TC(NN+2)
06950        TC(NN)=TC(NN)-TC(NN+3)
06960        IF(NM.EQ.NB)GO TO 11
06970        NZ=NN+2
06980        DO 10 M=NZ,NDF
06990    10  TC(M)=TC(M)-TC(M+2)
07000    11  CALL MINVER(NDF,GM)
07010        DO 13 M=1,NDF
07020        YDD(M)=0.
07030        DO 12 K=1,NDF
```

```
07040    12  YDD(M)=YDD(M)+GM(M,K)*(DD(K)-TC(K)+F(K))
07050        YD(M)=YD(M)+YDD(M)*DT
07060        Y(M)=YD(M)*DT
07070    13  X(M)=X(M)+Y(M)
07080        CS1=DCOS(X(NN+1))
07090        SN1=DSIN(X(NN+1))
07100        HO=ZCG+(DEPTH-X(NN))/CS1
07110        HOD=HO-HST
07120        RETURN
07130        END
READY
```

```
 SSD31.FORT
00010 C
00020 C     PROGRAM TO COMPUTE LOW FREQ. MOTION OF SUBSURFACE MOORED SYSTEMS.
00030 C     THIS ANALYSIS IS DONE IN THREE DIMENSIONS, AND INERTIA FORCES ARE
00040 C     INCLUDED. ABSOLUTE VELOCITY PROFILE IS INPUTTED AS AMPLITUDE AND
00050 C     FREQUENCY. A SINUSOIDAL SURFACE WAVE IS ALSO INPUTTED.
00060 C     ------------------------------------------------------------------
00070 C
00080       IMPLICIT REAL*8(A-H,O-Z)
00090 C*****BEGINING OF THE MAIN PROGRAM*****************************************
00100       COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00110      1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
00120      1(20),V(3,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3),AWX
00130      1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
00140      1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
00150      130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
00160      1VS(3),S2,TE,H2,IK,NI,J,NC,WS,AMP,NS,NP,DDT,E,ER,PII,T,SINT,AW,AWY,
00170      1DIA,ER1,XOLD(3),GR,WE,DT,T2,TMAX,IKK,I
00180 C*****BEGINING OF FORMAT STATEMENTS***************************************
00190  101  FORMAT(4I5,5F10.0)
00200  102  FORMAT(7F8.0)
00210  103  FORMAT(5F10.0)
00220  104  FORMAT(10F7.0)
00230  105  FORMAT(7F10.0)
00240  106  FORMAT(16F7.2)
00250  107  FORMAT(10F8.2)
00260 C*****GENERAL DATA*******************************************************
00270 C     DEPTH, DDT, ER, AND ER1 ARE IN METERS.
00280       READ(5,101)NI,NP,NS,IKK,DEPTH,DDT,ER,ER1
00290 C*****READ IN THE INSERTED INSTRUMENTS AND FLOATS SPECIFICATIONS********
00300 C     PI AND SI ARE IN METERS; ZM AND ZMV IN SLUGS; F IN POUNDS; AND
00310 C     CDIN, CDIT ARE IN F.P.S. UNITS.
```

```
00320       READ(5,102)(PI(J),SI(J),ZM(J),ZMV(J),F(3,J),CDIN(J),CDIT(J)
00330      1,J=1,NI)
00340       DO 1 I1=1,2
00350       DO 1 I2=1,NI
00360     1 F(I1,I2)=0.
00370 C*****READ IN THE PARAMETERS FOR DIFFERENT MOORING PARTS*****************
00380 C     DIAL IS IN INCHES; SLL IN METERS; AWL AND WWL IN LBS/M; TPL IN
00390 C     POUNDS OR LBS/(SQ.IN.); CO1 IS IN INCHES FOR JACKETED WIRE ROPE
00400       READ(5,103)(DIAL(I),SLL(I),AWL(I),WWL(I),TPL(I),I=1,NP)
00410       READ(5,105)(CO1(I),PO1(I),CO2(I),PO2(I),AO(I),CDN(I),CDT(I)
00420      1,I=1,NP)
00430 C*****READ THE FORCING CURRENT PROFILE************************************
00440 C     D ARE IN METERS; V IN MM/SEC; AND WE IN RAD/SEC.
00450       READ(5,105)(D(I),I=1,IKK)
00460       READ(5,105)((V(I,J),I=1,3),J=1,8)
00470       READ(5,105)WE,DT,T2,TMAX
00480 C*****READ IN THE SINUSOIDAL SURFACE WAVE.*****************************
00490 C     WS IS IN RAD/SEC; AMP IN MM.
00500       READ(5,105)WS,AMP,BETA
00510       CSB=DCOS(BETA)
00520       SNB=DSIN(BETA)
00530       PII=4.*DATAN(1.D0)
00540       RHO=1.9905
00550       GR=32.2
00560       CONS1=RHO*GR*PII*0.0057
00570       AW=WS*WS/GR
00580       AWX=AW*CSB
00590       AWY=AW*SNB
00600       SLT=0.
00610       DO 2 I=1,NP
00620       IF(PO1(I).EQ.0.0)GO TO 2
00630 C*****PERMANENT STRETCH FOR SYNTHETIC ROPES.****************************
```

```
00640       PSTRA(I)=((TPL(I)/(DIAL(I)**2)-200.0)/CO1(I))**(1.0/PO1(I))
00650     2 SLT=SLT+SLL(I)
00660 C*****SEGMENTATION OF THE MOORING LINE IN NPTS SEGMENTS*****************
00670       DO 3 I=1,NP
00680       NM(I)=(SLL(I)/SLT)*NS+0.5
00690       IF(NM(I).EQ.0)NM(I)=1
00700       SL(I)=SLL(I)/NM(I)
00710       SWA(I)=AWL(I)*SL(I)
00720       SWW(I)=WWL(I)*SL(I)
00730       SWD(I)=SWA(I)-SWW(I)
00740       CDIA(I)=SL(I)*(DIAL(I)**2)
00750     3 CONS(I)=CDIA(I)*CONS1
00760       NS1=NS+NP
00770 C*****INITIALIZE DYNAMIC VARIABLES************************************
00780       DO 5 I1=1,3
00790       DO 4 I2=1,NS1
00800       YDDL(I2,I1)=0.
00810       DO 4 I3=1,2
00820       YDL(I3,I2,I1)=0.
00830     4 YL(I3,I2,I1)=0.
00840       DO 5 I4=1,30
00850       YDDI(I4,I1)=0.
00860       DO 5 I5=1,2
00870       YDI(I5,I4,I1)=0.
00880     5 YI(I5,I4,I1)=0.
00890       CALL MOTION
00900       CALL EXIT
00910       END
00920       IMPLICIT REAL*8(A-H,O-Z)
00930       SUBROUTINE MOTION
00940 C     THIS SUBROUTINE CALCULATES THE DYNAMIC EQUILIBRIUM OF THE MOORING
00950 C     LINE FOR TMAX SECONDS.
```

```
C     ------------------------------------------------------------------
C
      COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
     1L(20),CDN(20),CDT(20),TPL(20),CD1(20),CD2(20),PD1(20),PD2(20),CONS
     1(20),V(3,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3),AWX
     1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
     1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
     130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
     1VS(3),S2,TE,H2,IK,NI,J,NC,WS,AMP,NS,NP,DDT,E,ER,PII,T,SINT,AW,AWY,
     1DIA,ER1,XOLD(3),GR,WE,DT,T2,TMAX,IKK,I
 101  FORMAT(5F10.2)
 102  FORMAT(1X,' SOLUTION IS TERMINATING AS THE ERROR IN MOORING LINE V
     1ELOCITY AND ACCELERATION CALCULATIONS IS EXCESSIVE'/1X,' ER1 IS EX
     1CEEDED BY=',F5.3)
      T=0.
      T3=0.
      VTRY=GR*AMP/WS
C*****BEGINNING OF MOORING LINE CALCULATIONS**************************
   1  SINT=DSIN(WE*T)
      SINTW=DSIN(WS*T)
      VS(1)=-VTRY*AWX*SINTW
      VS(2)=-VTRY*AWY*SINTW
      VS(3)=VTRY*AW*DCOS(WS*T)
      INDEX=0
   2  INDEX=INDEX+1
      S2=0.0
      X(3)=DDT
      K=1
      X(1)=0.
      X(2)=0.
      J=1
      TE=0.0
```

```
01280       THETA(1)=PII/2.
01290       THETA(2)=THETA(1)
01300       THETA(3)=0.
01310       DO 13 I=1,NP
01320       NK=NM(I)+K-1
01330       DO 12 N=K,NK
01340 C*****CHECK FOR ANY INSERTED INSTRUMENTS OR FLOATS*******************
01350       IF(J.GT.NI)GO TO 4
01360    3  IF((S2+SL(I)*0.5).GE.PI(J))CALL FORCES
01370       IF(J.GT.NI)GO TO 4
01380       IF((S2+SL(I)*0.5).GE.PI(J))GO TO 3
01390 C*****STRESS-STRAIN CALCULATIONS FOR A MOORING SEGMENT*****************
01400    4  IF(PO1(I).EQ.0.0)GO TO 6
01410 C*****SYNTHETIC ROPES*************************************************
01420       STR=TE/(DIAL(I)**2)
01430       IF(STR.LE.200.0)GO TO 5
01440       TSTRA=PSTRA(I)+((STR-200.0)/CO2(I))**(1.0/PO2(I))
01450       S1=AO(I)*SL(I)*(1.0+TSTRA/100.0)
01460       GO TO 7
01470    5  S1=SL(I)
01480       GO TO 7
01490 C*****WIRE ROPES-TPL IS ELASTIC MODULUS AND CO1 IS JACKET DIA***********
01500    6  TSTRA=TE/(.7854*TPL(I)*(DIAL(I)-CO1(I))**2)
01510       S1=SL(I)*(1.+TSTRA)
01520 C*****BEGINING OF THE NUMERICAL INTEGRATION PROCEDURE******************
01530    7  DIA=DSQRT(CDIA(I)/S1)
01540       S2=S2+SL(I)
01550       DO 8 IC=1,3
01560       SN(IC)=DSIN(THETA(IC))
01570       CS(IC)=DCOS(THETA(IC))
01580       X(IC)=X(IC)+S1*CS(IC)
01590    8  YL(2,N,IC)=X(IC)
```

```
01600         H2=X(3)-S1*CS(3)/2.
01610         CALL 3PEED
01620         DO 9 IC=1,3
01630         U(IC)=U(IC)/304.8-YDL(2,N,IC)
01640         UT(IC)=U(IC)*CS(IC)
01650         YDDT(IC)=YDDL(N,IC)*CS(IC)
01660       9 WN(IC)=SWW(I)*CS(IC)*CS(3)
01670         WN(3)=WN(3)-SWW(I)
01680         UTT=UT(1)+UT(2)+UT(3)
01690         YDDTT=YDDT(1)+YDDT(2)+YDDT(3)
01700         DTT=3.28*CDT(I)*S1*DIA*DABS(UTT)*UTT/12.
01710         TE=TE-SWW(I)*CS(3)+DTT-SWA(I)*YDDTT/GR
01720         TN=TE-H2*SWD(I)/S1
01730         DO 10 IC=1,3
01740         YDDN(IC)=YDDL(N,IC)-YDDTT*CS(IC)
01750      10 UN(IC)=U(IC)-UTT*CS(IC)
01760         VNM=DSQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01770         DO 11 IC=1,3
01780         DN(IC)=3.28*CDN(I)*S1*DIA*VNM*UN(IC)/12.+WN(IC)-((2.*SWA(I)-SWW(I)
01790        1)/GR)*YDDN(IC)
01800         IF(SN(3).EQ.0.)GO TO 11
01810         THETA(IC)=THETA(IC)-DN(IC)/(TE*SN(IC))
01820      11 CONTINUE
01830      12 CONTINUE
01840         K=NK+1
01850      13 CONTINUE
01860 C*****CHECK FOR ANY INSERTED INSTRUMENTS OR FLOATS BEFORE THE ANCHOR***
01870      14 IF(J.GT.NI)GO TO 15
01880         CALL FORCES
01890         GO TO 14
01900      15 E=DEPTH-X(3)
01910         IF(T.EQ.0.)GO TO 21
```

```
01920       IF(INDEX.EQ.1)GO TO 17
01930       IF(T.EQ.DT)GO TO 21
01940       DO 16 IE=1,2
01950       TRY=DABS((X(IE)-XOLD(IE))/X(IE))
01960       IF(TRY.GT.ER1)GO TO 25
01970    16 CONTINUE
01980       GO TO 21
01990    17 DO 18 IC=1,3
02000    18 XOLD(IC)=X(IC)
02010       DO 20 IC=1,3
02020       DO 19 I1=1,N
02030       YDL(2,I1,IC)=(X(IC)-YL(2,I1,IC)-YL(1,I1,IC))*3.28/DT
02040    19 YDDL(I1,IC)=(YDL(2,I1,IC)-YDL(1,I1,IC))/DT
02050       DO 20 I2=1,NI
02060       YDI(2,I2,IC)=(X(IC)-YI(2,I2,IC)-YI(1,I2,IC))*3.28/DT
02070    20 YDDI(I2,IC)=(YDI(2,I2,IC)-YDI(1,I2,IC))/DT
02080       WRITE(6,101)T,(X(KJ),KJ=1,3),E
02090       GO TO 2
02100    21 DO 23 IC=1,3
02110       DO 22 I1=1,N
02120       YL(1,I1,IC)=X(IC)-YL(2,I1,IC)
02130    22 YDL(1,I1,IC)=YDL(2,I1,IC)
02140       DO 23 I2=1,NI
02150       YI(1,I2,IC)=X(IC)-YI(2,I2,IC)
02160    23 YDI(1,I2,IC)=YDI(2,I2,IC)
02170       IF(T.LT.T3)GO TO 24
02180       X(3)=X(3)-DDT
02190       WRITE(6,101)T,(X(KJ),KJ=1,3),E
02200       IF(T.GT.TMAX)GO TO 26
02210       T3=T3+T2
02220    24 T=T+DT
02230       GO TO 1
```

```
02240    25  EE1=TRY-ER1
02250        WRITE(6,102)EE1
02260    26  RETURN
02270        END
02280        IMPLICIT REAL*8(A-H,O-Z)
02290        SUBROUTINE FORCES
02300 C      THIS SUBROUTINE CALCULATES THE FORCE EQUILIBRIUM FOR INSERTED
02310 C      INSTRUMENTS OR FLOATS.
02320 C      ------------------------------------------------------------------
02330 C
02340        COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
02350       1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
02360       1(20),V(3,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3),AWX
02370       1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
02380       1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
02390       130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
02400       1VS(3),S2,TE,H2,IK,NI,J,NC,WS,AMP,NS,NP,DDT,E,ER,PII,T,SINT,AW,AWY,
02410       1DIA,ER1,XOLD(3),GR,WE,DT,T2,TMAX,IKK,I
02420 C*****NUMERICAL INTEGRATION PROCEDURE*************************************
02430        S2=S2+SI(J)
02440        DO 1 IC=1,3
02450        SN(IC)=DSIN(THETA(IC))
02460        CS(IC)=DCOS(THETA(IC))
02470        X(IC)=X(IC)+SI(J)*CS(IC)
02480     1  YI(2,J,IC)=X(IC)
02490        H2=X(3)-SI(J)*CS(3)/2.
02500        CALL SPEED
02510        DO 2 IC=1,3
02520        U(IC)=U(IC)/304.8-YDI(2,J,IC)
02530     2  CONTINUE
02540        IF(SI(J).NE.0.)GO TO 4
02550        VNM=DSQRT(U(1)**2+U(2)**2+U(3)**2)
```

```
02560        DO 3 IC=1,3
02570     3  DD(IC)=TE*CS(IC)+CDIN(J)*VNM*U(IC)+F(IC,J)-(ZM(J)+ZMV(J))
02580       1*YDDI(J,IC)
02590        GO TO 8
02600     4  DO 5 IC=1,3
02610        YDDT(IC)=YDDI(J,IC)*CS(IC)
02620     5  UT(IC)=U(IC)*CS(IC)
02630        UTT=UT(1)+UT(2)+UT(3)
02640        YDDTT=YDDT(1)+YDDT(2)+YDDT(3)
02650        DTT=CDIT(J)*DABS(UTT)*UTT
02660        DO 6 IC=1,3
02670        YDDN(IC)=YDDI(J,IC)-YDDTT*CS(IC)
02680     6  UN(IC)=U(IC)-UTT*CS(IC)
02690        VNM=DSQRT(UN(1)**2+UN(2)**2+UN(3)**2)
02700        DO 7 IC=1,3
02710        DD(IC)=(TE+DTT)*CS(IC)+CDIN(J)*VNM*UN(IC)+F(IC,J)
02720       1-ZM(J)*YDDI(J,IC)-ZMV(J)*YDDN(IC)
02730     7  CONTINUE
02740     8  TE=DSQRT(DD(1)**2+DD(2)**2+DD(3)**2)
02750        DO 9 IC=1,3
02760     9  THETA(IC)=DARCOS(DD(IC)/TE)
02770        J=J+1
02780        RETURN
02790        END
02800        IMPLICIT REAL*8(A-H,O-Z)
02810        SUBROUTINE SPEED
02820 C      THIS SUBROUTINE CALCULATES THE TOTAL (PROFILE+SURFACE WAVE VEL.)
02830 C      ABSOLUTE VELOCITY FIELD; GIVEN VELOCITY AT FEW DEPTHS AND A SIN.
02840 C      SURFACE WAVE. STEPPED PROFILE (#2 OF VOLUME 1 CHHABRA(1973)) IS
02850 C      USED.
02860 C      ------------------------------------------------------------------
02870 C
```

```
02880        COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
02890       1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
02900       1(20),V(3,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3),AWX
02910       1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
02920       1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
02930       130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
02940       1VS(3),S2,TE,H2,IK,NI,J,NC,WS,AMP,NS,NP,DDT,E,ER,PII,T,SINT,AW,AWY,
02950       1DIA,ER1,XOLD(3),GR,WE,DT,T2,TMAX,IKK,I
02960        OEXP=DEXP(-AW*H2)
02970        M=1
02980        DO 5 JN=1,3
02990        IF(H2.LE.D(1))GO TO 3
03000        IF(H2.GE.D(IKK))GO TO 4
03010     1  IF(H2.GT.D(M).AND.H2.LE.D(M+1))GO TO 2
03020        M=M+1
03030        GO TO 1
03040     2  U(JN)=V(JN,M+1)*(1.+0.05*SINT)+VS(JN)*OEXP
03050        GO TO 5
03060     3  U(JN)=V(JN,1)*(1.+0.05*SINT)+VS(JN)*OEXP
03070        GO TO 5
03080     4  U(JN)=V(JN,IKK+1)*(1.+0.05*SINT)+VS(JN)*OEXP
03090     5  CONTINUE
03100        RETURN
03110        END
READY
```

```
 SSS3.FORT
00010 C
00020 C      PROGRAM TO COMPUTE LOW FREQ. MOTION OF SUBSURFACE MOORED SYSTEMS.
00030 C      ANALYSIS IS DONE IN THREE DIMENSIONS. INERTIA FORCES ARE NEGLECTED
00040 C      IN THIS PROGRAM. INPUT VELOCITY PROFILE IS GIVEN RELATIVE TO THE
00050 C      MOORING LINE AT EACH TIME STEP.
00060 C      ------------------------------------------------------------------
00070 C
00080 C*****BEGINING OF THE MAIN PROGRAM*****************************************
00090        COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00100       1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
00110       1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
00120       1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
00130       1WWL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ
00140       1,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,IKK
00150 C*****BEGINING OF FORMAT STATEMENTS*****************************************
00160  101   FORMAT(5I5,5F10.0)
00170  102   FORMAT(7F8.0)
00180  103   FORMAT(5F10.0)
00190  104   FORMAT(10F7.0)
00200  105   FORMAT(7F10.0)
00210  106   FORMAT(16F7.2)
00220  107   FORMAT(10F8.2)
00230 C*****GENERAL DATA*********************************************************
00240 C      DEPTH,DDT,AND ER ARE IN METERS.
00250        READ(5,101)NI,NP,NS,NPT,IKK,DEPTH,DDT,ER
00260 C*****READ IN THE INSERTED INSTRUMENT OR FLOAT SPECIFICATIONS*********
00270 C      PI,SI ARE IN METERS; F IN POUNDS; AND CDIN,CDIT IN F.P.S. UNITS.
00280        READ(5,102)(PI(J),SI(J),F(1,J),F(2,J),F(3,J),CDIN(J),CDIT(J)
00290       1,J=1,NI)
00300 C*****READ IN THE PARAMETERS FOR DIFFERENT MOORING PARTS****************
00310 C      DIAL IN INCHES, SLL IN METERS, AWL AND WWL IN POUNDS/M; TPL IN
```

```
00320 C      POUNDS/(SQ.IN.) OR POUNDS.CO1 IS JACKET DIA.(IN.) FOR WIRE ROPE.
00330        READ(5,103)(DIAL(I),SLL(I),AWL(I),WWL(I),TPL(I),I=1,NP)
00340        READ(5,105)(CO1(I),PO1(I),CO2(I),PO2(I),AO(I),CDN(I),CDT(I),I=1,NP
00350       1)
00360 C*****READ THE FORCING CURRENT PROFILE************************************
00370 C      D ARE IN METERS, AND V IN MM/SEC.
00380        READ(5,105)(D(I),I=1,IKK)
00390        D(3)=0.
00400        READ(10,106)(((V(L,I,J),I=1,2),J=1,8),L=1,NPT)
00410        PII=4.*ATAN(1.)
00420        RHO=1.9905
00430        GR=32.2
00440        CONS1=RHO*GR*PII*0.0057
00450        SLT=0.
00460        DO 1 I=1,NP
00470        IF(PO1(I).EQ.0.0)GO TO 1
00480 C*****PERMANENT STRETCH FOR SYNTHETIC LINES*****************************
00490        PSTRA(I)=((TPL(I)/(DIAL(I)**2)-200.0)/CO1(I))**(1.0/PO1(I))
00500     1  SLT=SLT+SLL(I)
00510 C*****SEGMENTATION OF THE MOORING LINE IN NS SEGMENTS*******************
00520        DO 2 I=1,NP
00530        NM(I)=(SLL(I)/SLT)*NS+0.5
00540        IF(NM(I).EQ.0)NM(I)=1
00550        SL(I)=SLL(I)/NM(I)
00560        SWA(I)=AWL(I)*SL(I)
00570        SWW(I)=WWL(I)*SL(I)
00580        SWD(I)=SWA(I)-SWW(I)
00590        CDIA(I)=SL(I)*(DIAL(I)**2)
00600     2  CONS(I)=CDIA(I)*CONS1
00610        CALL MOTION
00620        CALL EXIT
00630        END
```

```
00640       SUBROUTINE MOTION
00650 C     THIS SUBROUTINE CALCULATES THE DYNAMIC EQUILIBRIUM OF THE MOORING
00660 C     LINE AT NPT INSTANTS OF TIMES.
00670 C     ------------------------------------------------------------------
00680
00690       COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00700      1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
00710      1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
00720      1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
00730      1WWL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ
00740      1,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,IKK
00750  101  FORMAT(3F8.2)
00760 C*****BEGINNING OF MOORING LINE CALCULATIONS*****************************
00770       DO 14 IK=1,NPT
00780       S2=0.0
00790       X(3)=DDT
00800       K=1
00810       X(1)=0.
00820       X(2)=0.
00830       J=1
00840       TE=0.0
00850       DIA=0.
00860       THETA(1)=PII/2.
00870       THETA(2)=THETA(1)
00880       THETA(3)=0.
00890       DO 11 I=1,NP
00900       NK=NM(I)+K-1
00910       DO 10 N=K,NK
00920       INDEX=1
00930 C*****CHECK FOR ANY INSERTED INSTRUMENT OR FLOAT**************************
00940       IF(J.GT.NI)GO TO 2
00950     1 IF((S2+SL(I)*0.5).GE.FI(J))CALL FORCES
```

```
00960       IF(J.GT.NI)GO TO 2
00970       IF((S2+SL(I)*0.5).GE.PI(J))GO TO 1
00980     2 CONTINUE
00990 C*****STRESS-STRAIN CALCULATIONS FOR MOORING SEGMENT*********************
01000       IF(PO1(I).EQ.0.0)GO TO 4
01010 C*****SYNTHETIC ROPES*************************************************
01020       STR=TE/(DIAL(I)**2)
01030       IF(STR.LE.200.0)GO TO 3
01040       TSTRA=PSTRA(I)+((STR-200.0)/CO2(I))**(1.0/PO2(I))
01050       S1=AO(I)*SL(I)*(1.0+TSTRA/100.0)
01060       GO TO 5
01070     3 S1=SL(I)
01080       GO TO 5
01090 C*****WIRE ROPES-TPL IS ELASTIC MODULUS AND CO1 IS JACKET DIA************
01100     4 TSTRA=TE/(.7854*TPL(I)*(DIAL(I)-CO1(I))**2)
01110       S1=SL(I)*(1.+TSTRA)
01120 C*****BEGINING OF THE NUMERICAL INTEGRATION PROCEDURE*******************
01130     5 DIA=SQRT(CDIA(I)/S1)
01140       S2=S2+SL(I)
01150       DO 6 IC=1,3
01160       SN(IC)=SIN(THETA(IC))
01170       CS(IC)=COS(THETA(IC))
01180     6 X(IC)=X(IC)+S1*CS(IC)
01190       IF(SN(3).EQ.0.)SN(3)=0.000001
01200       H2=X(3)-S1*CS(3)/2.
01210       CALL SPEED
01220       DO 7 IC=1,3
01230       UT(IC)=U(IC)*CS(IC)
01240     7 WN(IC)=SWW(I)*CS(IC)*CS(3)
01250       WN(3)=WN(3)-SWW(I)
01260       UTT=UT(1)+UT(2)+UT(3)
01270       DTT=(2.94E-6)*CDT(I)*S1*DIA*ABS(UTT)*UTT
```

```
01280        TE=TE-SWW(I)*CS(3)+DTT
01290        T=TE-H2*SWD(I)/S1
01300        DO 8 IC=1,3
01310     8  UN(IC)=U(IC)-UTT*CS(IC)
01320        VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01330        DO 9 IC=1,3
01340        DN(IC)=(2.94E-6)*CDN(I)*S1*DIA*VNM*UN(IC)+WN(IC)
01350     9  THETA(IC)=THETA(IC)-DN(IC)/(TE*SN(IC))
01360    10  CONTINUE
01370        K=NK+1
01380    11  CONTINUE
01390 C*****CHECK FOR ANY INSTRUMENT OR FLOAT BEFORE ANCHOR*****************
01400        INDEX=1
01410    12  IF(J.GT.NI)GO TO 13
01420        CALL FORCES
01430        GO TO 12
01440    13  E=DEPTH-X(3)
01450        X(3)=X(3)-DDT
01460        WRITE(6,101)(X(KJ),KJ=1,3)
01470    14  CONTINUE
01480        RETURN
01490        END
01500        SUBROUTINE FORCES
01510 C      THIS SUBROUTINE CALCULATES THE FORCE EQUILIBRIUM FOR INSERTED
01520 C      INSTRUMENTS OR FLOATS.
01530 C      ----------------------------------------------------------------
01540 C
01550        COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
01560       1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
01570       1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
01580       1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
01590       1WWL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ
```

```
01600       1,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,IKK
01610 C*****NUMERICAL INTEGRATION PROCEDURE**********************************
01620       S2=S2+SI(J)
01630       DO 1 IC=1,3
01640       SN(IC)=SIN(THETA(IC))
01650       CS(IC)=COS(THETA(IC))
01660     1 X(IC)=X(IC)+SI(J)*CS(IC)
01670       IF(SN(3).EQ.0.)SN(3)=0.000001
01680       H2=X(3)-SI(J)*CS(3)/2.
01690       CALL SPEED
01700       IF(SI(J).NE.0.)GO TO 3
01710       VNM=SQRT(U(1)**2+U(2)**2+U(3)**2)
01720       DO 2 IC=1,3
01730     2 DD(IC)=TE*CS(IC)+CDIN(J)*VNM*U(IC)/92903.+F(IC,J)
01740       GO TO 7
01750     3 DO 4 IC=1,3
01760     4 UT(IC)=U(IC)*CS(IC)
01770       UTT=UT(1)+UT(2)+UT(3)
01780       DTT=CDIT(J)*ABS(UTT)*UTT/92903.
01790       DO 5 IC=1,3
01800     5 UN(IC)=U(IC)-UTT*CS(IC)
01810       VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01820       DO 6 IC=1,3
01830     6 DD(IC)=(TE+DTT)*CS(IC)+CDIN(J)*VNM*UN(IC)/92903.+F(IC,J)
01840     7 TE=SQRT(DD(1)**2+DD(2)**2+DD(3)**2)
01850       DO 8 IC=1,3
01860     8 THETA(IC)=ARCOS(DD(IC)/TE)
01870       J=J+1
01880       INDEX=2
01890       RETURN
01900       END
01910       SUBROUTINE SPEED
```

```
01920 C     THIS SUBROUTINE CALCULATES THE VELOCITY PROFILE GIVEN THE VELOCITY
01930 C     AT A FEW DEPTHS. HERE THE STEPPED PROFILE (#2 IN VOLUME 1
01940 C     (CHHABRA,1973)), IS USED.
01950 C     ------------------------------------------------------------------
01960 C
01970       COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
01980      1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
01990      1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
02000      1,DN(3),WN(3),CS(3),SN(3),DT(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
02010      1WWL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ
02020      1,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,IKK
02030       M=1
02040       DO 5 JN=1,2
02050       IF(H2.LE.D(1))GO TO 3
02060       IF(H2.GE.D(IKK))GO TO 4
02070     1 IF(H2.GT.D(M).AND.H2.LE.D(M+1))GO TO 2
02080       M=M+1
02090       GO TO 1
02100     2 U(JN)=V(IK,JN,M+1)
02110       GO TO 5
02120     3 U(JN)=V(IK,JN,1)
02130       GO TO 5
02140     4 U(JN)=V(IK,JN,IKK+1)
02150     5 CONTINUE
02160       RETURN
02170       END
READY
```

```
 SSD3.FORT
00010 C
00020 C      PROGRAM TO COMPUTE LOW FREQ. MOTION OF SUBSURFACE MOORED SYSTEMS.
00030 C      THIS ANALYSIS IS DONE IN THREE DIMENSIONS, AND INCLUDES INERTIA
00040 C      FORCES. INPUT VELOCITY PROFILE RELATIVE TO THE MOORING SYSTEM IS
00050 C      READ AT EACH TIME STEP.
00060 C      ------------------------------------------------------------------
00070 C
00080 C*****BEGINING OF THE MAIN PROGRAM*****************************************
00090       COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00100      1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
00110      1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
00120      1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
00130      1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
00140      130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
00150      1TSTRA,I,S2,TE,H2,IK,NI,J,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,GR,IKK
00160 C*****BEGINING OF FORMAT STATEMENTS*************************************
00170  101   FORMAT(5I5,5F10.0)
00180  102   FORMAT(7F8.0)
00190  103   FORMAT(5F10.0)
00200  104   FORMAT(10F7.0)
00210  105   FORMAT(7F10.0)
00220  106   FORMAT(16F7.2)
00230  107   FORMAT(10F8.2)
00240 C*****GENERAL DATA*****************************************************
00250 C      DEPTH, DDT, AND ER ARE IN METERS.
00260       READ(5,101)NI,NP,NS,NPT,IKK,DEPTH,DDT,ER
00270 C*****READ IN THE INSERTED INSTRUMENT OR FLOAT SPECIFICATIONS*********
00280 C      PI, SI ARE IN METERS; ZM, ZMV IN SLUGS; F IN POUNDS; AND CDIN,
00290 C      CDIT IN F.P.S. UNITS.
00300       READ(5,102)(PI(J),SI(J),ZM(J),ZMV(J),F(3,J),CDIN(J),CDIT(J)
00310      1,J=1,NI)
```

```
00320       DO 1 I1=1,2
00330       DO 1 I2=1,NI
00340    1  F(I1,I2)=0.
00350 C*****READ IN THE PARAMETERS FOR DIFFERENT MOORING PARTS*****************
00360 C     DIAL IN INCHES, SLL IN METERS; AWL AND WWL IN LBS/M; AND TPL IN
00370 C     POUNDS OR POUNDS/(SQ.IN.). CO1 IS IN INCHES FOR JACKETED WIRE ROPE
00380       READ(5,103)(DIAL(I),SLL(I),AWL(I),WWL(I),TPL(I),I=1,NP)
00390       READ(5,105)(CO1(I),PO1(I),CO2(I),PO2(I),AO(I),CDN(I),CDT(I),I=1,NP
00400      1)
00410 C*****READ THE FORCING CURRENT PROFILE*************************************
00420 C     D ARE IN METERS; AND V ARE IN MM/SEC.
00430       READ(5,105)(D(I),I=1,IKK)
00440       U(3)=0.
00450       READ(10,106)(((V(L,I,J),I=1,2),J=1,8),L=1,NPT)
00460       PII=4.*ATAN(1.)
00470       RHO=1.9905
00480       GR=32.2
00490       CONS1=RHO*GR*PII*0.0057
00500       SLT=0.
00510       DO 2 I=1,NP
00520       IF(PO1(I).EQ.0.0)GO TO 2
00530 C*****PERMANENT STRETCH FOR SYNTHETIC LINES.*****************************
00540       PSTRA(I)=((TPL(I)/(DIAL(I)**2)-200.0)/CO1(I))**(1.0/PO1(I))
00550    2  SLT=SLT+SLL(I)
00560 C*****SEGMENTATION OF THE MOORING LINE IN NS SEGMENTS*******************
00570       DO 3 I=1,NP
00580       NM(I)=(SLL(I)/SLT)*NS+0.5
00590       IF(NM(I).EQ.0)NM(I)=1
00600       SL(I)=SLL(I)/NM(I)
00610       SWA(I)=AWL(I)*SL(I)
00620       SWW(I)=WWL(I)*SL(I)
00630       SWD(I)=SWA(I)-SWW(I)
```

```
00640        CDIA(I)=SL(I)*(DIAL(I)**2)
00650     3  CONS(I)=CDIA(I)*CONS1
00660        NS1=NS+NP
00670 C*****INITIALIZE ALL DYNAMIC VARIABLES.*******************************
00680        DO 5 I1=1,3
00690        DO 4 I2=1,NS1
00700        YDDL(I2,I1)=0.
00710        DO 4 I3=1,2
00720        YDL(I3,I2,I1)=0.
00730     4  YL(I3,I2,I1)=0.
00740        DO 5 I4=1,30
00750        YDDI(I4,I1)=0.
00760        DO 5 I5=1,2
00770        YDI(I5,I4,I1)=0.
00780     5  YI(I5,I4,I1)=0.
00790        CALL MOTION
00800        CALL EXIT
00810        END
00820        SUBROUTINE MOTION
00830 C      THIS SUBROUTINE CALCULATES THE DYNAMIC EQUILIBRIUM OF THE MOORING
00840 C      LINE AT NPT INSTANTS OF TIMES.
00850 C      ------------------------------------------------------------------
00860 C
00870        COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00880       1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
00890       1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
00900       1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),FSTRA(20),AO(20),D(10),
00910       1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
00920       130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
00930       1TSTRA,I,S2,TE,H2,IK,NI,J,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,GR,IKK
00940  101  FORMAT(3F8.2)
00950        DO 16 IK=1,NPT
```

```
C*****BEGINNING OF MOORING LINE CALCULATIONS*****************************
      S2=0.0
      X(3)=DDT
      K=1
      X(1)=0.
      X(2)=0.
      J=1
      TE=0.0
      THETA(1)=PII/2.
      THETA(2)=THETA(1)
      THETA(3)=0.
      DO 11 I=1,NP
      NK=NM(I)+K-1
      DO 10 N=K,NK
C*****CHECK FOR ANY INSERTED INSTRUMENT OR FLOAT ************************
      IF(J.GT.NI)GO TO 2
    1 IF((S2+SL(I)*0.5).GE.PI(J))CALL FORCES
      IF(J.GT.NI)GO TO 2
      IF((S2+SL(I)*0.5).GE.PI(J))GO TO 1
C*****STRESS-STRAIN CALCULATIONS FOR MOORING SEGMENT*********************
    2 IF(PO1(I).EQ.0.0)GO TO 4
C*****SYNTHETIC ROPES*****************************************************
      STR=TE/(DIAL(I)**2)
      IF(STR.LE.200.0)GO TO 3
      TSTRA=PSTRA(I)+((STR-200.0)/CO2(I))**(1.0/PO2(I))
      S1=AO(I)*SL(I)*(1.0+TSTRA/100.0)
      GO TO 5
    3 S1=SL(I)
      GO TO 5
C*****WIRE ROPES-TPL IS ELASTIC MODULUS AND CO1 IS JACKET DIA************
    4 TSTRA=TE/(.7854*TPL(I)*(DIAL(I)-CO1(I))**2)
      S1=SL(I)*(1.+TSTRA)
```

```
01280 C*****BEGINING OF THE NUMERICAL INTEGRATION PROCEDURE*******************
01290     5  DIA=SQRT(CDIA(I)/S1)
01300        S2=S2+SL(I)
01310        DO 6 IC=1,3
01320        SN(IC)=SIN(THETA(IC))
01330        CS(IC)=COS(THETA(IC))
01340        X(IC)=X(IC)+S1*CS(IC)
01350     6  YDL(1,N,IC)=X(IC)
01360        IF(SN(3).EQ.0.)SN(3)=0.000001
01370        H2=X(3)-S1*CS(3)/2.
01380        CALL SPEED
01390        DO 7 IC=1,3
01400        UT(IC)=U(IC)*CS(IC)
01410        YDDT(IC)=YDDL(N,IC)*CS(IC)
01420     7  WN(IC)=SWW(I)*CS(IC)*CS(3)
01430        WN(3)=WN(3)-SWW(I)
01440        UTT=UT(1)+UT(2)+UT(3)
01450        YDDTT=YDDT(1)+YDDT(2)+YDDT(3)
01460        DTT=(2.94E-6)*CDT(I)*S1*DIA*ABS(UTT)*UTT
01470        TE=TE-SWW(I)*CS(3)+DTT-SWA(I)*YDDTT/GR
01480        T=TE-H2*SWD(I)/S1
01490        DO 8 IC=1,3
01500        YDDN(IC)=YDDL(N,IC)-YDDTT*CS(IC)
01510     8  UN(IC)=U(IC)-UTT*CS(IC)
01520        VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01530        DO 9 IC=1,3
01540        DN(IC)=(2.94E-6)*CDN(I)*S1*DIA*VNM*UN(IC)+WN(IC)-((2.*SWA(I)-SWW(I
01550       1))/GR)*YDDN(IC)
01560     9  THETA(IC)=THETA(IC)-DN(IC)/(TE*SN(IC))
01570    10  CONTINUE
01580        K=NK+1
01590    11  CONTINUE
```

```
01600 C*****CHECK FOR ANY INSERTED INSTRUMENT OR FLOAT BEFORE ANCHOR********
01610   12  IF(J.GT.NI)GO TO 13
01620       CALL FORCES
01630       GO TO 12
01640   13  E=DEPTH-X(3)
01650       X(3)=X(3)-DDT
01660       WRITE(6,101)(X(KJ),KJ=1,3)
01670 C*****CALCULATIONS FOR VELOCITIES AND ACCELERATIONS OF THE MOORING.
01680       DO 15 IC=1,3
01690       DO 14 I1=1,N
01700       YL(1,I1,IC)=YL(2,I1,IC)
01710       YL(2,I1,IC)=X(IC)-YDL(1,I1,IC)
01720       YDL(1,I1,IC)=YDL(2,I1,IC)
01730       YDL(2,I1,IC)=(YL(2,I1,IC)-YL(1,I1,IC))*0.003645
01740       YDDL(I1,IC)=(YDL(2,I1,IC)-YDL(1,I1,IC))/900.
01750   14  IF(IK.LE.2)YDDL(I1,IC)=0.
01760       DO 15 I2=1,NI
01770       YI(1,I2,IC)=YI(2,I2,IC)
01780       YI(2,I2,IC)=X(IC)-YDI(1,I2,IC)
01790       YDI(1,I2,IC)=YDI(2,I2,IC)
01800       YDI(2,I2,IC)=(YI(2,I2,IC)-YI(1,I2,IC))*0.003645
01810       YDDI(I2,IC)=(YDI(2,I2,IC)-YDI(1,I2,IC))/900.
01820   15  IF(IK.LE.2)YDDI(I2,IC)=0.
01830   16  CONTINUE
01840       RETURN
01850       END
01860       SUBROUTINE FORCES
01870 C     THIS SUBROUTINE CALCULATES THE FORCE EQUILIBRIUM FOR INSERTED
01880 C     INSTRUMENTS OR FLOATS.
01890 C     ----------------------------------------------------------------
01900 C
01910       COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
```

```
01920      1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
01930      1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
01940      1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
01950      1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
01960      130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
01970      1TSTRA,I,S2,TE,H2,IK,NI,J,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,GR,IKK
01980 C*****NUMERICAL INTEGRATION PROCEDURE**********************************
01990       S2=S2+SI(J)
02000       DO 1 IC=1,3
02010       SN(IC)=SIN(THETA(IC))
02020       CS(IC)=COS(THETA(IC))
02030       X(IC)=X(IC)+SI(J)*CS(IC)
02040     1 YDI(1,J,IC)=X(IC)
02050       IF(SN(3).EQ.0.)SN(3)=0.000001
02060       H2=X(3)-SI(J)*CS(3)/2.
02070       CALL SPEED
02080       IF(SI(J).NE.0.)GO TO 3
02090       VNM=SQRT(U(1)**2+U(2)**2+U(3)**2)
02100       DO 2 IC=1,3
02110     2 DD(IC)=TE*CS(IC)+CDIN(J)*VNM*U(IC)/92903.+F(IC,J)-(ZM(J)+ZMV(J))*Y
02120      1DDI(J,IC)
02130       GO TO 7
02140     3 DO 4 IC=1,3
02150       YDDT(IC)=YDDI(J,IC)*CS(IC)
02160     4 UT(IC)=U(IC)*CS(IC)
02170       UTT=UT(1)+UT(2)+UT(3)
02180       YDDTT=YDDT(1)+YDDT(2)+YDDT(3)
02190       DTT=CDIT(J)*ABS(UTT)*UTT/92903.
02200       DO 5 IC=1,3
02210       YDDN(IC)=YDDI(J,IC)-YDDTT*CS(IC)
02220     5 UN(IC)=U(IC)-UTT*CS(IC)
02230       VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
```

```
02240        DO 6 IC=1,3
02250     6  DD(IC)=(TE+DTT)*CS(IC)+CDIN(J)*VNM*UN(IC)/92903.+F(IC,J)
02260       1-ZM(J)*YDDI(J,IC)-ZMV(J)*YDDN(IC)
02270     7  TE=SQRT(DD(1)**2+DD(2)**2+DD(3)**2)
02280        DO 8 IC=1,3
02290     8  THETA(IC)=ARCOS(DD(IC)/TE)
02300        J=J+1
02310        RETURN
02320        END
02330        SUBROUTINE SPEED
02340 C      THIS SUBROUTINE CALCULATES THE VELOCITY PROFILE, GIVEN THE
02350 C      VELOCITY AT A FEW DEPTHS. HERE THE STEPPED PROFILE (#2 IN
02360 C      VOLUME 1 CHHABRA(1973)) IS USED.
02370 C      ------------------------------------------------------------
02380 C
02390        COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
02400       1L(20),CDN(20),CDT(20),TPL(20),CD1(20),CD2(20),PD1(20),PD2(20),CONS
02410       1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
02420       1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
02430       1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
02440       130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
02450       1TSTRA,I,S2,TE,H2,IK,NI,J,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,GR,IKK
02460        M=1
02470        DO 5 JN=1,2
02480        IF(H2.LE.D(1))GO TO 3
02490        IF(H2.GE.D(IKK))GO TO 4
02500     1  IF(H2.GT.D(M).AND.H2.LE.D(M+1))GO TO 2
02510        M=M+1
02520        GO TO 1
02530     2  U(JN)=V(IK,JN,M+1)
02540        GO TO 5
02550     3  U(JN)=V(IK,JN,1)
```

```
02560          GO TO 5
02570      4   U(JN)=V(IK,JN,IKK+1)
02580      5   CONTINUE
02590          RETURN
02600          END
READY
```

REFERENCES

CHHABRA, N. K. (1973) Mooring Mechanics a Comprehensive Computer Study, Vol. 1, C. S. Draper Laboratory, Inc., Report R-775.

CHHABRA, N. K., J. M. DAHLEN and M. R. FROIDEVAUX (1974) Mooring Dynamics Experiment-Determination of a Verified Dynamic Model of the WHOI Intermediate Mooring, C. S. Draper Laboratory, Inc., Report R-823.

CHHABRA, N. K. (1976) Correction of Vector Averaging Current Meter Records from the MODE-1 Central Mooring for the Effects of Low-Frequency Mooring Line Motion, Accepted Deep-Sea Research.

CHUNG, J. S. (1976) Motion of a Floating Structure in Water of Uniform Depth, Journal of Hydronautics, Vol. 10, 65-73.

GOODMAN, T. R., and J. P. BRESLIN (1976) Statics and Dynamics of Anchoring Cables in Waves, Journal of Hydronautics, Vol. 10, 113-120.

LAMB, H. (1945) Hydrodynamics, Sixth edition, Dover Publications, New York, N.Y., Article 68.

NEWMAN, J. N. (1963) The Motions of a Spar Buoy in Regular Waves, David Taylor Model Basin, Washington, D.C., Report 1499.

RUDNICK, P. (1967) Motion of a Large Spar Buoy in Sea Waves, Journal of Ship Research, December 1967, 257-267.

VACHON, W. A. (1973) Scale Model Testing of Drogues for Free Drifting Buoys, C. S. Draper Laboratory, Inc., Report R-769.

VACHON, W. A. (1975) Instrumented Full-Scale Tests of a Drifting Buoy and Drogue, C.S. Draper Lab., Inc., Report R-947.

WUNSCH, C. and J. M. DAHLEN (1974) A Moored Temperature and Pressure Recorder, Deep-Sea Research, 21, 145-154.

MANDATORY DISTRIBUTION LIST

FOR UNCLASSIFIED TECHNICAL REPORTS, REPRINTS, & FINAL REPORTS PUBLISHED BY OCEANOGRAPHIC CONTRACTORS OF THE OCEAN SCIENCE AND TECHNOLOGY DIVISION OF THE OFFICE OF NAVAL RESEARCH

1 Director of Defense Research
and Engineering
Office of the Secretary of Defense
Washington, DC 20301
ATTN: Office, Assistant Director
(Research)

Office of Naval Research
Arlington, Virginia 22217
1 ATTN: (Code 460)
1 ATTN: (Code 102-OS)
1 ATTN: (Code 200)
6 ATTN: (Code 102IP)

1 ONR Resident Representative
M.I.T. Room E19-628

National Space Technology Laboratories
Bay St. Louis, Miss. 39520
3 ATTN: (Code 400)*

Director
Naval Research Laboratory
Washington, DC 20375
6 ATTN: Library, Code 2620

12** Defense Documentation Center
Cameron Station
Alexandria, VA 22314

Commander
Naval Oceanographic Office
Washington, DC 20390
1 ATTN: Code 1640
1 ATTN: Code 70

1 NODC/NOAA
Rockville, MD 20882

Total Required - 35 Copies

*Add one separate copy of Form DD-1473

**Send with these 12 copies 2 completed forms DDC-50, one self-addressed back to contractor, the other addressed to NSTL, Code 400.

DISTRIBUTION

C.S. DRAPER LABORATORY, INC.

10	Library	1	Pruszenski, Anthony S.
1	Araujo, Richard K.	1	Reid, Robert W.
1	Bowditch, Philip N.	1	Scholten, James
35	Chhabra, Narender K.	1	Shepard, G. Dudley
1	Cummings, Damon E.	1	Shillingford, John
6	Dahlen, John M.	1	Toth, William E.
1	Lozow, Jeffery	1	Vachon, William
1	Morey, Ronald L.		

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1	Abkowitz, Martin A.	
1	Milgram, J. H.	Dept. of Ocean Engineering
1	Newman, J. N.	
1	Wunsch, Carl I.	Dept. of Earth & Planetary Sciences
1	Heinmiller, Robert	
1	Mollo-Christensen, Erik L.	Dept. of Meteorology
1	Stommel, Henry M.	

EXTERNAL

1 Christensen, Thomas, Code 450
1 Gregory, John B., Code 450
1 Lange, Edward, Code 410
1 Wilson, Stanley, Code 410
NORDA,
National Space Technology Laboratories
Bay St. Louis, Miss. 39520

1 Director
1 Canada, Raymond
1 DeBok, Donald
1 Kerut, Edmund G.
NOAA Data Buoy Office
National Space Technology Laboratories
Bay St. Louis, Miss. 39520

1 Brooks, David
Sperry Support Services
National Space Technology Laboratories
Bay St. Louis, Miss. 39520

DISTRIBUTION (Cont.)

1 Silva, Eugene A.
Ocean Engr. & Constr. Project Office
Navy Facilities Engineering Command
Chesapeake Division
Washington Navy Yard, Bldg. 200
Washington, DC 20374

1 Beardsley, Robert
1 Berteaux, Henri
1 Briscoe, Melbourne A.
1 Eriksen, Charles C.
1 Fofonoff, Nicolas P.
1 McCullough, James
1 Moller, Donald
1 Saunders, Peter
1 Schmitz, William
1 Walden, Robert
1 Webster, Ferris
Woods Hole Oceanographic Institution
Woods Hole, Mass. 02543

1 Collins, Curtis
1 Jennings, Feenan D.
IDOE Office
National Science Foundation
Washington, DC 20550

1 Swensen, Richard
USN Undersea Center
Code 2212
New London, CT 06320

1 Baker, D. James
School of Oceanography
University of Washington
Seattle, WA 98195

1 Halpern, David
Pacific Marine Environmental Laboratory/NOAA
Seattle, WA 98195

DISTRIBUTION (Cont.)

1 Wenstrand, David
John Hopkins University
Applied Physics Laboratory
John Hopkins Road
Laurel, MD 20810

1 Mesecar, Rod
1 Nath, John H.
1 Niiler, Peter
1 Smith, R. L.
School of Oceanography
Oregon State University
Corvallis, OR 97331

1 Savage, Godfrey H.
Professor and Director
Engineering Design & Analysis Lab
University of New Hampshire
Durham, NH 03824

1 Kirwan, A. D., Jr.
1 Nowlin, Worth D., Jr.
1 Cochrane, J.
Texas A&M University
College of Geosciences
College Station, TX 77843

1 Bonde, Les
Washington Analytical Services Center,Inc.
Hydrospace-Challenger Group
2150 Fields Road
Rockville, MD 20850

1 Bernstein, Robert L.
1 Cox, Charles
1 Davis, Russ
1 Sessions, Meredith
Scripps Institution of Oceanography
University of California, San Diego
La Jolla, CA 92037

DISTRIBUTION (Cont.)

1 Albertsen, Norman D.
Code L44
U.S. Civil Engineering Lab
Port Hueneme, CA 93093

1 Kalvaitis, A.
NOAA/OMT
Test and Evaluation Lab
Rockville, MD 20852

1 Harvey, Robert
Department of Oceanography
University of Hawaii
Hawaii

1 McGorman, Robert E.
Bedford Institute, Room 602
Dartmouth, Nova Scotia
CANADA

1 Bourgault, Thomas
NUSC,
Newport, RI 02840

1 Earle, Marshall
USN Oceanographic Office
Code 6110
Washington, DC 20373

1 Bedard, Philip P.
Oceanography Lab
Nova University
8000 North Ocean Drive
Dania, FL 33004

1 Coudeville, J. M.
Centre Oceanologique de Bretagne
Boite Postale 337
29.273 Brest CEDEX
FRANCE

DISTRIBUTION (Cont.)

1 Froidevaux, Michel
Aerospatiale
Subdivision Systemes, SYX/E
78130, Les Mureaux
FRANCE

1 J. E. Bowker Associates, Inc.
Statler Office Building
Boston, Mass. 02116

1 Evans-Hamilton, Inc.
8100 Kirkwood, Suite 130
Houston, TX 77072

1 Leedham, Clive D.
General Motors Corporation
6767 Hollister Avenue
Goleta, CA 93017